POLITECNICO DI MILANO

SALAMINA GRAIN WORKSHOP

KINETIC THEORY APPLIED TO PRESSURE-CONTROLLED SHEAR FLOWS OF FRICTIONLESS SPHERES BETWEEN RIGID, BUMPY PLANES

D. VESCOVI and D. BERZI

Dept. of Civil and Environmental Engineering, Politecnico di Milano, Italy

dalila.vescovi@polimi.it

SSLiP

European Innovation Council — Funded by the European Union

MOTIVATION

In the most of real engineering problems and natural phenomena involving systems of flowing particles, boundaries strongly affect the local behaviour of the granular material, generating spatial heterogeneities.

A theoretical continuum model must be supplemented with appropriate boundary conditions in order to deal with **boundary-valued problems**.

GOAL

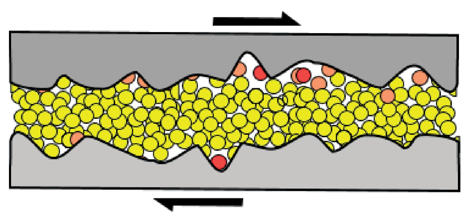
Test the reliability of **kinetic theory of granular gases** and its **boundary conditions** in steady, pressure-controlled bounded shear flows (in the absence of gravity)

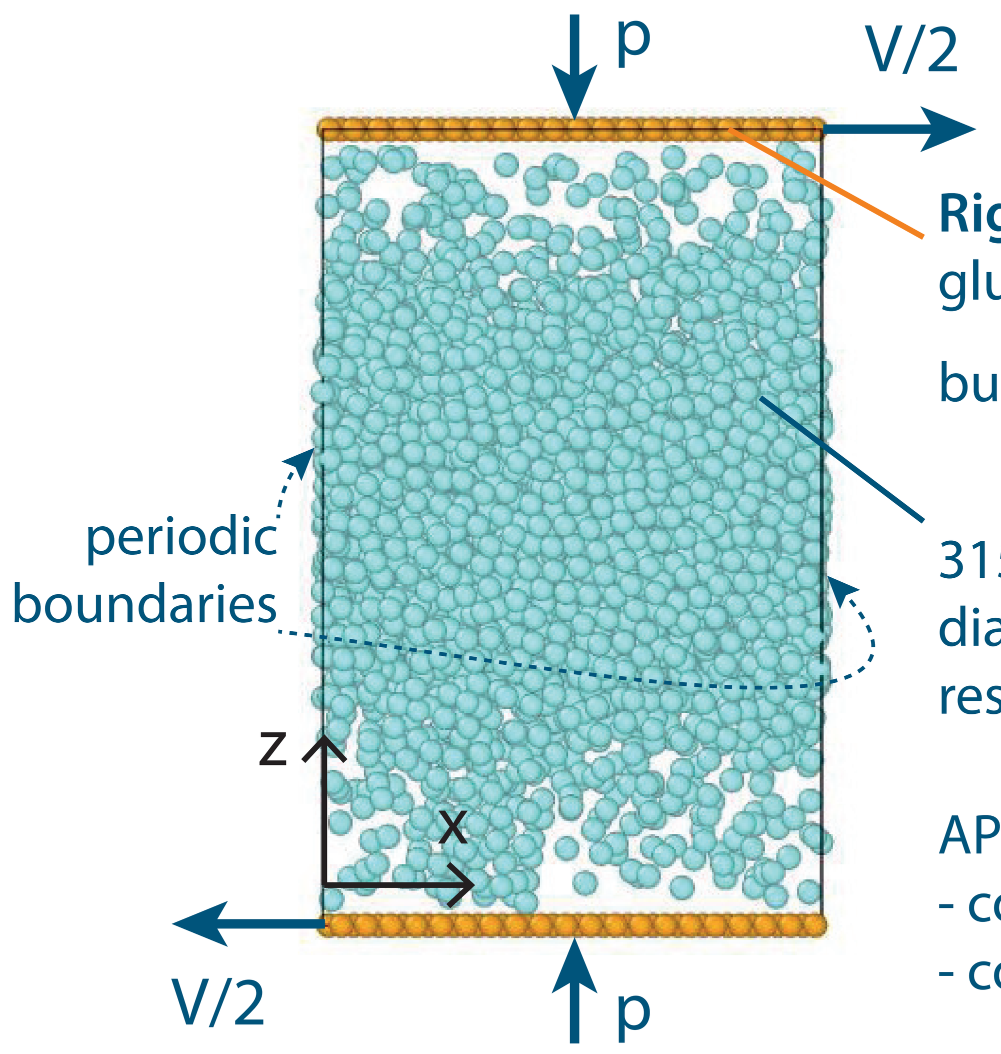
TOOLS

- 3D Discrete Element Method (DEM) simulations
- Kinetic theory of granular gases

