Particle motion in a bed under a rigid plate, submerged and oscillated over its surface, and bed, morphologies induced by flexible plates

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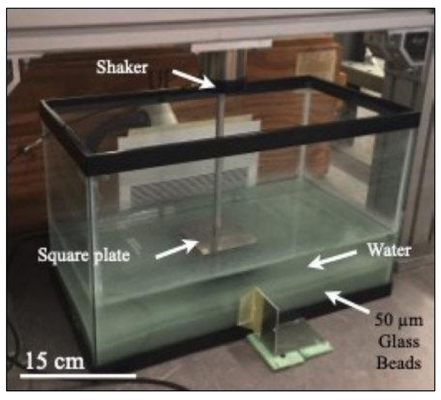
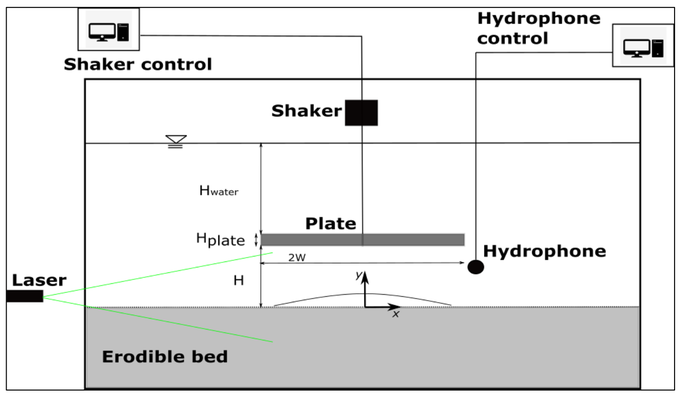
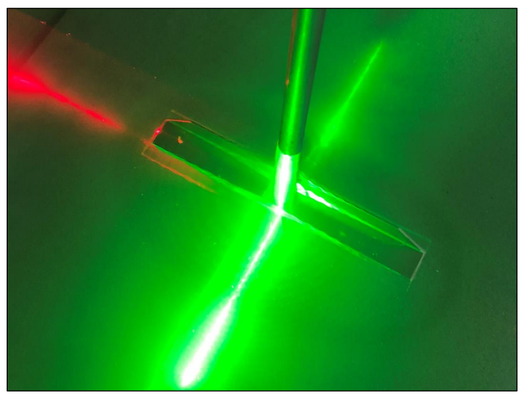
The problem

We study the behaviour of a particle bed immersed in water when a plate is oscillating above it.

- square, rigid plate (1a);
- rectangular, rigid plate (1b);
- rectangular, flexible plate (2).





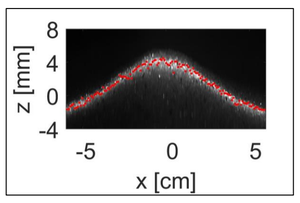


Experimental data

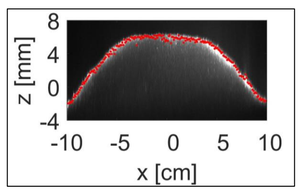
Measurements of the evolution of bed morphology, of the water pressure at the bed, and of the plate deformation.

1) Rigid plates

1a) Square plate — 30 Hz

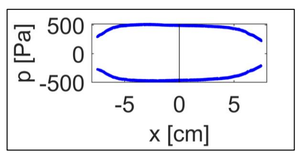


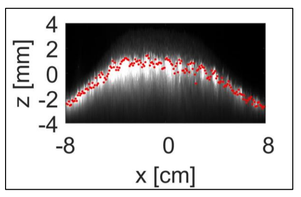
1b) Rectangular plate — 30 Hz



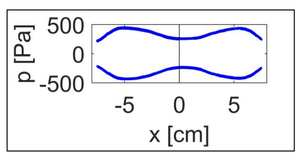
2) Flexible plate

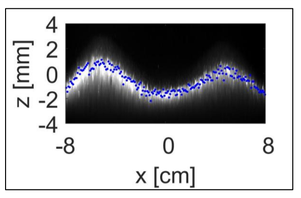
Water pressure at the bed | Final bed morphology



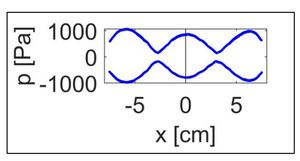


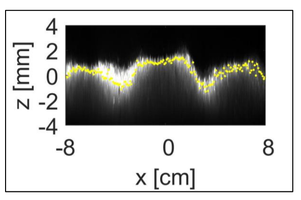
30 Hz





60 Hz





90 Hz

Theoretical model

The goal of the model: prediction of the heap's height in the regime when local avalanches do not occur.

