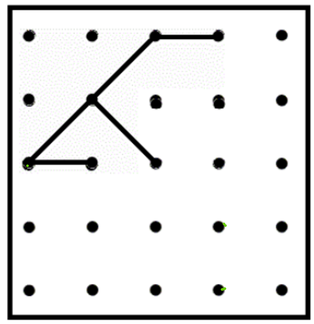
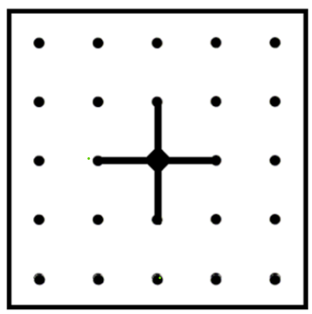
Module 8 Plane and Solid Figures

Section 8.1 Translation Symmetry
Practice Problems 8.1
For Problem 1-5, follow the instructions given to solve the problem.

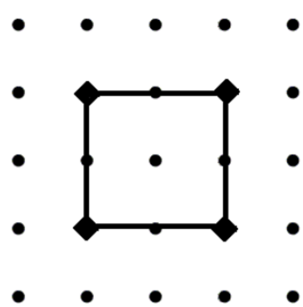
1. Move the basic element of design two spaces right and two spaces down in order to cover the grid? Will there be a gap or overlap in the basic design? What one piece can be removed to fix this?



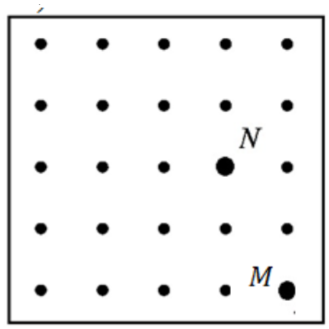
2. Move the basic element of design two spaces right, left, up, or down to cover the grid. Will there be any gaps or overlaps in the final design? What do you think the final design will look like?



3. The basic element of design, when translated two spaces right, left, up or down will create the design in Problem 2. Explain why this basic element is not preferred for a wallpaper border.



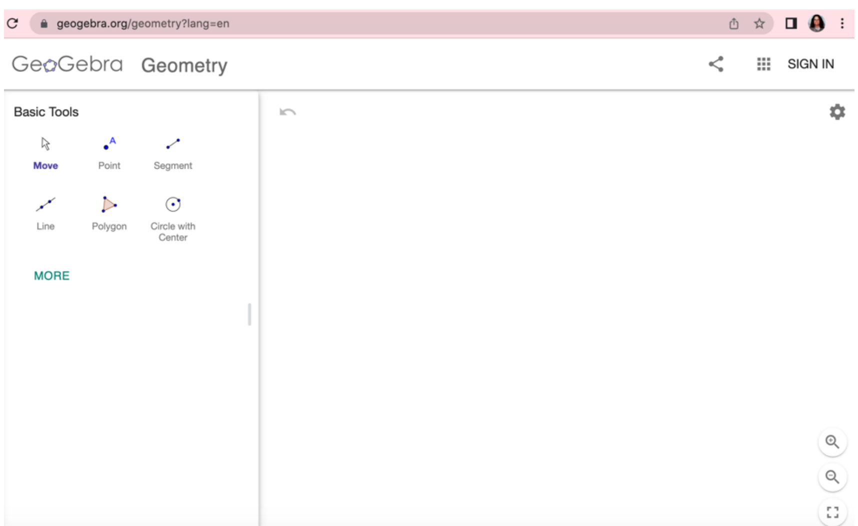
4. Translate the points *N* and *M* two units up and three units left. Draw the image of the new points and label them *N'* and *M'* (Read: "*N* prime" and "*M* prime"); *N* and *M* are called the pre-images and *N'* and *M'* are called the images. Pre-image means before so it is the original point. An image is what you see in the mirror, so the image point is the one that is drawn after the original is moved.



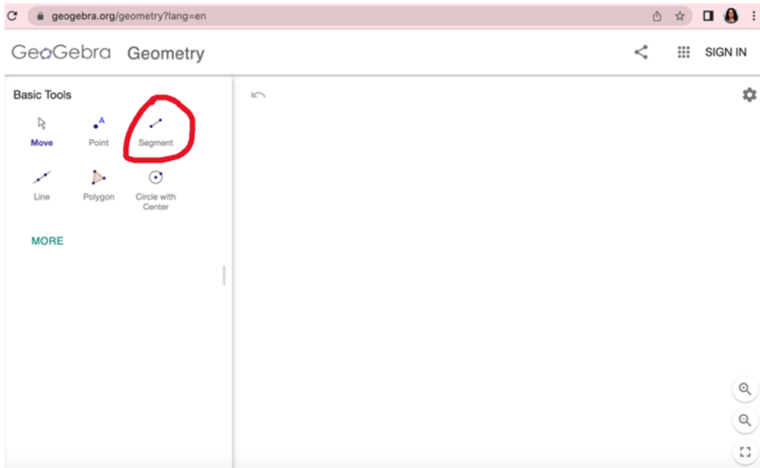
5. Connect points N and M and connect points N' and M' in Problem 4. Are the lines parallel or perpendicular? How do you know?

Now that you have studied making basic design elements and translating points, we are going to use technology to make and translate our own design element.

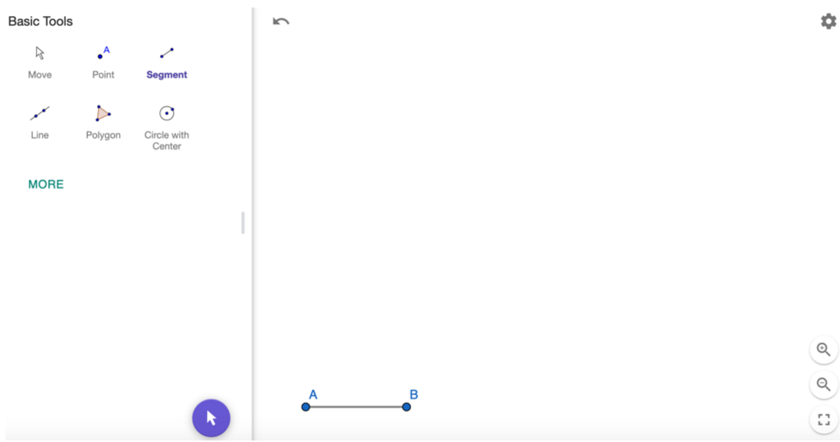
Go to https://www.geogebra.org/geometry?lang=en. You should see a page as the one shown below:



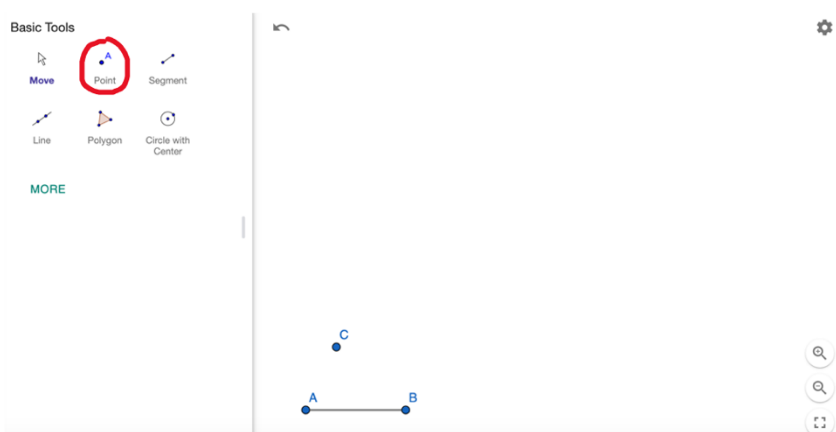
Click “Segment” under “Basic Tools” on the lefthand menu (circled in red).



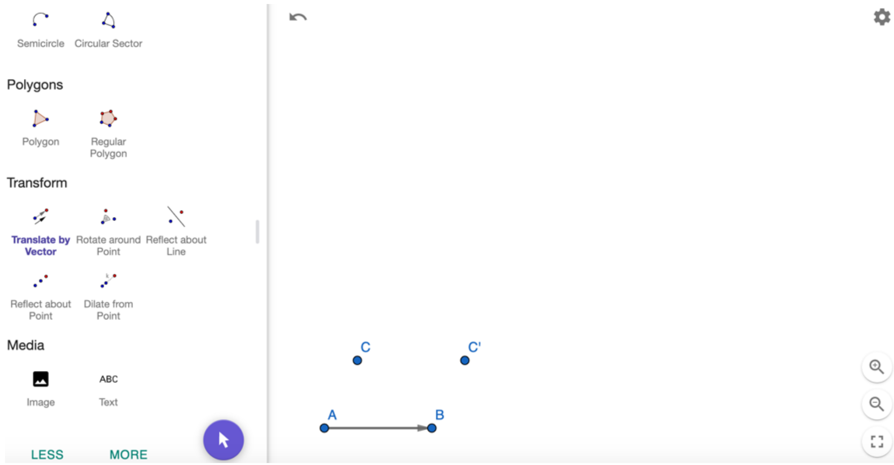
Construct a segment AB in the bottom left corner of the page as shown below.



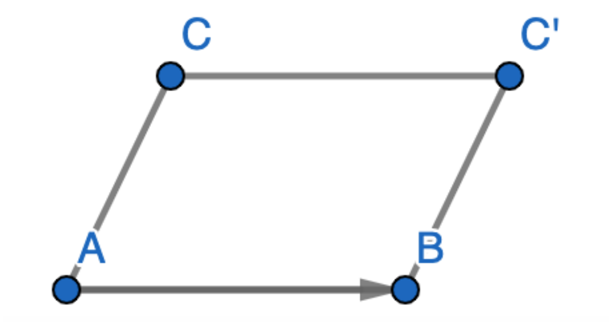
Click “Point” (circled in red) under “Basic Tools” and construct a point C above segment AB closer to A.



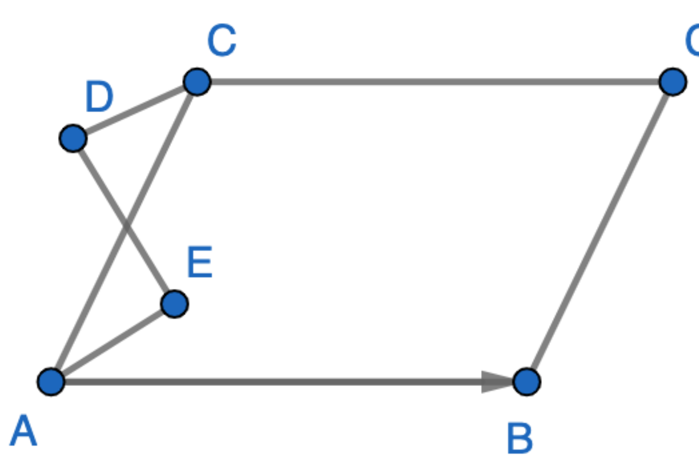
Click "MORE" on the lefthand tab and scroll down to and click "Translate by Vector" under "Transform." Then click point C, point A, point B in that order. What you should see is shown below.