GRANULAR SYSTEM SHEARED BETWEEN BUMPY PLANES

DEM SIMULATIONS





Rigid bumpy walls made of glued spheres of diameter d_w

bumpiness $\psi = \sin^{-1}\left(\frac{d_w}{d_w + d}\right)$

3150 moving **frictionless spheres** diameter d, density ρ_p, stiffness k, restitution coefficient e_n

APPLY:
- constant relative wall velocity V
- constant pressure p

$\left(\begin{array}{l} d = \rho_p = V = 1 \\ k = 2 \times 10^5 \end{array}\right)$

KINETIC THEORY OF GRANULAR GASES

INGREDIENTS:
- Fluctuating energy balance
- Constitutive relations
- Boundary conditions

PARAMETERS:
- micro: d, ρ_p, e_n
- MACRO: $\nu_{rcp} = 0.64$ (random close packing)
  $\nu_f = 0.49$ (freezing point)

↓

SET of 4 DIFFERENTIAL EQUATIONS to compute
- shear stress s, flow height H
- z-distribution of concentration ν and granular temperature T (...)

CONCLUSIONS

- Lubrication effect due to the tendency of the particles to accumulate in a dense, slow-moving core squeezed in between 2 regions of high shear and high agitation near the bumpy planes.

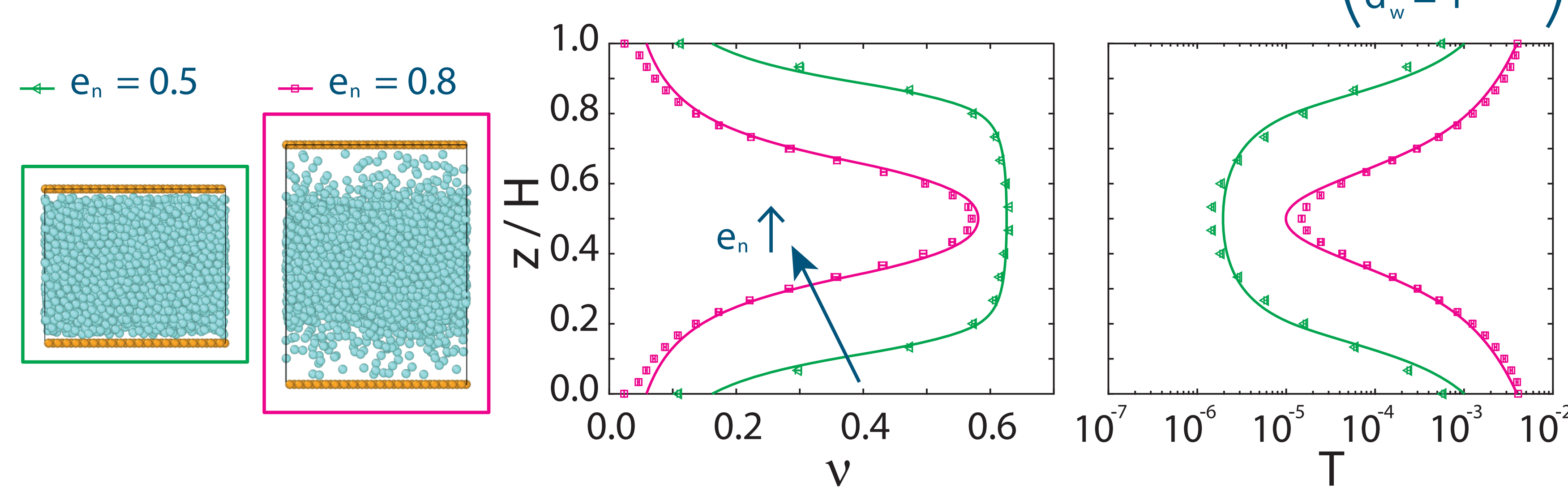
- The theoretical predictions are in **excellent** qualitative and quantitative **agreement** with the simulations for all control inputs investigated. The theory does not require any parameter calibration.

