Frictional hyperspheres in hyperspace

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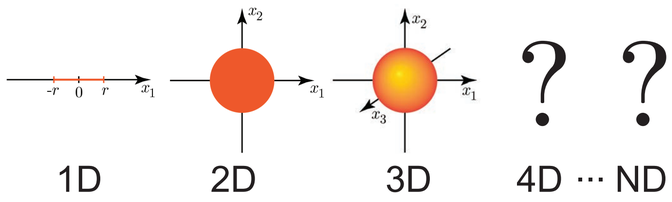
1. Introduction

Discrete element simulations of frictional grains in arbitrary number of spatial dimensions N



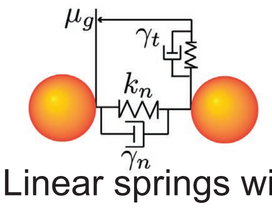
Consider granular problems in higher dimensions:

- Packing
- Flows
- Segregation
- ...

Provides insights into lower dimensions.

2. Simulation method and visualisation

Contacts

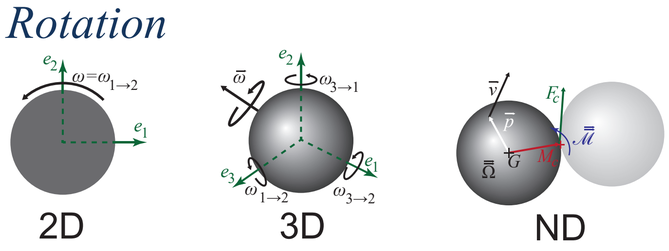


Linear springs with friction & dissipation

Translation

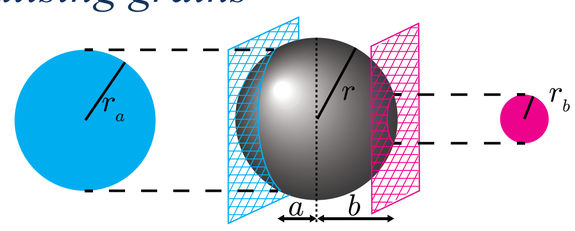
Trivial generalisation to higher dimensions

Rotation



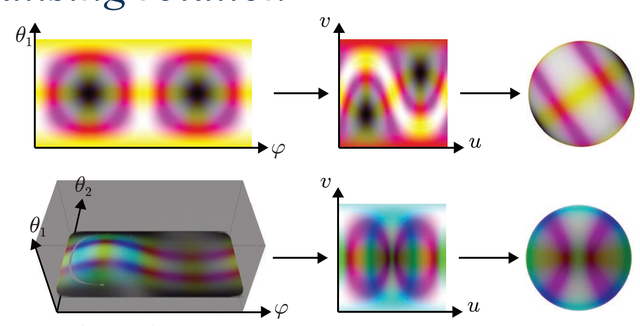
$N\times(N-1)/2$ degrees of freedom of rotation
Generalisation to ND using rotation matrices

Visualising grains



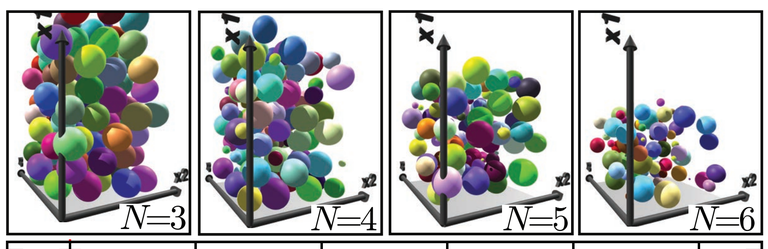
Two or three dimensional slice through the higher dimensional space