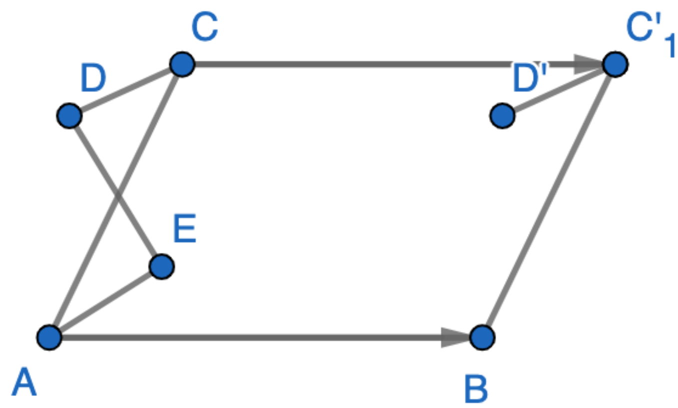
Scroll back up to the top of the lefthand tab and click "Segment" under "Basic Tools." Construct segments from point A to point C, point C to point C', and point C' to point B.



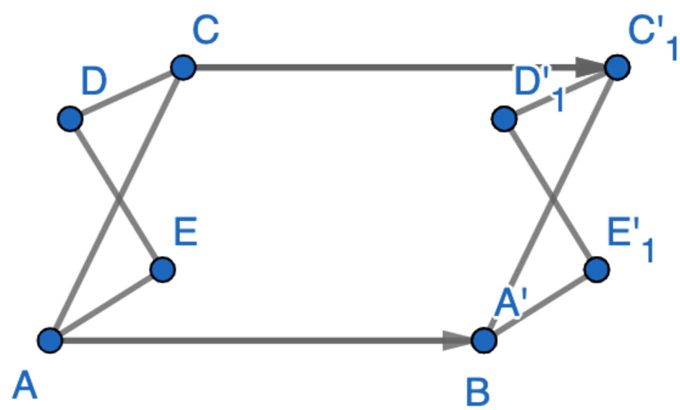
Construct points D and E as shown below and connect them with segments.



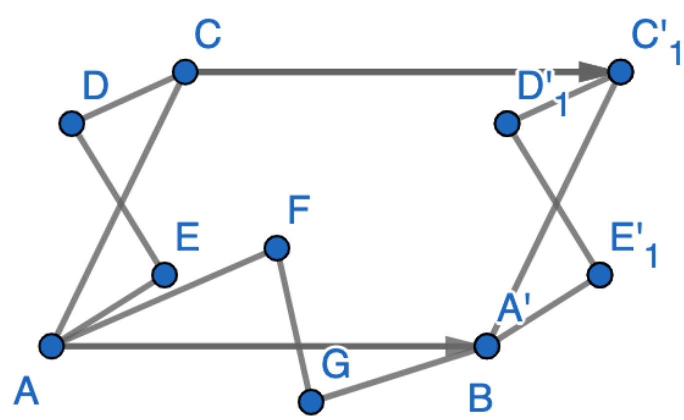
Scroll back down to and click "Translate by Vector." Click segment CD, then point C, and then point C' in that order.



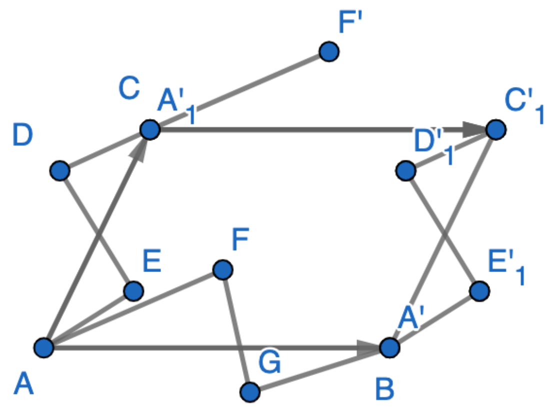
Click "Translate by Vector," then segment DE, then point C, and then point C'. Click "Translate by Vector," then segment EA, then point C, and then point C'.



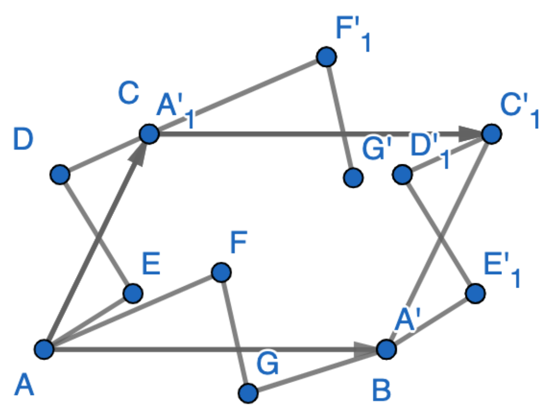
Construct points F and G and then connect them with segments as shown below.



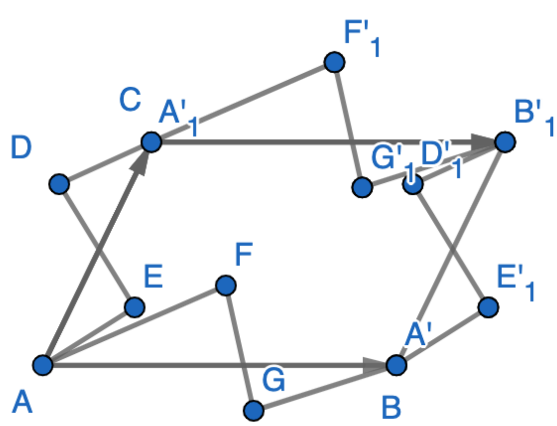
Scroll down and click “Translate by Vector.” Click segment AF, click point A, and then click point C.



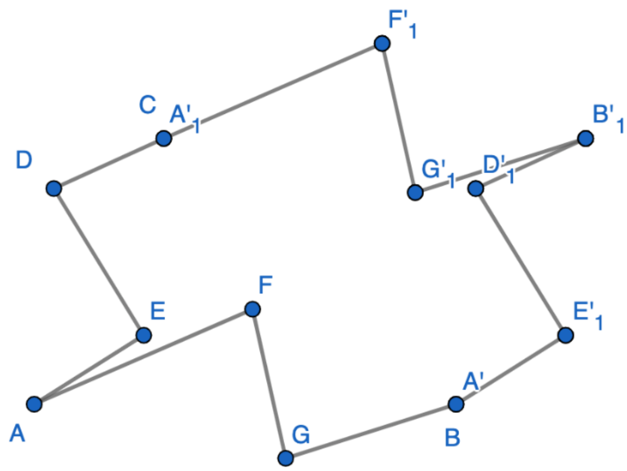
Click “Translate by Vector.” Click segment FG, click point A, and then click point C.



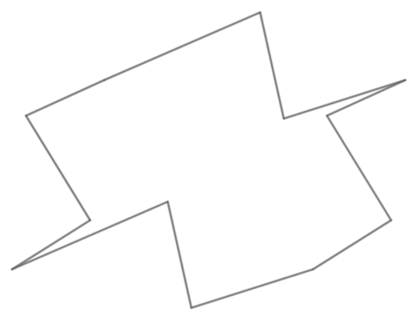
Click “Translate by Vector.” Click segment GB, click point A, and then click point C.



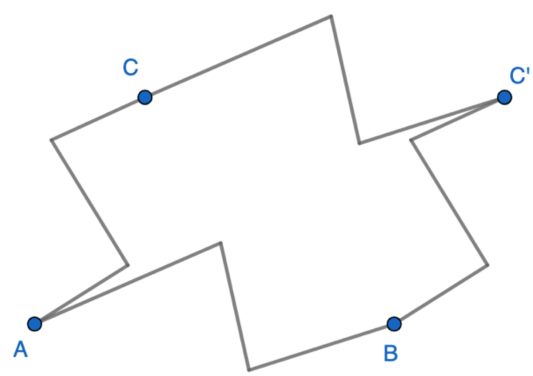
You are now going to construct the irregular polygon interior so that you can translate the new design without the original polygon.
Scroll to “Edit” and click “Select Objects.” Then click the segments: AC; CC’; C’B; BA. Then click “Show/Hide Object.” Keep repeating this process until your screen looks as below:



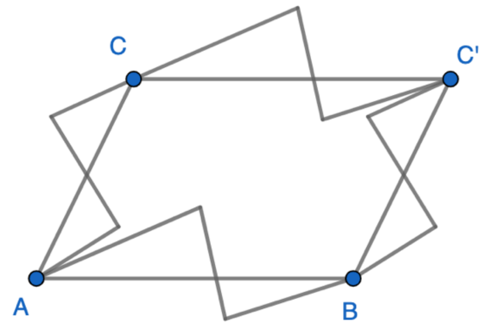
Click “Select Objects.” Beginning with A, go clockwise and click each point. Then click “Show/Hide Object.” Keep repeating this process until your screen looks as below:



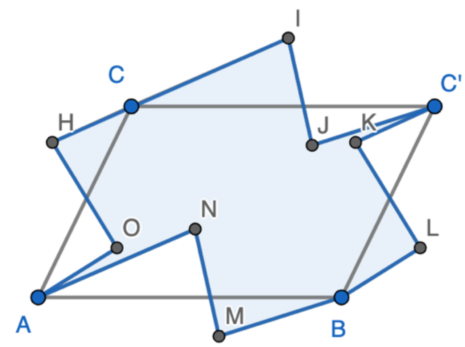
Click “Show/Hide Object” and then click the original four points: A; B; C; C’.



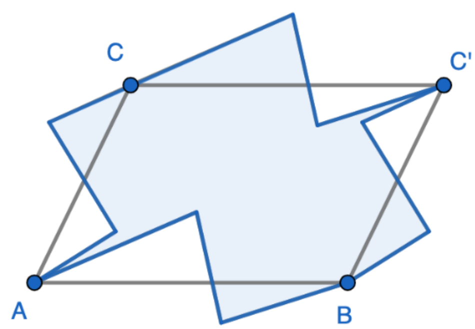
Connect the original four points with segments.



