Equilibrium and particles trajectories in sheared dense suspension

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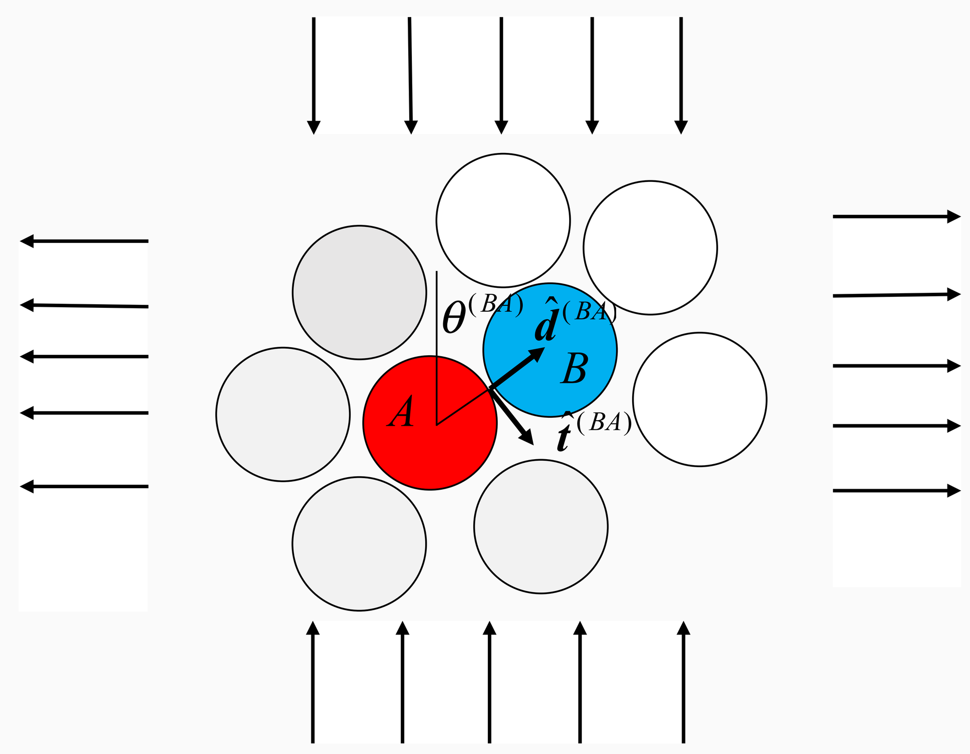
We focus on simple extensional flow of a dense suspension in which the interaction between closely pairs of particles, with radius a, is governed by a lubrication force and by a short-range repulsion force responsible for the variations in approaching and departing velocities. We employ force and moment equilibrium to determine normal and tangential velocities of a typical pair of particles that are integrated into a flux condition incorporating a radial distribution. The problem is then governed by a differential equation that is solved by the method of characteristic with a boundary condition applied to the radial distribution function that approaches unity at far distance. Trajectories are derived and compared with those obtained through Stokesian dynamics simulations.

Characterization of the Flow



Kinematics

In a pure shearing flow the average rate of deformation $\mathbf{D}$ has components $D_{11} = -D_{22} = \dot{\gamma}$. The relative motion of the center of particle B with respect to the center of particle A is

$$v_\alpha^{(BA)} = v_s \hat{d}_\alpha^{(BA)} + v_\theta{}^{(BA)} \hat{t}_\alpha^{(BA)},$$

where $v_s = \dot{s}$, $v_\theta = 2a\dot{\theta}$ and s is the separation of the edges along the line of centers and an over-dot indicates a derivative with respect to time. The relative velocity of their points of near contact is, then,

$$v_\alpha^{(BA)} + a(\omega^{(A)} + \omega^{(B)})\hat{t}_\alpha^{(BA)},$$

where ω is the angular velocity of the sphere. The interaction of A with $k-1$ near contacting neighbors n, other than B, is treated differently; the sphere n is assumed to move relative to A with the average flow. Then, the relative velocity of centers of pair nA is