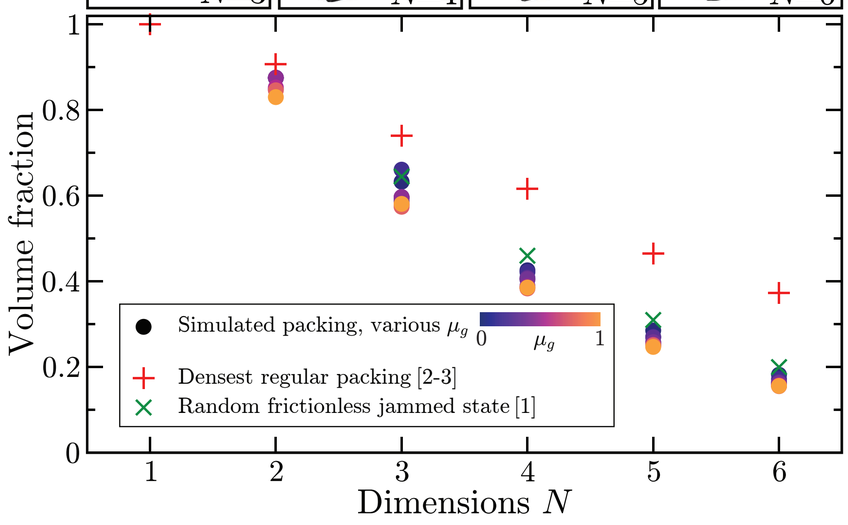
Visualising rotation

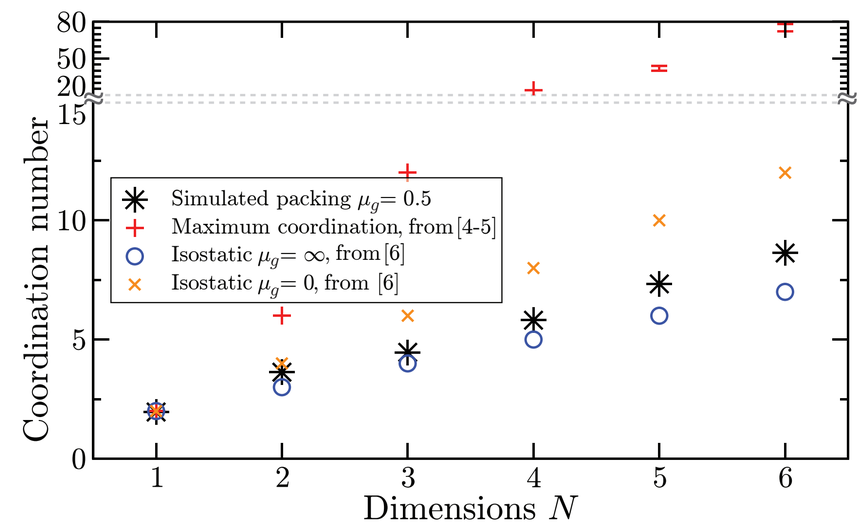


Mapping $(N-1)$ dimensional hypersphere surface to a color palette.