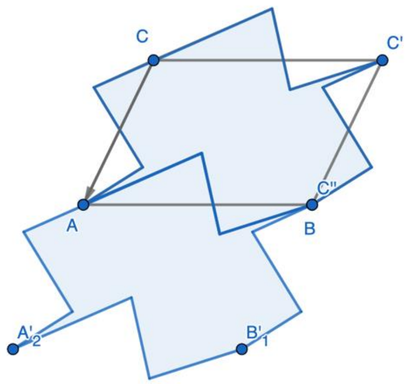
Click "Polygon" under "Polygon." Click each point around the outside of your shape until your screen looks as below:



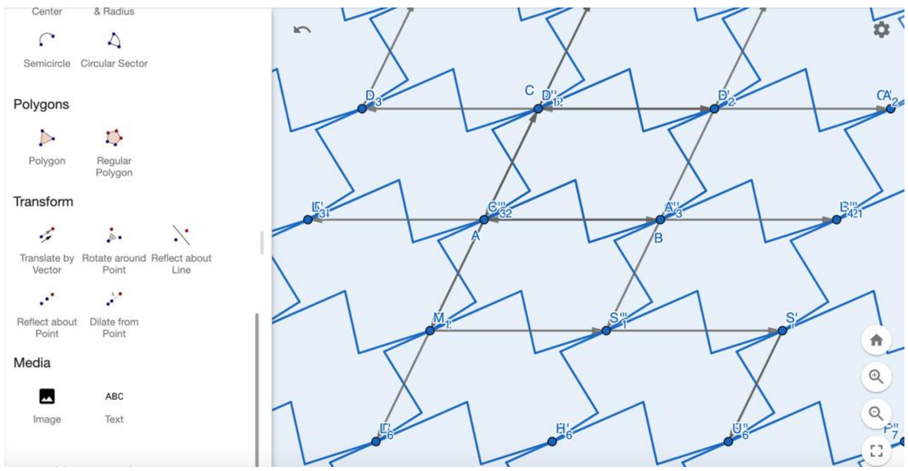
Click "Select Objects" under "Edit." Click all the grey points. Click "Show/Hide Object."



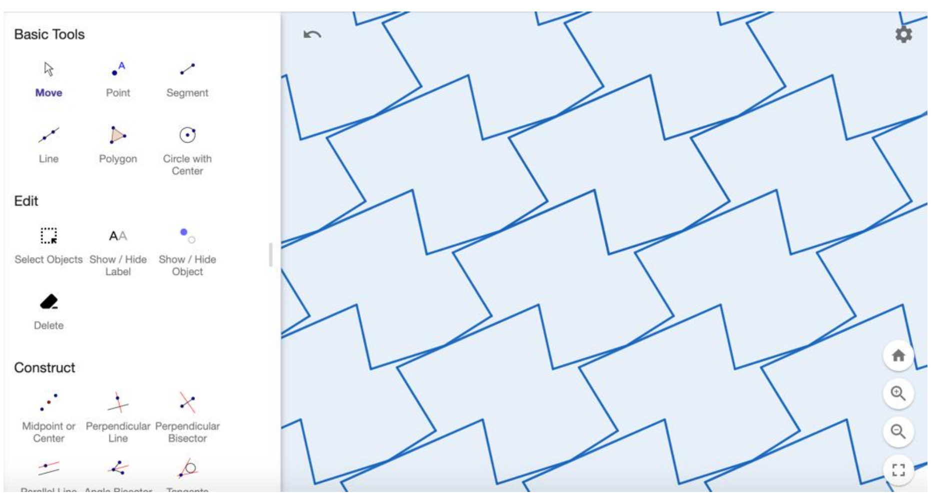
Click “Translate by Vector.” Click the middle of the polygon. Click point C and then click point A.



Continue this process until your page is covered.



Click “Select Objects” under “Edit.” Click each object that is not part of a blue polygon going clockwise and then click “Show/Hide Object.” Keep doing this until you can only see the blue polygon pattern shown below.

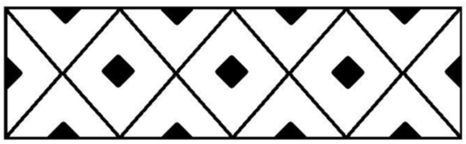


Section 8.2 Reflection Symmetry

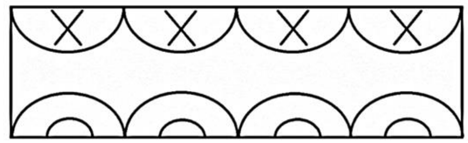
Practice Problems 8.2

For Problem 1-3, trace the design given on a piece of tracing paper, find all the fold lines so the design is superimposed onto itself, and mark the lines of reflection on the design. Then tell how you know they are the lines of reflection.

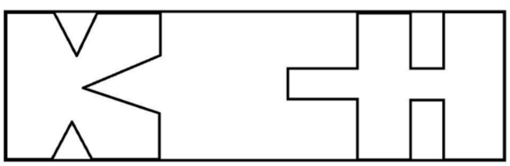
1.



2.

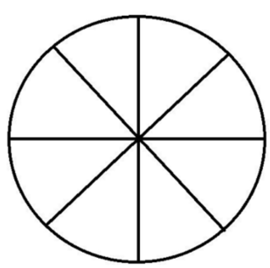


3.

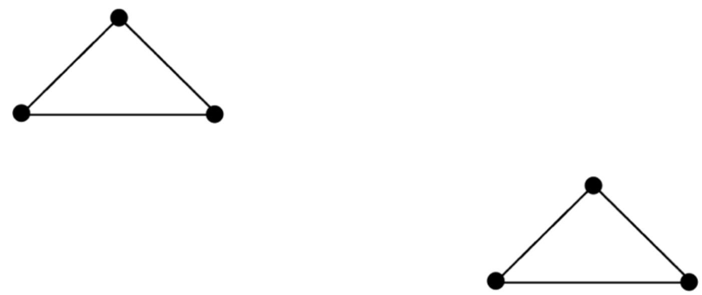


For Problem 4-10, use the diagram given to solve the problem.

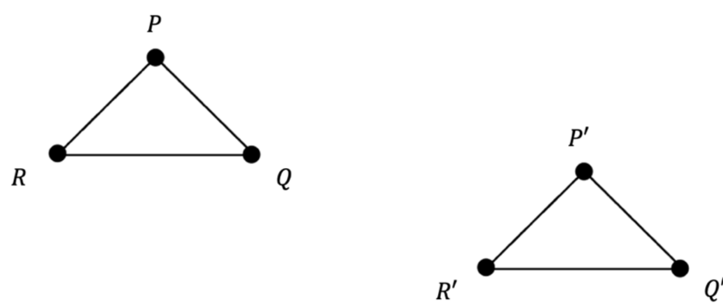
4. Draw all the lines of symmetry in the wheel of a bicycle shown below.



5. Is the upper triangle a reflection or translation of the lower triangle?

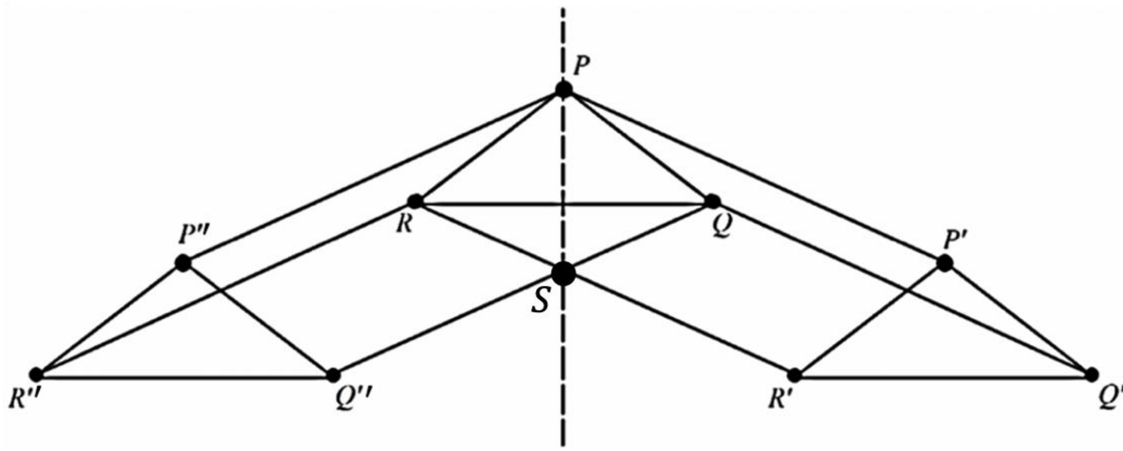


6. Draw lines from triangle PQR to triangle $P'Q'R'$ so the points of the pre-image are connected to the translated points of the image.



7. In Problem 6, measure the distance in millimeters from the pre-image points of the triangle PQR to the image points of the triangle $P'Q'R'$. Are these distances the same or different? Why do you think this is so?

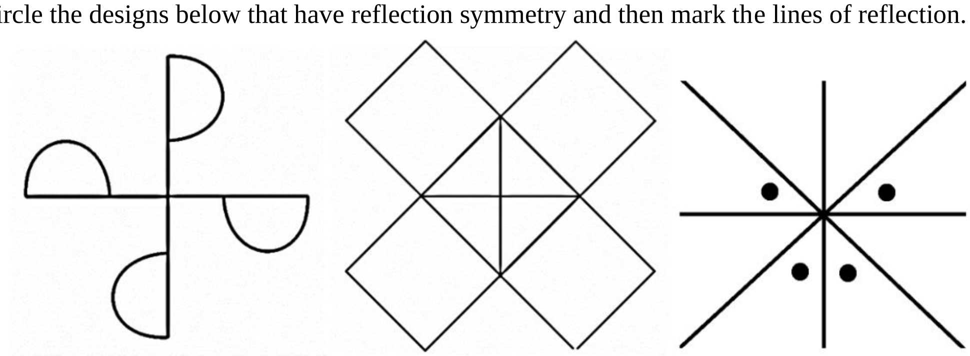
8. If you translate triangle PQR so it is translated the same distance as triangle $P'Q'R'$ but to the southwest of triangle PQR rather than the southeast, you get a design that looks like the one below. The line of reflection is the dashed line through point P.



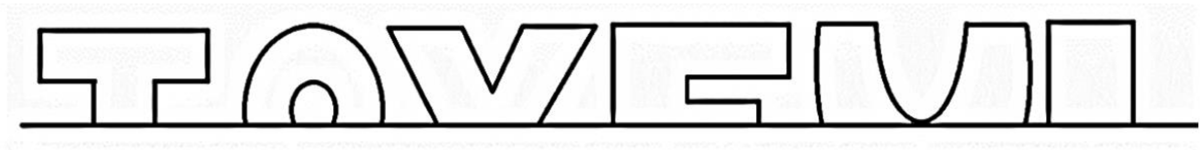
a) Measure from point R to the line of reflection, and from point Q to the line of reflection. What do you notice?

b) Measure from point P'' (read: "P double prime") perpendicular to the line of reflection at point S. Measure from point P' perpendicular to the line of reflection at point S. What do you notice?

9. Circle the designs below that have reflection symmetry and then mark the lines of reflection.



10. Reflect the top of the word “JOYFUL” across the line given.



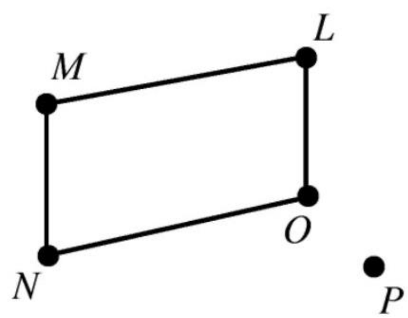
It does not spell “JOYFUL” anymore because the printed word does not have reflection symmetry. However, a joyful attitude is a reflection of Christ!