RESULTS AND COMPARISONS

DEM results: symbols
Theory predictions: lines

Influence of the restitution coefficient e_n

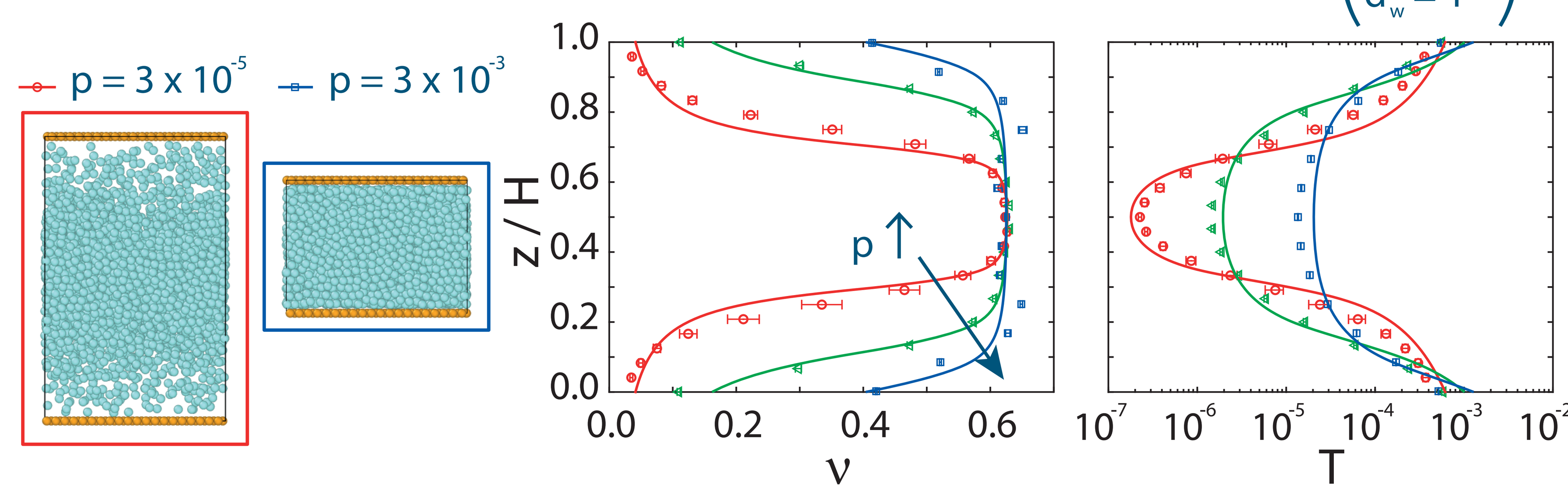
$\left(\begin{array}{l} p = 3 \times 10^{-4} \\ d_w = 1 \end{array}\right)$



- for all e_n: dense, "cold" core region surrounded by 2 dilute, "hot" layers
- as e_n decreases: the size of the dense core increase
- the **kinetic theory can remarkably reproduce the DEM results**

Influence of the imposed pressure p

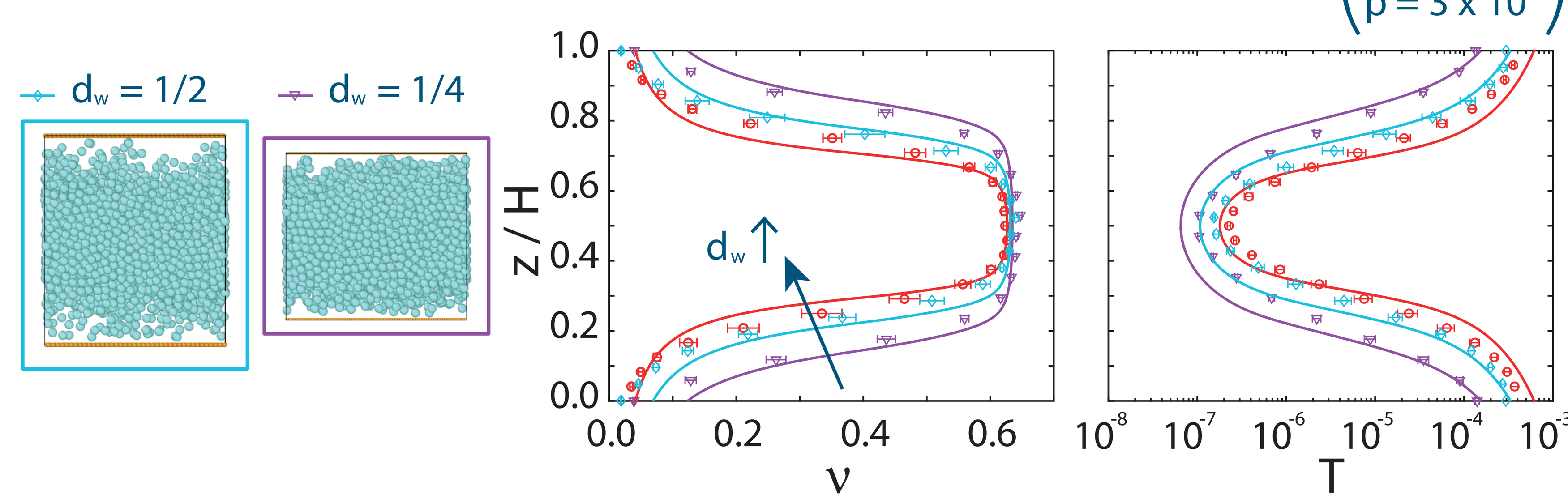
$\left(\begin{array}{l} e_n = 0.5 \\ d_w = 1 \end{array}\right)$