$$v_\alpha^{(nA)} = 2aD_{\alpha\beta}\hat{d}_\beta^{(nA)}$$

and the relative velocity of the points of near contact nA is

$$v_\alpha^{(nA)} + a\omega^{(A)}\hat{t}_\alpha^{(nA)}.$$

Equilibrium

Viscous Force

The dimensionless force $\mathbf{F}^{(BA)}$ exerted by sphere B on sphere A is

$$F_\alpha^{(BA)} = \frac{3}{2}\frac{1}{\dot{\gamma}}\frac{v_s}{s}\hat{d}_\alpha + \frac{1}{\dot{\gamma}}\left[\ln\left(\frac{1}{s}\right) + 3.84\right] v_\theta - \frac{\hat{F}}{s}\hat{d}_\alpha - 9.54\sin 2\theta \hat{t}_\alpha$$
$$+ \frac{1}{\dot{\gamma}}\left[\ln\left(\frac{1}{s}\right) - 0.96\right]\omega^{(A)}\hat{t}_\alpha^{(BA)} + \frac{1}{\dot{\gamma}}\ln\left(\frac{1}{s}\right)\omega^{(B)}\hat{t}_\alpha^{(BA)},$$

Force equilibrium for particle A $\quad F_\alpha^{(BA)} + \sum_{n\neq B}^{N^{(A)}} F_\alpha^{(nA)} = 0;$

Moment equilibrium for particle A $\quad \varepsilon_{\kappa\alpha}F_\alpha^{(BA)} + \varepsilon_{\kappa\alpha}\sum_{n\neq B}^{N^{(A)}} F_\alpha^{(nA)} = 0$

Similar equations for particle B

Solutions

Normal component velocity

$$v_s = \frac{2}{3}\hat{F}\left(1 + \frac{4bs}{\bar{s}}\right) + \frac{4bs}{\bar{s}}\cos 2\theta\dot{\gamma}$$

Tangential component velocity

$$v_\theta = (2+s)c_2 \sin 2\theta\dot{\gamma}$$

with $\hat{F}$ the dimensionless repulsive force and $\bar{s}$ the average distance between particles edges, and

$$c_2 = \frac{6b/\left[\bar{s}\left(4b - k + 1\right)\right]}{\ln\left(1/\bar{s}\right) - 0.96} \qquad b = -\frac{3\sqrt{3}(k-1)}{16\pi}.$$

Flux Condition

$$\nabla \cdot (\mathbf{v}g_{12}) = 0$$

$$(s+2)v_s\frac{\partial g_{12}}{\partial s} + v_\theta\frac{\partial g_{12}}{\partial \theta} = -g_{12}\left[\frac{\partial v_\theta}{\partial \theta} - (s+2)\frac{\partial v_s}{\partial s} - v_s\right]$$

b.c. $\quad g_{12}(s=2,\theta) = 1$

Method of Characteristics

$$\frac{ds}{d\tau} = (s+2)v_s \qquad s(\tau=0,\theta_0) = 2$$

$$\frac{d\theta}{d\tau} = v_\theta \qquad \theta(\tau=0,\theta_0) = \theta_0$$

$$\frac{dg_{12}}{d\tau} = -g_{12}\frac{\partial v_\theta}{\partial \theta} - (s+2)g_{12}\frac{\partial v_s}{\partial s} - v_s g_{12} \qquad g_{12}(\tau=0,\theta_0) = 1$$

Solutions

$$g_{12} = \left(\frac{s}{2}\right)^{-(1/q+1)} \times e^{(1-s/2)} \times \left(\frac{\tan\theta}{\tan\theta_0}\right)^{-(4q+1)\hat{F}/3} \times e^{\left[-4q\hat{F}/(3\sin^q 2\theta_0)\right]\int_{\theta_0}^{\theta} 1/(\sin 2\xi)^{q+1}d\xi}$$