CHALLENGE: List all the letters of the alphabet that have reflection symmetry. Draw the line of symmetry through each letter.

Section 8.3 Rotation Symmetry

Practice Problems 8.3

For Problem 1-10, use the given diagram to solve the problem.



1. Let point P be the center of rotation.

2. Connect point P and L with a colored line and let it be the axis of rotation.

3. What is the shape of the pre-image LMNO?

4. Let the angle of rotation be 90°. How many rotations are in 360°?

5. How many images will result in the full circle?

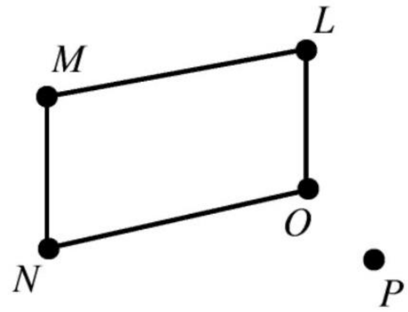
6. Does the length of PL ever change as the pre-image is rotated?

7. What will be the shape of all the images?

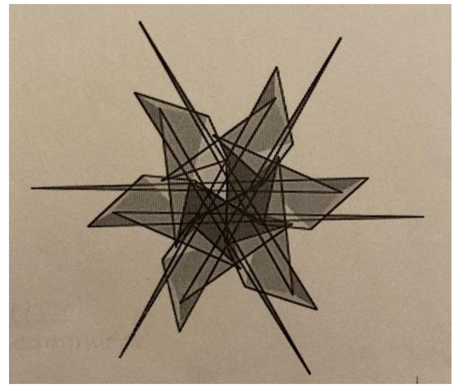
8. Draw the completed design for four 90° rotations of the pre-image in the diagram above.

9. If you rotated LMNO about point P at an angle of 120°, how many images would be in the completed design of 360°?

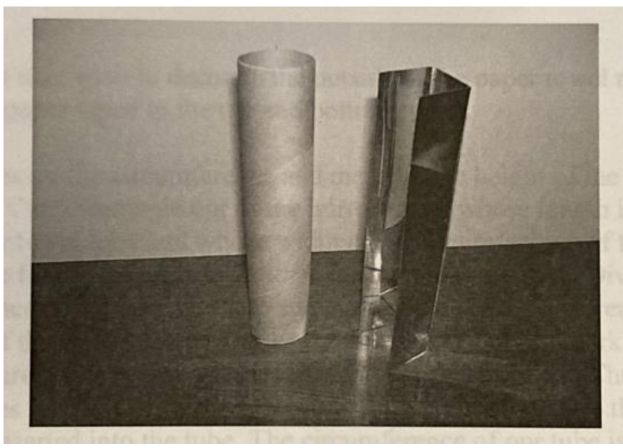
10. Draw the completed design for three 120° rotations in the diagram below.



Kaleidoscopes have rotation symmetry. In Geometry and Trigonometry, you will use technology to make a kaleidoscope. For now, you can follow these instructions to make one by hand.

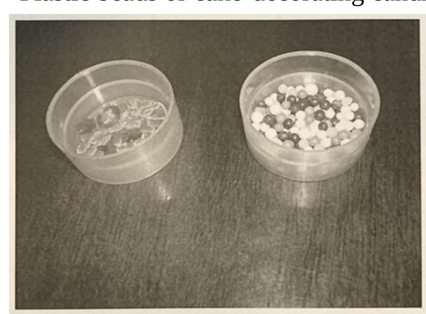


You can purchase the materials at any crafts store, such as Hobby Lobby®, etc. If you cannot find the materials at a store near you, they can be purchased online, though they may need to be purchased individually.



Below are the materials you will need:

- Paper towel roll
- Mirrored cellophane paper
- Marker/pen
- Basic plastic dish with a slightly larger diameter and circumference than the paper towel roll
- Clear plastic file folder (A clear plastic circle or cover will need to fit in the top of the plastic dish over the filler beads).
- Plastic eye-hole cap (This can also be made from a piece of cardboard with a small hole cut in the center).
- Plastic beads or cake-decorating candies

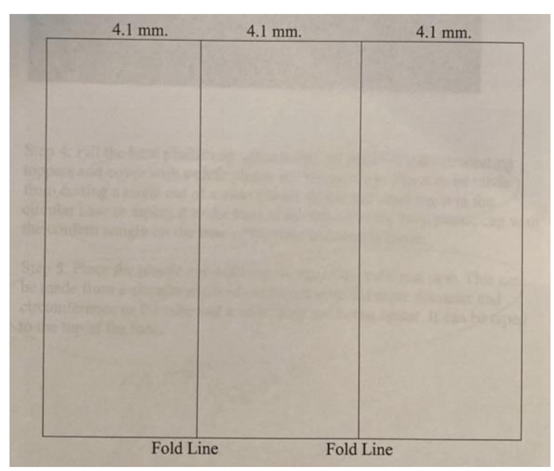


Step 1: You may wish to decorate the outside of the paper towel roll with cellophane paper taped to the top and bottom edges.

Step 2: Measure the circumference and height of the paper towel roll. Cut a rectangle out of the mirror paper that has a length the height of the paper towel roll and a width the circumference of the paper towel roll. This rectangle will be folded into a triangle to be inserted in the paper towel roll.

Divide the circumference of the paper towel roll by three and make and mark these three measurements along the width of the mirror paper. Use a straightedge or ruler to mark lines at these measurements parallel to the length of the mirror paper. These will be the fold lines for the mirror paper so you will be looking down into the triangle when it is inserted into the tube.

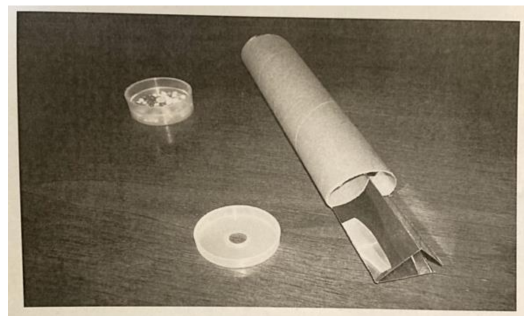
The circumference of our tube is 12.4 mm so each of the three sections are 4.1 mm wide.



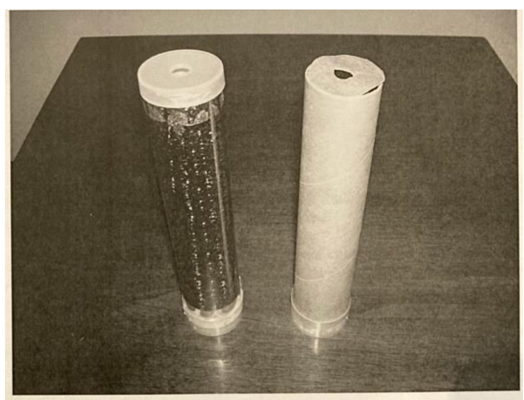
Step 3: Tape the right and left edge of the mirror paper once so it is a tri-fold and insert it into the paper towel roll.

Step 4: Fill the base plastic dish with confetti or beads or cake-decorating toppers and cover it with a clear plastic piece (This can be made by cutting a circle out of a clear plastic folder and inserting it in the circular base or taping it to the base of the paper towel roll before attaching the plastic dish). Put the base plastic dish with the filler snugly on the base of the paper towel roll and tape it in place.

Step 5: Place the plastic eye-hole cap on top of the paper towel roll and tape it in place (This can be made from a circular piece of cardboard with the same diameter and circumference as the tube and a small hole cut in the center). This can be taped to the top of the tube.



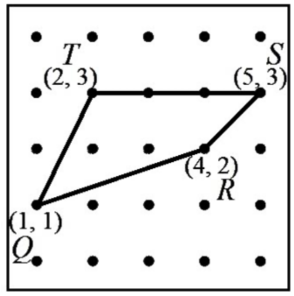
Step 6: Look through the eye-hole and spin the paper towel roll. If you have a plastic tube on bottom that is snug but not taped, you could hold the paper towel roll still and turn it the dish to see your kaleidoscope design.



Section 8.4 The Distance Formula

Practice Problems 8.4

For Problem 1-5, use the given diagram to solve the problem.



Find the lengths of the sides of the quadrilateral using the distance formula. Then find the perimeter of the quadrilateral and estimate it to the nearest length.

1. QR

2. RS

3. ST

4. TQ

5. Perimeter of $QRST$

For Problem 6 and 7, use the given information to solve the problem.
The diameter of a circle has endpoints $(4, -2)$ and $(-4, 5)$ on the coordinate grid.

6. How long is the diameter of the circle?

7. What is the approximate area of the circle?

For Problem 8 and 9, find the distance between the pair of points given.

8. $O(2, 5)$ and $P(7, 9)$

9. $S(-3, -2)$ and $T(5, 0)$

For Problem 10, use the given information to solve the problem.
A game piece is on the point $(4, -2)$ and slides to the point $(8, 8)$ on its next turn.

10. How many units did the game piece slide?

Section 8.5 The Midpoint Formula

Practice Problems 8.5

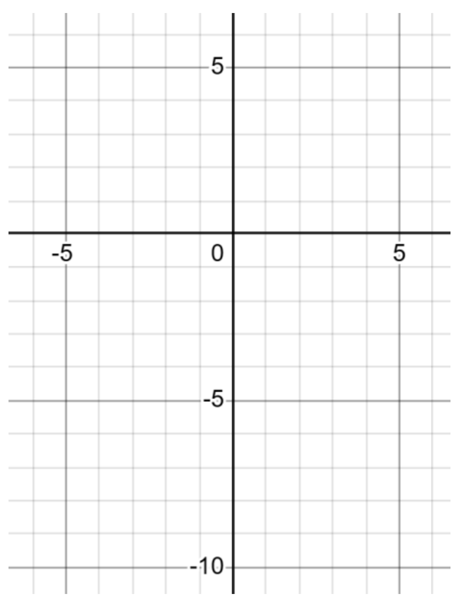
For Problem 1-4, solve the word problem given.

1. K'Lynn used the distance formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ to find the distance between points $(-1, -7)$ and $(2, -9)$. Graham said it was not correct that the distance formula is: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Graham is correct but both K'Lynn and Graham got the same solution. Why?

2. Jeffrey found the midpoint of the two points in Problem 1 using the formula: $(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2})$. Elysia said that Jeffrey was not correct and used the formula: $(\frac{x_2-x_1}{2}, \frac{y_2-y_1}{2})$. They both got two different solutions. Who was correct?

3. Devon said that if you drew a triangle using the two points as the endpoints of the hypotenuse, then Elysia's formula in Problem 2 gives half the lengths of each leg. Is Devon correct?

4. Demonstrate Devin's solution on the coordinate grid below.



For Problem 5-8, find the midpoints of the segments with the endpoints given.

5. $P(0, 6)$ and $Q(-1, 10)$

6. $R(0, 2)$ and $S(-5, -8)$

7. $B(1, 11)$ and $C(5, -2)$

8. $P(0, 1)$ and $Q(-4, 0)$

For Problem 9 and 10, solve the word problem given.