3. Packing



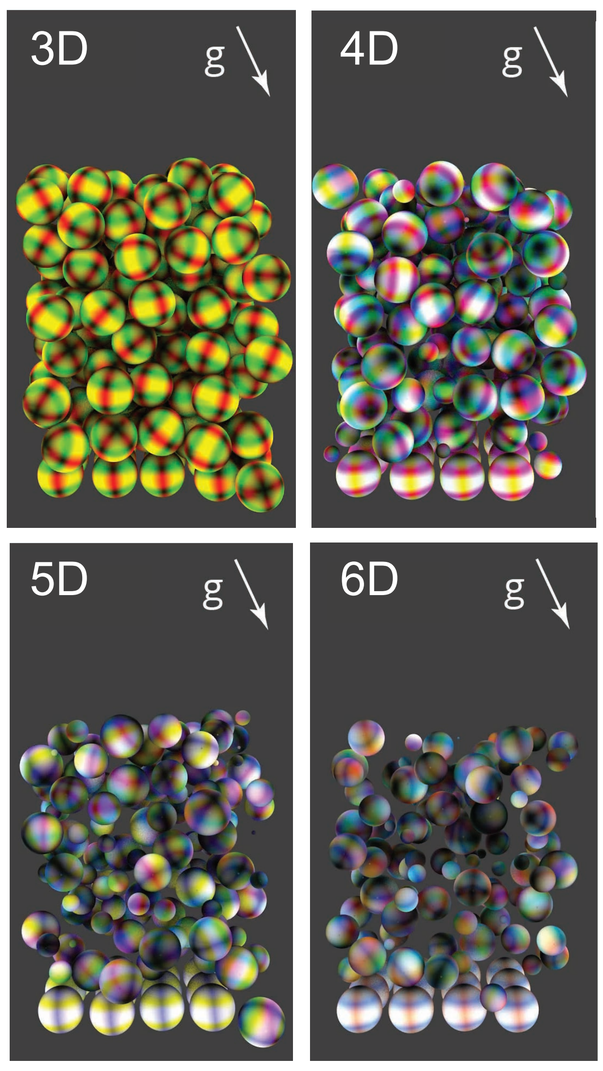




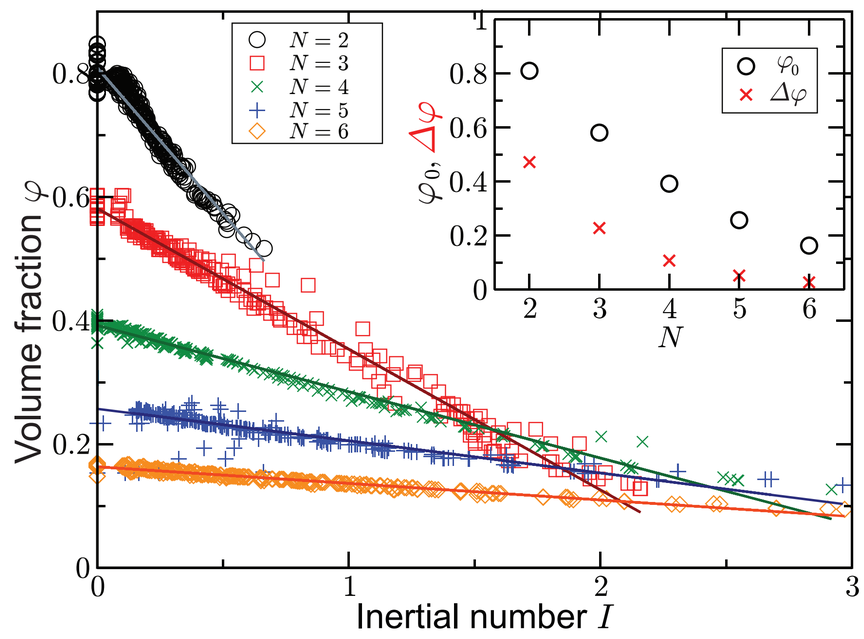
- Random close packing fraction decreases with N
- Small effect of interparticle friction
- Coordination number departing rapidly from theoretical maximum

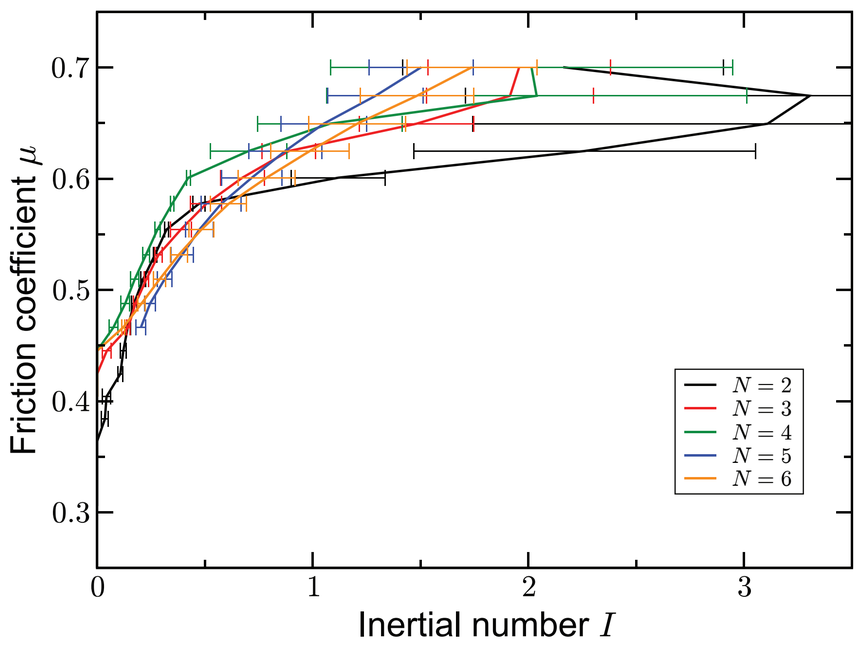
4. Flow

Inclined plane flow of mono-disperse hyperspheres.