in which

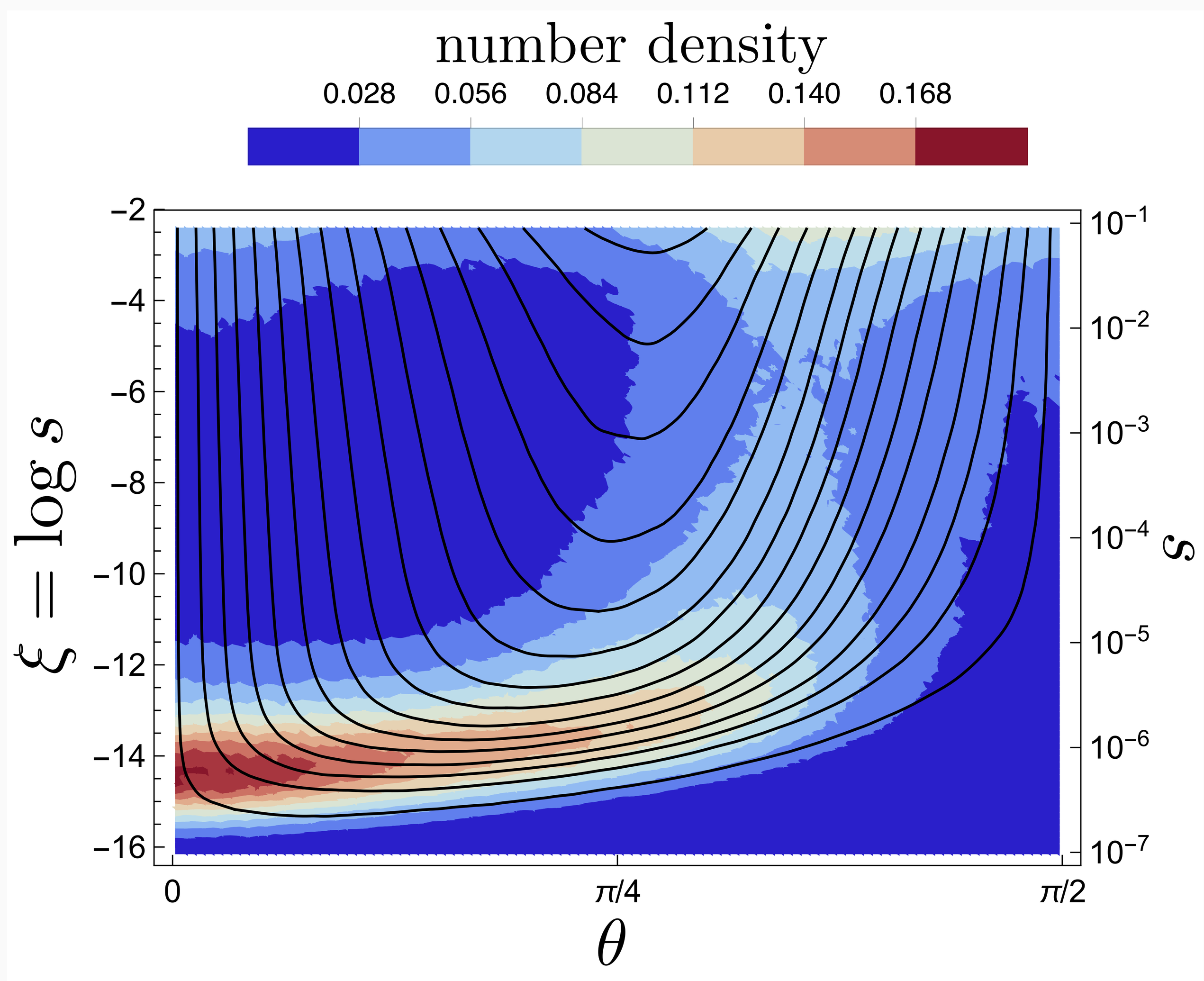
$$q = -\frac{1}{3}\left[\frac{3\sqrt{3}(k-1)}{4\pi} + k - 1\right]\left[\ln\left(\frac{1}{\bar{s}}\right) - 0.96\right]; \quad \theta_0 = \frac{1}{2}\sin^{-1}\left[\left(\frac{2}{s}\right)^{1/q}\sin(2\theta)\right]$$

and

$$s = 2\left(\frac{\sin 2\theta}{\sin 2\theta_0}\right)^q + \hat{F}\frac{1}{(\sin 2\theta)^{-q}}\left\{\frac{2}{3c_2}\int_{\theta_0}^{\theta}\left[1 + 4qc_2\frac{\sin(2\xi)^q}{\sin(2\theta_0)^q}\right]\sin(2\xi)^{(-q-1)}d\xi\right\}$$

Particles trajectories at closest proximity, area fraction $\nu = 0.64$

Numerical simulations



Theoretical prediction: solid lines($\hat{F} = 10^{-4}$), dashed lines ($\hat{F} = 0$)