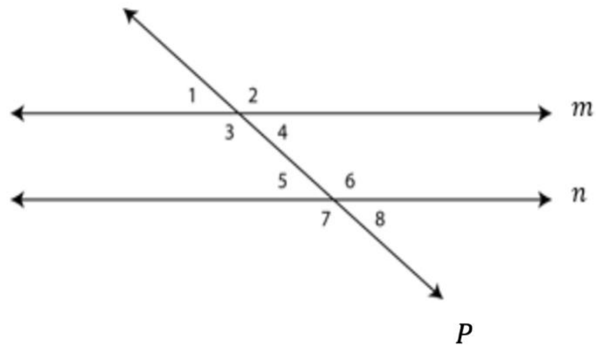
9. A circle has endpoints $(4, 2)$ and $(-2, -1)$ that lie on the diameter of the circle. What is the length of the diameter?

10. What is the midpoint of the diameter of the circle in Problem 9?

Section 8.6 Angle Relationships

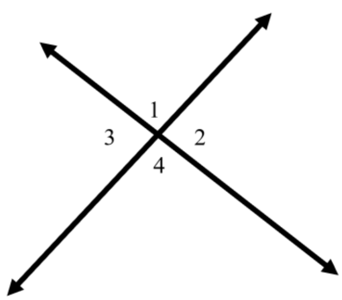
Practice Problems 8.6

For Problem 1-3, use the given diagram and instructions to solve the problem.



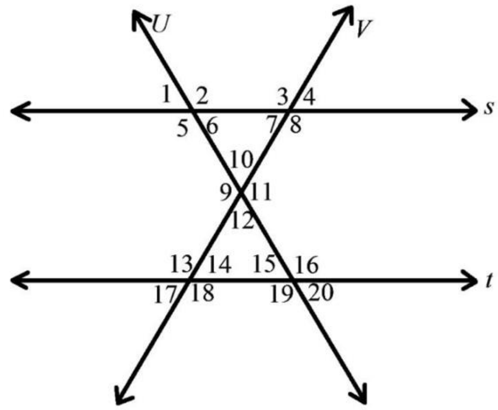
1. Lines m and n are parallel. They are cut by the transversal P. Trace $\angle 1$ with tracing paper and slide it down the left side of the transversal to cover $\angle 5$ and demonstrate they are equal. Color these angles using the same color. These are called corresponding angles; they correspond to one another; they are both on the left side of the traversal above the parallel lines. There are three other pairs of corresponding angles in the diagram. Find them and color each pair a different color.

2. Take a piece of tracing paper. Fold the bottom left corner up to the top right corner and crease it to make a line. Fold the bottom right corner up to the top left corner and crease it to make intersecting lines. Label the angles 1, 2, 3, 4 and darken the lines. Fold $\angle 1$ onto $\angle 4$. What do you notice?



3. In Problem 2, $\angle 1$ and $\angle 4$ are called vertical angles. They are formed when two lines cross or intersect. The two intersecting lines form V shapes opposite the point of intersection. They are not adjacent because they do not share a common side, but they do share a common point. Find one other pair of vertical angles in Problem 2 and all of the pairs of vertical angles in the diagram in Problem 1.

For Problem 4-10, use the given information and diagram to solve the problem.
Line s is parallel to t ($s \parallel t$). Both s and t are cut by the transversals U and V. The symbol for congruent is "≅," which means equal.



4. Angle 4 is congruent to ∠____________ by corresponding angles.

Angle 14 is congruent to ∠____________ by vertical angles.

If ∠4 ≅ ∠____________ and ∠14 ≅ ∠17, then ∠4 ≅ ∠____________ by the transitive property, which states that if two angles are congruent to the same ______________________, then they are ______________________ to each other.

5. Angle 19 and ∠20 are _____________________ because they add up to 180°. Angle 19 and ∠___________ are also supplementary because they lie on a straight line. If ∠20 is acute, then so is ∠____________ because they are congruent by vertical _____________________. When two lines _______________________ each other, they form vertical angles and vertical angles are always _____________________. If ∠20 and ∠15 are acute, then ∠19 must be _____________________ because the two angles have a sum of ________________ because they lie on a straight _____________________.

6. What do the measures of ∠6, ∠7, ∠10 add up to? ________________. These angles have this sum because they form a _____________________.

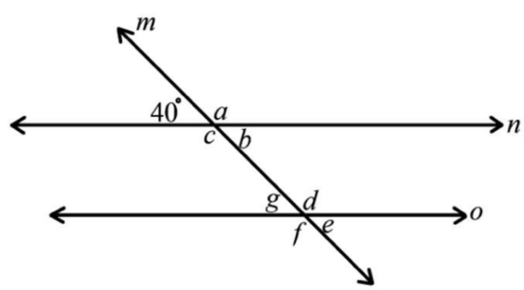
7. If ∠1 is acute, what other angles are acute? Why?

8. There are not any complementary angles in the diagram. How do we know this?

9. What is the Alternate Exterior Angle that pairs with $\angle 19$?

10. What is the Alternate Interior Angle that pairs with $\angle 5$?

For Problem 11-14, use the given parallel lines and transversal to solve the problem.



$n \parallel o$

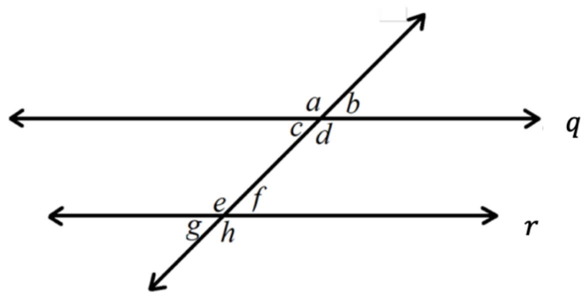
11. What is the measure of $\angle a$? Explain why.

12. Name all angles that are 40° and explain how you know.

13. What other angles are equal to $\angle a$? Explain why.

14. What is the sum of angles d, e, f, and g?

For Problem 15-18, use the given parallel lines and transversal to name all the angles.



$q \parallel r$

15. Name two pairs of Alternate Exterior Angles.

16. Name two pairs of Alternate Interior Angles.

17. Name four pairs of vertical angles.

18. Name four pairs of corresponding angles.

Section 8.7 Classifying Triangles and Quadrilaterals

Practice Problems 8.7

For Problem 1 and 2, solve the word problem given.

1. Why is it not possible to have more than one right angle or more than one obtuse angle in a triangle?

2. A regular polygon has all its sides congruent and all its angles congruent. What is another name for a regular triangle?

For Problem 3-5, fill in the blank(s) to solve the problem.

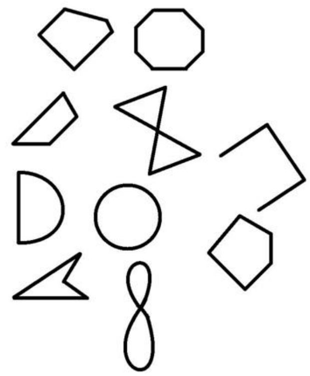
3. In a regular triangle, all the sides are congruent. Therefore, each of the angles measure __________ degrees.

4. A square is a special ____________________ because ____________________ have opposite sides parallel and equal and all four angles are 90°, but all sides may not be equal. If all sides of a ___________________ are equal, then it is a square.

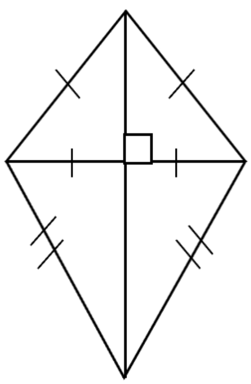
5. A square is a special ____________________ because a ____________________ has opposite sides parallel and all sides equal, and opposite angles equal, but all angles may not be 90°. If all angles of a rhombus are 90°, then it is a ____________________.

For Problem 6, use the given information and diagram to solve the problem.

6. A polygon is a closed figure made up of segments. Circle the polygons and put an X through the figures that are not polygons.



For Problem 7-10, use the given diagram to the left below to solve the problem.

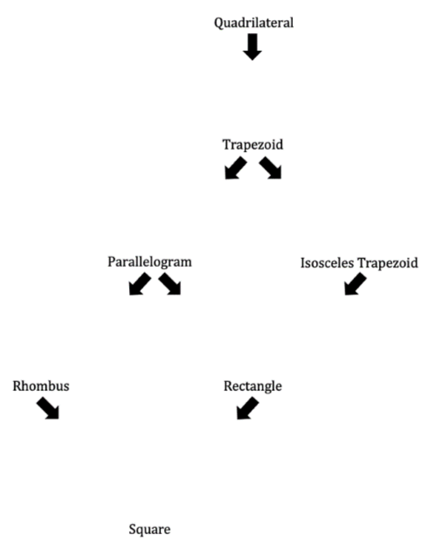


7. A kite has two pairs of equal sides, but the sides are not opposite each other, but ______________________ to each other.

8. If all the adjacent sides of a kite are equal, then the kite is a _________________________.

9. True or False: A square is also a kite.

10. Using the flowchart from the Lesson Notes, where would a kite be in the flowchart?

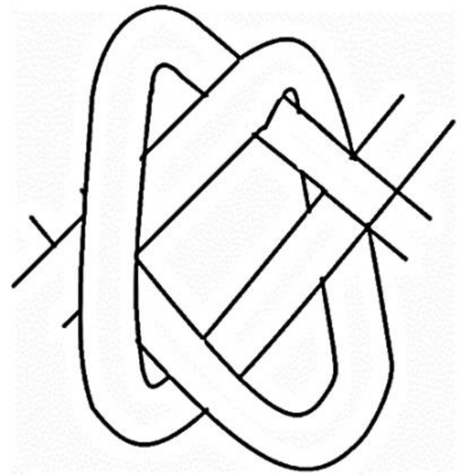


Section 8.8 Angle Sums of Polygons

Practice Problems 8.8

For Problem 1-5, follow Steps 1-5 to make a paper hexagon.

1. Cut out two long strips of paper that have equal width like you did in the Lesson Notes when you made the paper pentagon.

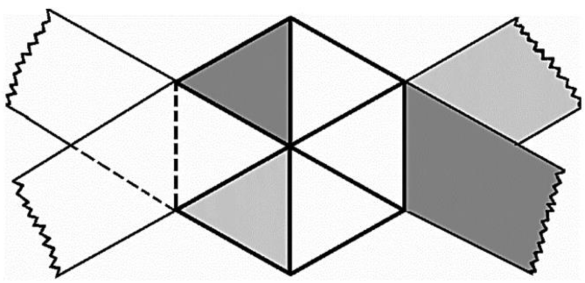


2. Tie a square knot as shown in the diagram to the right.

3. Tuck the ends of each strip into the loop of the other strip.

4. Pull tight by the ends and flatten the crease.

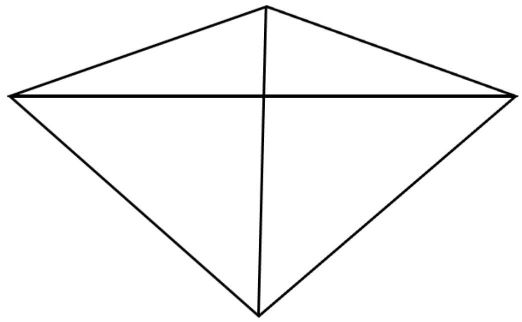
5. Cut off the lengths hanging out from the sides of the hexagon as shown in the diagram below.