Darcy's law (pressure p – velocity $\widetilde{U}, \tilde{V}$)

Characteristic time τ

$$\tau = \frac{H\nu_f}{32\,\kappa\, Af^2}$$

Depth-averaged particle mass balance

$$\int_{-1}^{\tilde{h}(\tilde{x},\tilde{y},\tilde{t})} \left[\frac{W}{Af\tau}\frac{\partial c}{\partial \tilde{t}} + \frac{\partial (c\widetilde{U})}{\partial \tilde{x}} + \frac{\partial (c\tilde{V})}{\partial \tilde{y}}\right] d\tilde{y} = 0$$

Particle mass balance (function of $\tilde{h}$)

$$\frac{\partial \tilde{h}}{\partial \tilde{t}} - \frac{2}{\pi} e^{\left(\frac{\pi\tilde{h}}{2}\right)} \sin\left(\frac{\pi}{2}\tilde{x}\right)\frac{\partial \tilde{h}}{\partial \tilde{x}} = e^{\left(\frac{\pi\tilde{h}}{2}\right)} \cos\left(\frac{\pi}{2}\tilde{x}\right)$$

Solution of the model: heap height in time

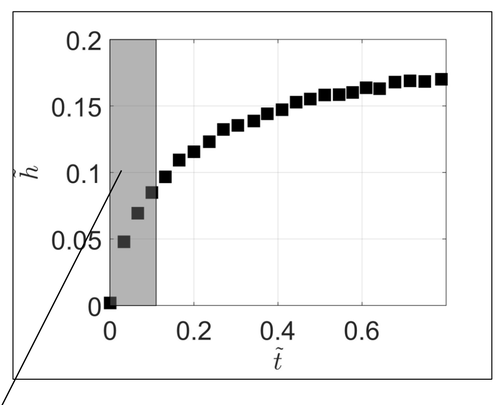
$$\tilde{h} = \ln\left[\frac{2e^{\tilde{t}}}{1 + e^{2\tilde{t}} + \left(1 - e^{2\tilde{t}}\right)\cos\left(\frac{\pi}{2}\,\tilde{x}\right)}\right]$$

Nomenclature. A: plate's oscillation amplitude; f: plate's oscillation frequency; W: half of the plate's width; $\tilde{h}$: dimensionless heap's height; $t = \widetilde{t}/\tau$: dimensionless time; ν_f: fluid kinematic viscosity; ρ_f: fluid density; κ: bed permeability; p: water pressure; c: bed concentration, $\widetilde{U}, \tilde{V}$: horiz. and vert. dimensionless particle velocity; H: distance between bed and plate.

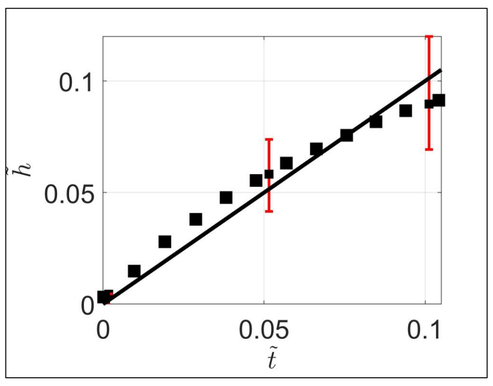
Conclusions

- Rigid plates → single heap
- Flexible plate → multiple heaps (→ 1° and 2° mode of vibration)

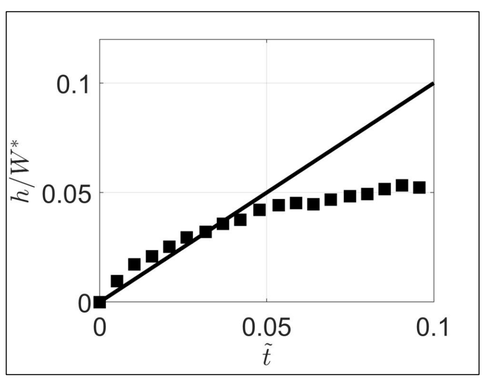
Comparison between the experimental data before the onset of local avalanches that is, without material loss.

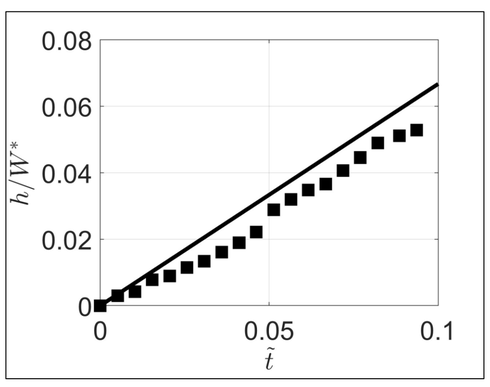


Area of interest for the comparison with the theoretical model



Rigid plates





Flexible plate

Legend:
— theoretical model
■ experimental data