Movies

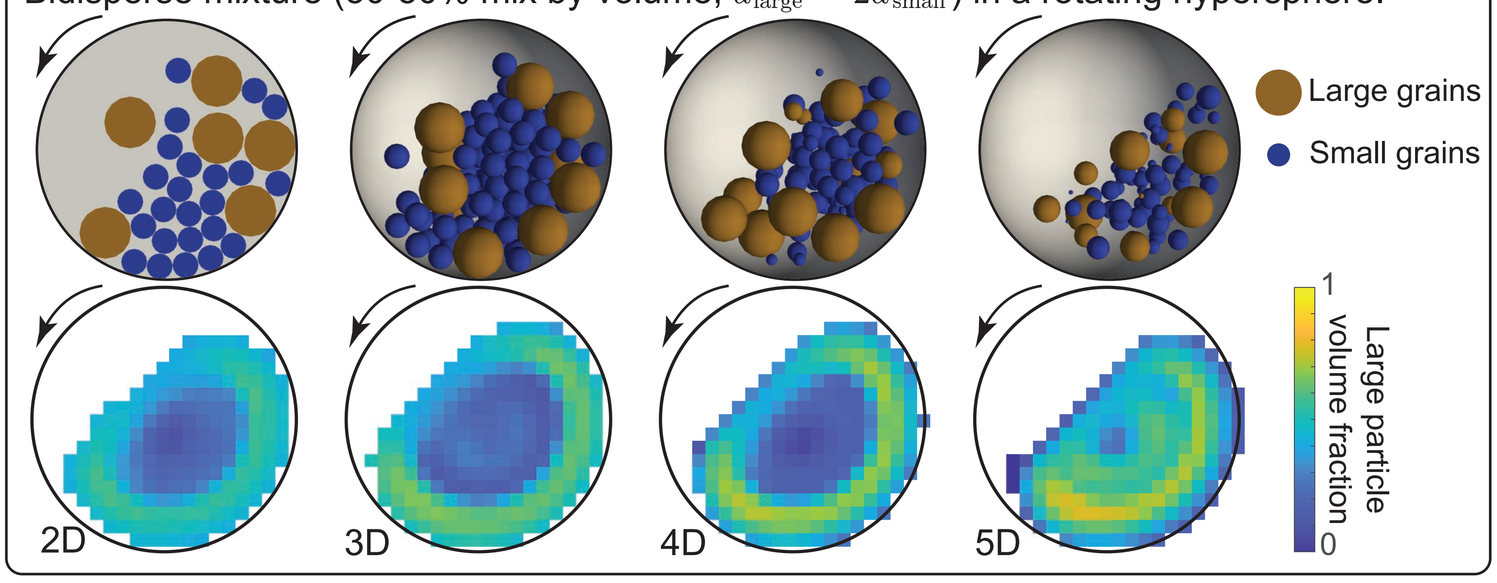




- Linear decrease of flow volume fraction with inertial number I
- Rheology $\mu(I)$ similar for all dimensions >2.

5. Segregation

Bidisperse mixture (50-50% mix by volume, $d_{\text{large}} = 2d_{\text{small}}$) in a rotating hypersphere.



6. Going further

Paper

Github

Live demos
Teaching resources

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[1] Skoge et al, PRE 74, 041127 (2006).
[2] Hales, Ferguson, Discrete Comput. Geom. 36, 21 (2006).
[3] Conway, Sloane, Sphere Packings, Lattices and Groups, 290 (Springer ed., 1999).
[4] Odlyzko, Sloane, J. Comb. Theory A 26, 210 (1979).
[5] Musin, Ann. Math. 168, 1 (2008).
[6] Song, Wang, Makse, Nature 453, 629 (2008).