For Problem 6, complete the chart.

Number of Sides of Polygon n	Number of Triangles in Polygon $n-2$	Sum of Interior Angles $180°(n-2)$	Degree of Each Interior Angle $\frac{180°(n-2)}{n}$
5	3	540°	108°
6			
7			
8			
9			
10			
11			
12			
N			

For Problem 7-10, use the given diagram of a kite to solve the problem.



7. A kite is made up of two isosceles triangles joined base to base. What does this say about opposite angles formed where the two triangles meet?

8. If the base angles of the top triangle are each 20° and the base angles of the bottom triangle are each 40°, what is the measure of each opposite angle in the kite?

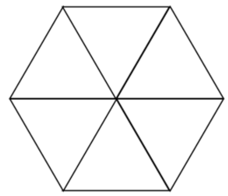
9. The diagonals of a kite connect opposite vertices. The kite is symmetrical about its main diagonal. Color this diagonal. The two diagonals meet at right angles. Mark the symbol for the right angle where they meet.

10. The main diagonal bisects the other diagonal. If the bisected diagonal is 10 units, mark the length on either side of where it intersects the main diagonal.

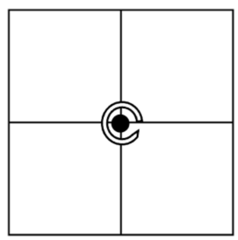
Section 8.9 Regular Tessellations and More

Practice Problems 8.9

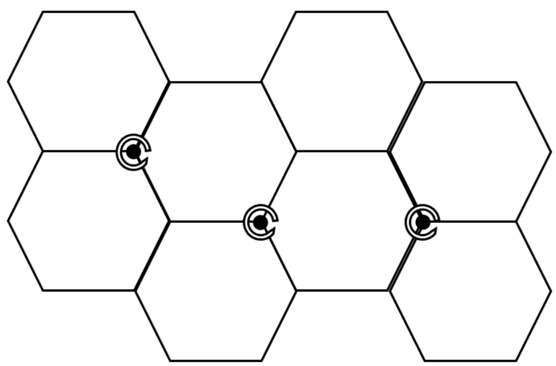
A regular tessellation uses one regular polygon to cover a plane. Each angle surrounding the vertex is congruent. In a tessellation, the polygon is named by the number of sides it has. The angles that surround a vertex must sum up to 360°.



A triangle has three sides, so it is called a "3." There are six triangles surrounding the vertex point in which all the triangles meet. Therefore, a triangle tessellation is a 3.3.3.3.3.3.



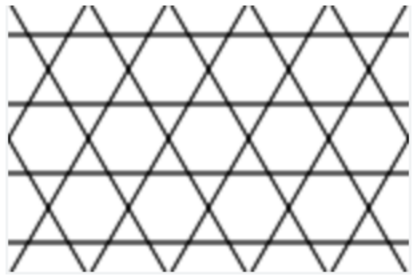
A square has four sides, so it is called a "4." There are four squares surrounding the vertex point in which all the squares meet. Therefore, a square tessellation is a 4.4.4.4.



A hexagon has six sides, so it is called a "6." There are three hexagons surrounding the vertex point in which all the hexagons meet. Therefore, a hexagon tessellation is a 6.6.6. It is not a 6.6.6.6.6.6. because only three six-sided hexagons surround each vertex, not six.

Part I:

A semi-regular tessellation is made up of two or more regular polygons surrounding a vertex in the same order at each vertex. The arrangement of polygons at each vertex is the same. A semi-regular tessellation is named in order beginning with the least number of sides first. For example, the diagram below shows a 3.6.3.6, not a 6.3.6.3.



1. There is another semi-regular tessellation that is a 3._____6._____. The angles of the triangle and hexagon are 60° and 120°, which sums up to 180°. How many degrees are left for the missing two angles? The missing polygons are the same. What are the missing shapes? Put the number representing the number of sides in the blanks above.

2. Another semi-regular tessellation has three regular polygons around a vertex and ends with a dodecagon (12 sides).

   a) If a dodecagon has an interior angle of 150°, how many degrees are left for the two interior angles of the other two polygons that surround that vertex?

   b) Two different regular polygons have single interior angles that give the sum from part a). Name these two polygons.

   c) Fill in the blanks for the name of the semi-regular tessellation: _____._____.12.

3. Another semi-regular tessellation has three regular polygons around a vertex and ends with two dodecagons (12 sides).

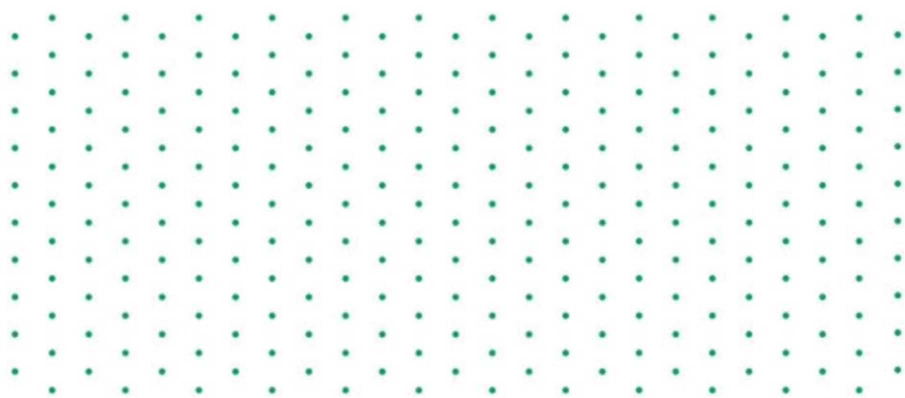
   a) If a dodecagon has an interior angle of 150°, how many degrees are left for the interior angle of the other regular polygon that surrounds that vertex?

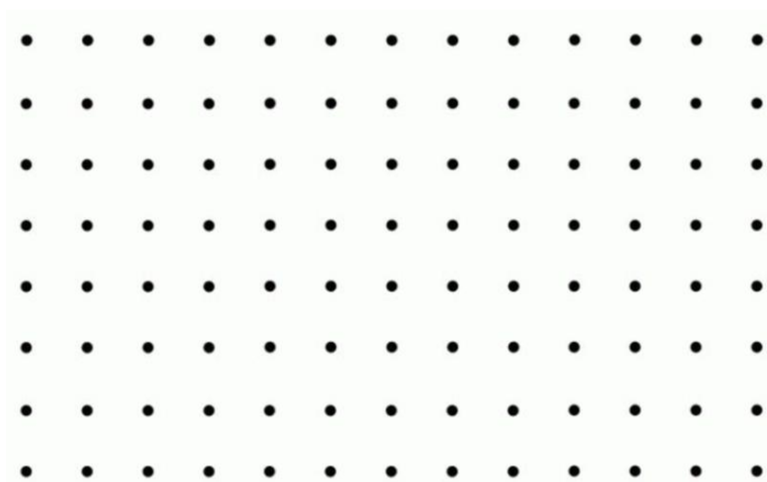
b) One regular polygon has an interior angle that gives the sum from part a). Name this polygon.

c) Fill in the blanks for the name of the semi-regular tessellation: _____.12.12.

You now know four of the eight semi-regular tessellations. For Problem 4-6, use the hints below to find the other four. (Use the isometric paper or the dot grid paper below to figure out the semi-regular tessellations.)

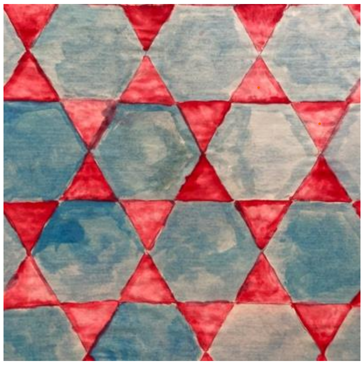
4. Two of them have three regular triangles as the polygons that surround a vertex.
5. One has four out of five polygons that are the same.
6. One has three polygons surrounding a vertex and uses octagons.



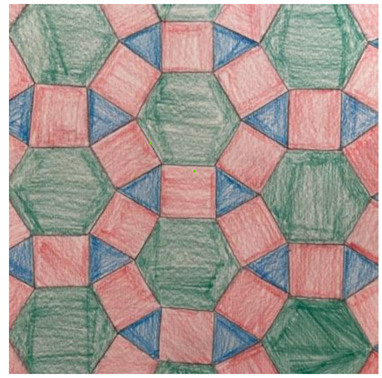


Below are pictures of former students' projects using watercolor and pastel. For Problem 7-10, name the tessellation shown.

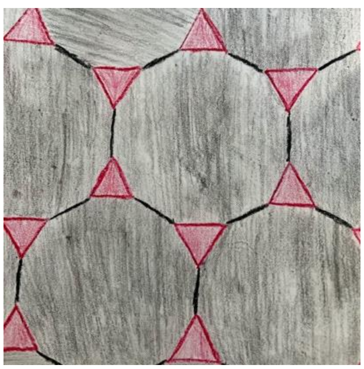
7.



8.



9.



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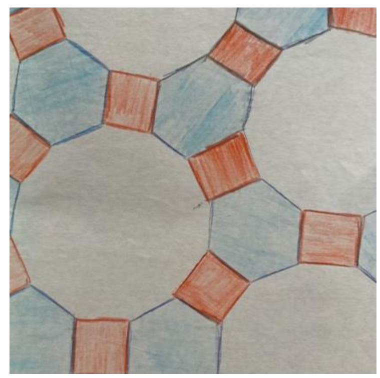
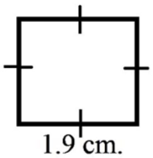


Section 8.10 Area of Polygons and Solids

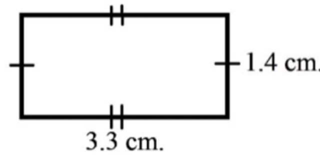
Practice Problems 8.10

For Problem 1-7, find the area of the shape given.

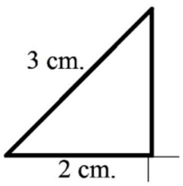
1.



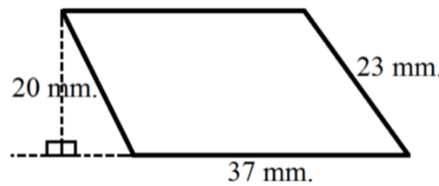
2.