- the size of the dense core increases with the pressure
- ν in the dense core and T at the boundaries are independent of p
- T in the dense core and ν at the boundaries increase with p

Influence of the bumpiness d_w

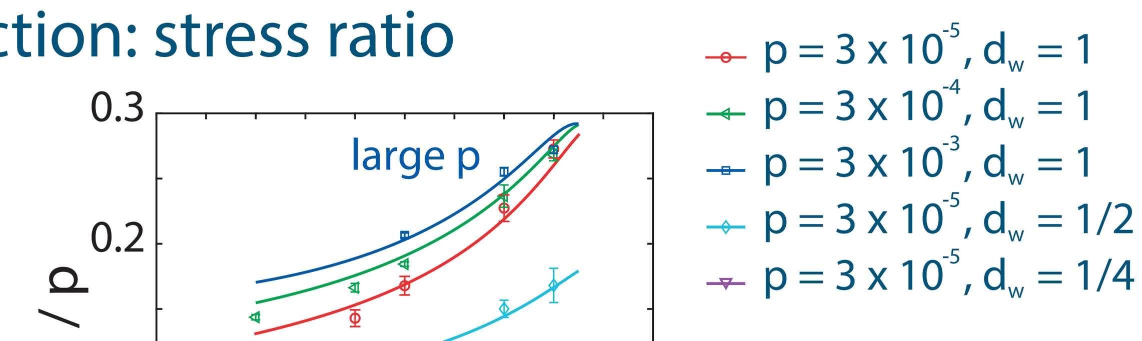
$\left(\begin{array}{l} e_n = 0.5 \\ p = 3 \times 10^{-5} \end{array}\right)$



- the reduction of d_w (ψ) yields a wider and colder dense core region
- less bumpy boundaries are less effective in transferring particle momentum from the flow, x-, to the gradient, z-direction

Macroscopic friction: stress ratio

capability of the frictionless moving spheres to act as a **lubricant** with respect to the bumpy planes

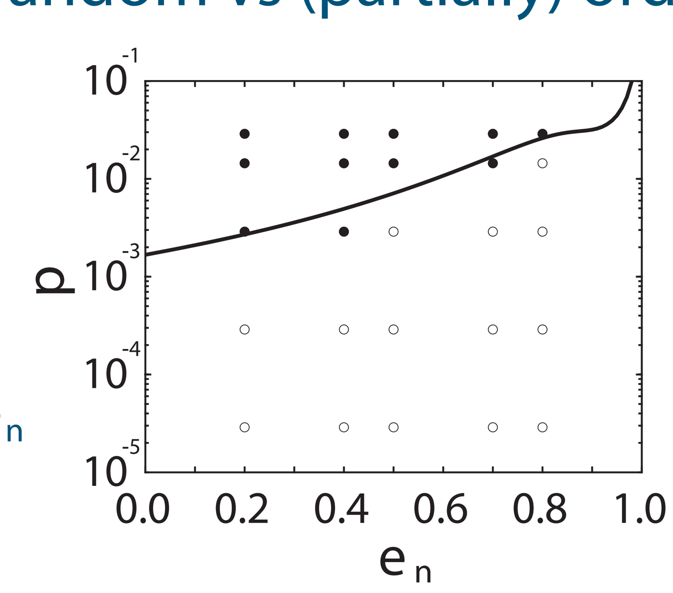


lubrication due to strong spatial heterogeneity of ν

Phase diagram: random vs (partially) ordered states

CRYSTALLIZATION

at large p and small e_n (here for $d_w = 1$)



- • crystallized
- ∘ not crystallized
- — theory prediction: phase-transition to an ordered state when ν > 0.49 (freezing point) at the bumpy walls

ACKNOWLEDGEMENTS

SSLiP project, HORIZON-EIC-2021-PATHFINDEROPEN-0. Funded by the European Union.