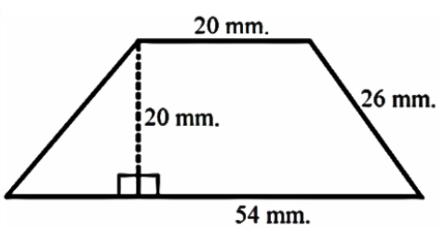
3.



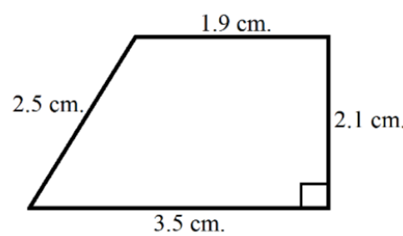
4.



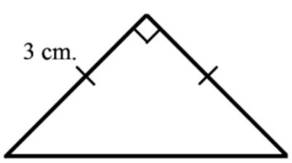
5.



6.



7.



8. A rhombus with side length 4 cm. and height 3.2 cm.

For Problem 9-15, use the given right triangular prism to solve the problem. The side lengths are measured in inches.

9. a) How many faces are there including bases?

b) Fill in the blanks: There are ____________ bases that are right triangles and ____________ faces that are rectangles.

c) Fill in the blanks: How many edges are legs of the right triangles? ____________ Theses edges are also the widths of the ____________________ faces.

10. What is the area of the bottom triangle?

11. What is the total area of the two bases?

12. Find the area of the rectangular face whose width is 6.2 inches, the larger leg of the triangle.

13. Find the area of the rectangular face whose width is 5 inches, the shorter leg of the triangle.

14. The last rectangular face in the back of the prism has a length of 7 inches and a width that is unknown. The unknown width is the hypotenuse of the right triangle of the base. How do we find the hypotenuse? Find the hypotenuse and use this width to find the area of the rectangle in back of the prism.

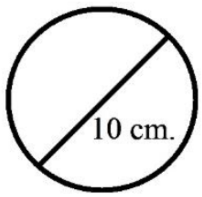
15. Add the areas of the two triangular bases and the three rectangular faces to find the total surface area of the right triangular prism.

Section 8.11 Area and Circumference of a Circle

Practice Problems 8.11

For Problem 1 and 2, find the area and circumference of the circle given.

1.



2.

For Problem 3-10, solve the word problem given.

3. Find the circumference of a circle whose radius is 7.07 inches.

4. Find the area of a circle whose diameter is 14 inches.

5. Find the formula for the circumference of a circle, $C = \pi d$, in terms of diameter and then in terms of π.

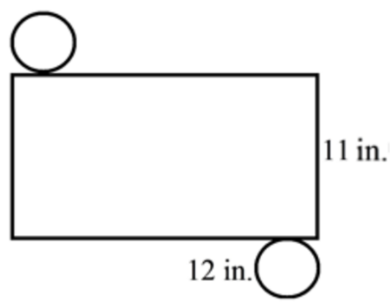
6. Find the area of a circle if the circumference of the circle is 22.6 inches.

7. Find the formula for the area of a circle in terms of r. Does the formula for r give you the radius in Problem 6?

8. Find the radius of a circle if you know the area is 16.4 square units.

9. Find the formula for the area of a circle in terms of π. Does this formula along with the area and radius from Problem 8 give you π?

10. A design engineer creates a template for a box manufacturer. What is the area of the cylindrical spaghetti box (shown below)? The circumference of the lid and the base is 12 inches.

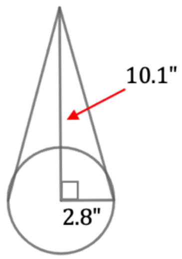


Section 8.12 Finding the Volume of Solids

Practice Problems 8.12

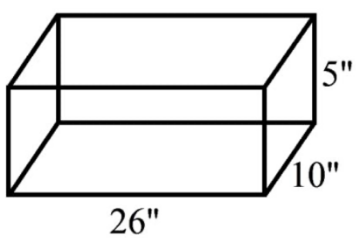
For Problem 1-10, solve the word problem given.

1. Find the volume of the cone.



2. Find the volume of a toy cone that is 5" high and whose base has a diameter of 1.4 inches.

3. A mathematics teacher orders nine mini-Rubik® cubes. They are $1" \times 1" \times 1"$. How many of these cubes will fit in the box shown below? Answer questions a)-e).



a) What shape is the base (B)?

b) What is the area formula you will use for the base?

c) What is the area of the base?

d) How many one-inch base layers will you stack up?

e) What is the volume of the box? (Use $V = Bh$ or $V = l \times w \times h$)

4. Find the volume of a box that has a length of 43", a width of 15", and a height of 6".

5. If a box has a volume of 2,520 cm^3, a length of 36 cm, and a height of 7 cm, what is the width of the box?

6. If a box has a volume of 13,860 mm^3, a length of 42 mm, and a width of 22 mm, what is the height of the box?

7. What polygon is a base for a rectangular prism and pyramid?

8. What is the formula for the area of the base of a rectangular prism and pyramid?

9. It will take three rectangular pyramids full of water to fill a rectangular prism with the same base and height. If the formula for a rectangular prism is $V = Bh$, fill in the blanks to find the formula for a rectangular pyramid:

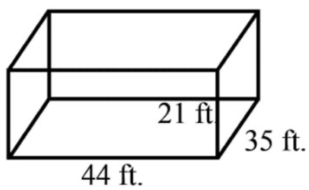
$$V = \underline{\hspace{2cm}} Bh$$

$$V = \underline{\hspace{2cm}} \times (\underline{\hspace{2cm}} \times \underline{\hspace{2cm}})h$$

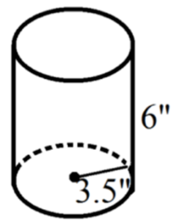
10. Find the volume of a rectangular pyramid with the same base and height as the box in Problem 3.

For Problem 11-14, find the volume and surface area of the shape given.

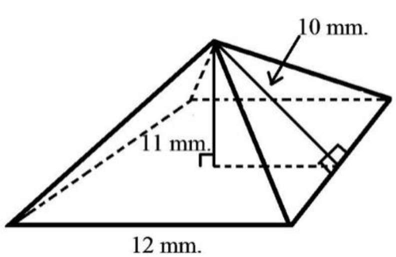
11.



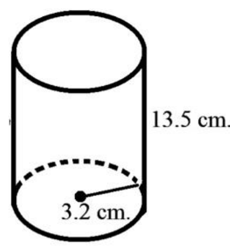
12.



13. Square Pyramid



14.



Section 8.13 Challenge Problems

Now that you have made some hexaflexagons on your own, here are some challenge problems to work through at your own pace.

1. The Race is On

There were three boys in a race: Brandon, Kevin, and Scott. Their fathers were Mr. Laughlin, Mr. Parks, and Mr. Sauerbeck, not necessarily in that order. Brandon broke his ankle at the start of the race. Mr. Sauerbeck's son wore blue and white shorts. Kevin had three trophies from previous races. Mr. Laughlin's son almost won. The winner had on red shorts. This was the first race that Mr. Park's son had ever run. Who won the race and what was his last name?

2. A Messy Desk

One of four girls wrote on their desk during lunch. Their statements are as follows:

Bertie: Deb did it.
Deb: Jay did it.
Fish: I didn't do it.
Jay: Bertie lied when she said Deb did it.

If only one of the statements is true, which one of the girls will be staying after school to clean desks (who wrote on the desk)?

3. Easy Answers

One of five students decided to borrow the teacher's answer key to do the homework assignment. Each student gave three statements when questioned:

Eloina: 1) I didn't take it. 2) I have never stolen anything in my life. 3) Glenn did it.
Slape: 4) I didn't take it. 5) My dad teaches Algebra and has the book 6) Ike knows who did it.
Ruschi: 7) I didn't take it. 8) I didn't know Ike before I enrolled here. 9) Glenn did it.
Glenn: 10) I'm not guilty. 11) Ike did it. 12) Eloina was lying when said I did it.
Ike: 13) I didn't take it. 14) Slape is the guilty party. 15) Ruschi can vouch for me because she has known me my whole life.

Each student gave two true statements and one false one. Assuming this is so, who took the answer key?

4. Chronic Truth-Tellers and Compulsive Liars

Lamont was at a convention of chronic truth-tellers and compulsive liars. He met a new friend, and she told him she overheard a man tell someone he was a liar in order to reveal his identity. Is Lamont's newfound friend a truth-teller or a liar? In other words, how long will Lamont's new friend be a true friend?

Assume the man is a truth-teller, what would he tell others?
Assume the man is a liar, what would he tell others?

5. Women at Work

In a restaurant, Gina West, Samantha Sedgebeer, and Ingrid Lambert are the hostess, waitress, and cashier, but not necessarily in that order. There are three customers in the restaurant: Mrs. West, Mrs. Sedgebeer, and Mrs. Lambert.

1) Mrs. Sedgebeer lives in Columbus.
2) The waitress lives exactly halfway between Cleveland and Columbus.
3) Mrs. Lambert earns exactly $20,000.00 a year.
4) The waitress' nearest neighbor, one of the customers, earns exactly three times as much as the other waitress.
5) Gina beats the hostess at cards.
6) The customer whose name is the same as the waitress lives in Cleveland.

Based on the facts, what is each girls' job?

6. The Flash Drive Deal

Dionne has packs of flash drives and sells them for $99.00 each. She gains 10% on one and loses 10% on the other. What is her profit or loss?

Section 8.14 Art Project: Escher Tessellations

The basic design element for a tiling that you created using technology in Section 8.1 is called an irregular tessellation. For this art project, you will be manually making a design using that process.

First, we will begin with a bit of history about M.C. Escher, the artist who is well known for his work in mathematics as well, namely Escher tessellations. During his lifetime, he created over 2,000 sketches and drawings, and 448 woodcuts, engravings, and lithographs.

Born Maurits Cornelis Escher, our man was a Dutch artist who lived from 1898 to 1972. Although he illustrated books and designed tapestries and murals, his principal work was as a printmaker. Escher would have never been a printmaker had not his teacher at Haarlem convinced him to switch over from architecture.

Later in his life, after getting married and starting a family, Escher found himself in Rome where he often drew countryside scenes. However, World War II would force he and his family to move across Europe and eventually back to his place of birth, Holland.

What started as drawing the Italian countryside turned into Escher drawing from multiple perspectives, looking up and down at the same time or going up and down stairs at the same time. This made him the master of impossible spaces and these “metamorphosis” designs became games involving complex architectural mazes.

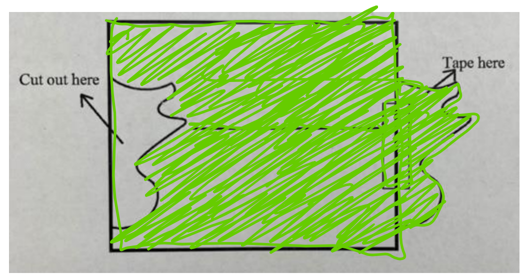
The National Gallery of Art in Washington D.C. houses impressions of 330 of the artist’s 450 prints, which is the largest collection of his work outside of Holland. Perhaps someday you will be able to visit, if not (for now), you can see M.C. Escher’s work on the official Escher website.

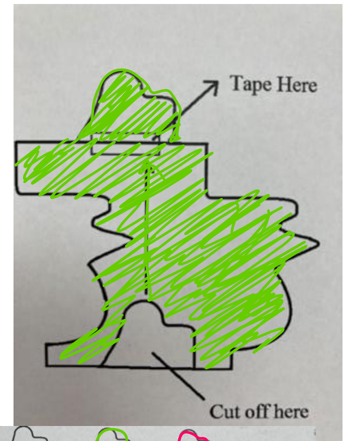
Your work for this section is to create your own irregular polygon and translate it across your page. See if your design looks like an architectural piece or maze. Cover as much of your paper as you can and design a picture when you are finished!

The instructions below will give you some idea for the irregular tessellation you will create using a basic design element. Once you try the example below, create some of your own. If you create a colorful masterpiece, you may want to pass it on as a gift!

1. Start with a 2" × 2" square on a white piece of paper.

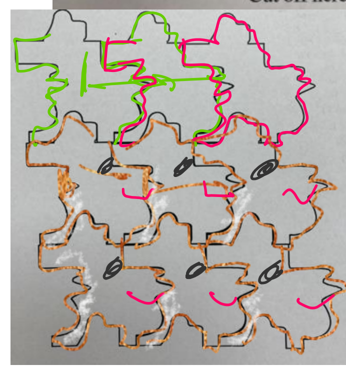
2. Cut out a piece of paper from the left side of your square and slide it straight across to the right side of your square and tape the edges together.

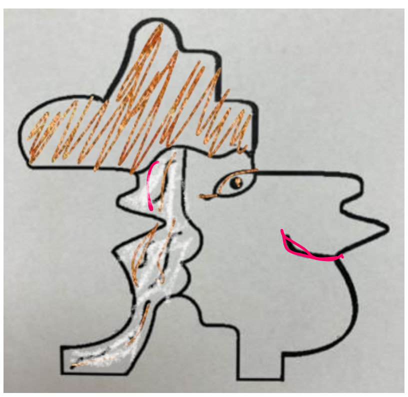




3. Cut out a piece of paper from the bottom of your square and slide it straight to the top of your square and tape the edges together.

4. You have created the irregular shape that you will use to tessellate the plane. Slide this shape across the page and trace it so each shape fits together like pieces of a puzzle. Try to cover your paper in your tessellation.

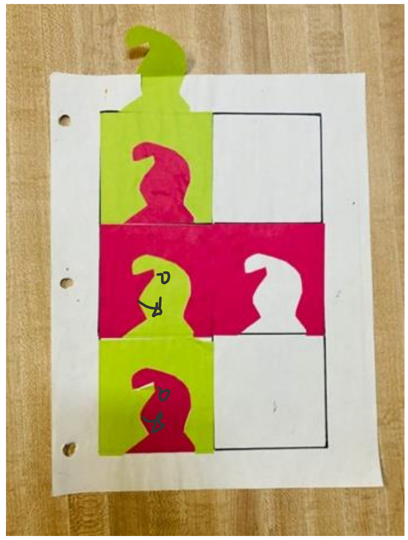




5. Once you have completely covered (tessellated) the paper (plane) with the design, decide what the basic design element looks like.

I think this one looks like one of the founding fathers of the United States of America who served in the Revolutionary War.

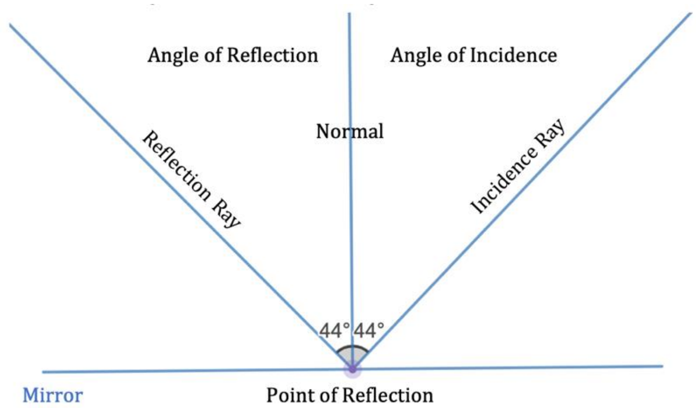
6. Create your own irregular tessellations. You may want to make prints out of them or use pastels to color them.



Then you can give them as gifts!

Section 8.15 Games: Pool Tables and Putt-Putt Golf Courses

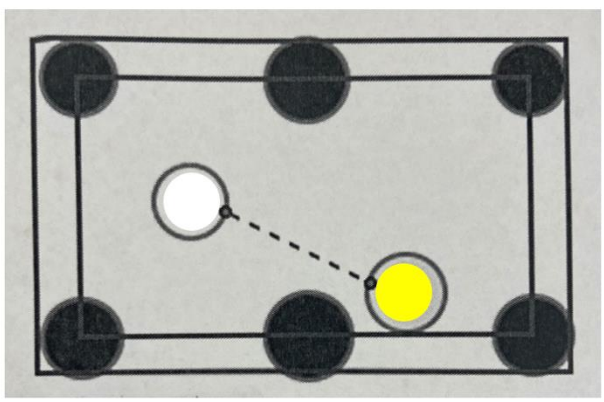
The diagram below shows how light bounces off a flat surface.



When a ray of light hits a mirror, it bounces off at the Point of Reflection. The Incoming Ray is called the "Incidence Ray." The Outgoing Ray is called the "Reflected Ray." The perpendicular line to the Point of Reflection is called the "Normal." The Angle of Incidence (Incoming Angle) formed between the Incidence Ray and the Normal is equal to the Angle of Reflection (Outgoing Angle) formed between the Reflection Ray and the Normal.

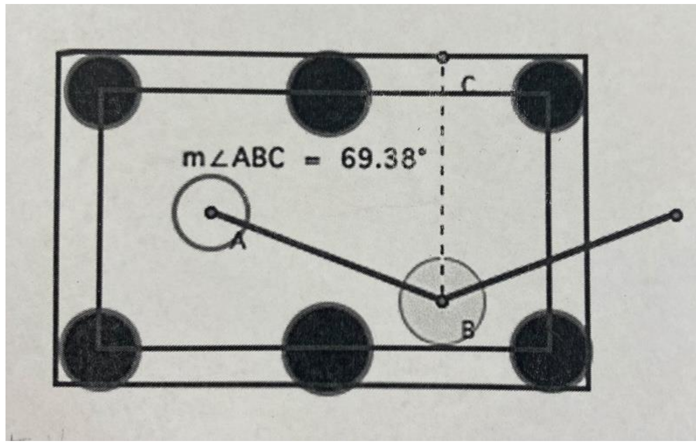
This information can help you know where to hit a ball if you are playing a game of pool or putt-putt golf.

Look at the pool table below. If the cue ball, which is white, is hit with a pool stick and hits the yellow ball, will the yellow ball bounce off the side of the table and go into the upper right-hand pocket?

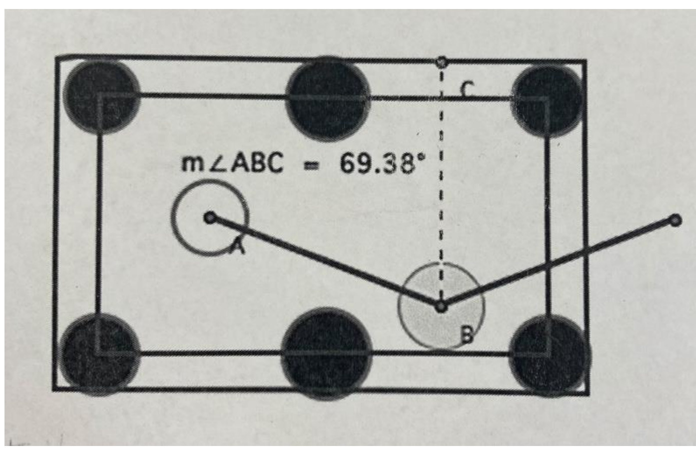


If we construct the Normal from the yellow ball perpendicular to the upper side of the pool table and measure the Angle of Incidence from the white to Normal that intersects the yellow ball, we see it is almost 70°.

If we measure 70° from the Normal at the yellow ball to the right side of the ball, we will have the angle that reflects over the Normal and the ray will intersect the right side of the pool table. This will not work. We can see below that the white ball will hit the yellow ball into the right side of the table, not the upper right-hand pocket.



Part I:
Follow the steps to find where the white ball must be placed on the pool table to hit the yellow ball into the upper right-hand corner.



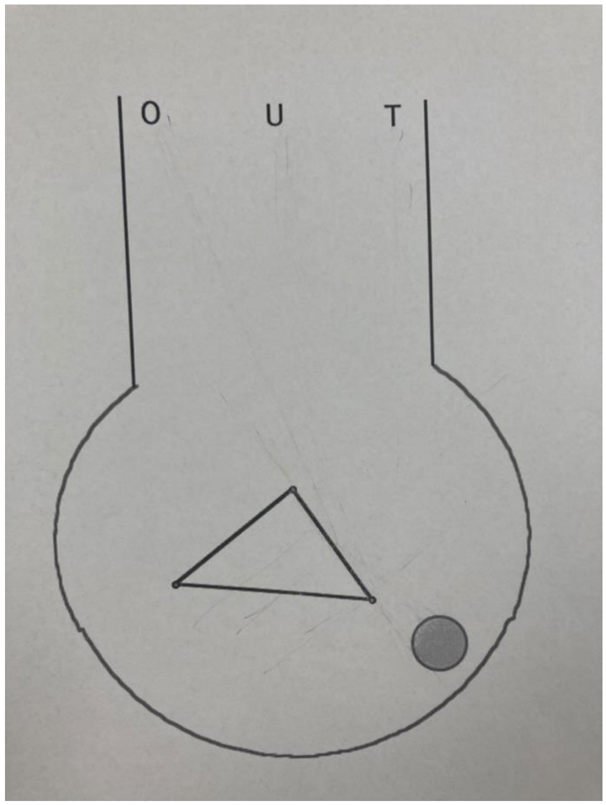
Step 1: Draw a line from the middle of the upper right-hand pocket to the yellow ball. Measure the angle to the Normal. What is the Outgoing Angle (Angle of Reflection)?

Step 2: What other angle is equal to the angle of reflection? Draw this angle from the normal to the left side of the yellow ball.

Step 3: Place the white ball where the ray of the Incoming Angle, or Angle of Incidence, crosses the top of the pool table.

Part II:

The diagram below shows the final hole in a putt-putt course. When the ball goes into the hole, you cannot retrieve it; the game is over. You would like to bank the ball off the triangular rock in the middle and into the hole to make a hole-in-one and win the championship. Where will you place the ball, on the O, the U, or the T?



What are the measures of the incoming and outgoing angles that allow you to make a hole-in-one?