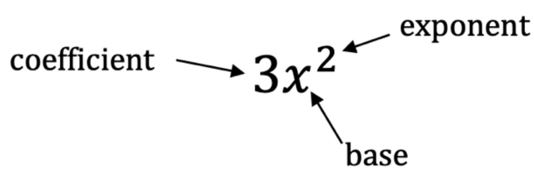
Module 6 Algebraic Reasoning

Section 6.1 Simplifying and Evaluating Expressions

Looking Back 6.1

We have previously learned that simplifying expressions means to combine like terms. Like terms have similar bases.

Suppose the terms are all numerical. In that case, they could be combined with one number. We would simply add, subtract, multiply, or divide the constants, which is what the mathematical operation requires. However, when using variables, we can only add or subtract variables that are alike (have the same base) and have the same exponent. The coefficient, which is the number in front of the variable, gets added or subtracted.



- To simplify expressions, do everything that uses grouping symbols first, such as exponents and parenthesis.
- Simplify any fractions by combining like terms in the numerator and denominator.
- Add or subtract monomial terms that are alike. These terms are separated by addition and subtraction signs.

Looking Ahead 6.1

Example 1: Simplify the expressions below and combine like terms.

a) $3x^2 + 2x^2 + x$

b) $-2x + 3y + 4x$

c) $2^3 + (8 - 4)7$

Remember, expressions do not have equal signs.

Example 2: Simplify the expressions below.

a) $\frac{2^2+8}{2(1+5)}$

b) $\frac{2x-5x}{6y-3y}$

c) $3(2 - 6) + 3(2^2)$

To evaluate an expression means to solve the expression. When you know the value of the variable for an expression, substitute it in for the variable and solve the problem. You will get a numerical answer.

Example 3:	Evaluate $-6x + \frac{1}{4}y$ when $x = -4$ and $y = 8$.

Example 4:	Evaluate $3x^2 - 2y^2$ when $x = -1$ and $y = -3$.

Rules for Integer Operations

To add integers with the same signs: Add the numbers and keep the sign the same.

$$A + B = +(A + B) \quad -A + -B = -(A + B)$$

To add integers with different signs: Subtract the numbers and keep the sign of the number whose absolute value is greater.

$$A + (-B) = A - B$$

To subtract integers: Change the operation to addition and change the sign of the following integer to its opposite. Then use the rules for addition to complete the operation.

$$A - B = A + (-B)$$

To multiply or divide two integers: Multiply or divide the numbers. Use the following rules to determine the sign of the product or quotient: If the signs of the two integers are the same, the product or quotient is positive. If the signs of the two integers are opposites, the product or quotient is negative.

$$(A)(B) = (-A)(-B) = AB \quad -(A)(B) = A(-B) = -AB$$

$$\frac{-A}{-B} = \frac{A}{B} \qquad \frac{-A}{B} = \frac{A}{-B} = -(\frac{A}{B})$$

To multiply more than two integers: Multiply the numbers and then count the number of negative signs. If you count an odd number of negative signs, the solution is negative. If you count an even number of negative signs, the solution is positive.

$$(-A)(-B)(-C) = -(ABC) \quad (A)(-B)(-C) = +(ABC)$$

Section 6.2 Algebraic Properties

Looking Back 6.2

Previously, in General Math, we learned many properties of numbers that made working with variables and constants easier. Now, let us review these properties as they will be needed in Algebra, especially when solving equations.

Review of Properties from General Mathematics

a) Reflexive Property: $a =$ ___________

b) Identity Property for Addition: $a + 0 =$ ___________

c) Inverse Property for Addition: $a + ($___________$) = 0$

d) Multiplication Property of Zero: $a \cdot 0 =$ ___________

e) Multiplication Property of -1: $a \cdot (-1) =$ ___________

f) Inverse Property of Multiplication: $a \cdot \frac{1}{a} = 1$

The Inverse Property of Multiplication is also called the Property of Reciprocals.

g) Transitive Property of Equality: If $a = b$ and $b = c$, then ___________ $=$ ___________.

h) Identity Property for Multiplication: $a \cdot 1 =$ ___________

i) $a + b = b + a$: _______________________ _______________________ of _____________________

j) $a + (b + c) = (a + b) + c$: ______________________ ______________________ of

____

k) $a \cdot b = b \cdot a$: ______________________ ______________________ of ______________________

l) $a(bc) = (ab)c$: ______________________ ______________________ of ______________________

m) $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$: ____________________ ____________________

n) $ab + ac = a(b + c)$ and $ab - ac = a(b - c)$ undoing the ___________________ ___________________

Looking Ahead 5.2

Special Properties Concerning Zero

A) If $A \cdot B = 0$, then either $A =$ ___________ or $B =$ ___________

B) $\frac{A}{0}$ or $\frac{B}{0}$ does not exist; it is undefined

C) $\frac{A}{0}$ = undefined; however, your calculator will say: “UNDEF”

D) $\frac{0}{A}$ or $\frac{0}{B} =$ _______________

E) $\frac{0}{0}$ is _________________________

Example 1: Fill in the blank and name the property.

a) $5 \cdot 1 =$ ____________

b) $-3 + 0 =$ ____________

c) $-2(x + 4) = -2x - 8$

Example 2: Fill in the blanks given the property.

a) Transitive Property of Equality

If $0.5 = \frac{1}{2}$ and $\frac{1}{2} = 50\%$, then ____________ = ____________

b) Commutative Property of Multiplication

$(3 \cdot 2) \cdot 4 = 4 \cdot ($____________ $\cdot$ ____________$)$

c) Property of Reciprocals

$-4 \cdot ($____________$) = 1$

Example 3: Tell whether the statement is true or false.

a) $a \cdot 0 = a$

b) $-3 + 3 = 1$

c) $\frac{1}{-2} \cdot -2 = 1$

Section 6.3 Solving One Variable Equations

Looking Back 6.3

Last year we balanced scales to solve equations. We also learned the Golden Rule of Algebra, which states that whatever one does to one side of an equation one must do to the other side of the equation. This is to keep the equation balanced. God says in Leviticus 19:36: "You shall have just balances, just weights, a just ephah (a dry measurement in Ancient Hebrew culture), and a just hin (a liquid measurement in Ancient Hebrew culture). I am the Lord your God, who brought you out from the land of Egypt." Proverbs 11:1 tells us: "A false balance is an abomination to the Lord, but a just weight is His delight." God wants us to be fair. When you go to the grocery, the clerk weighs your fruit because you pay for fruit by the pound. If the clerk adds a pound in order to charge more, they are tipping the scales; this is unfair.

God has designed our bodies and our world to keep things balanced. When we study air, we will learn among how important it is that the amount of oxygen in the air is balanced with other elements. If there is too much or too little it would cause our earth to be unhabitable (unable to sustain human life). We will also learn why God designed our bodies to sweat when too hot: as perspiration evaporates, we are cooled down. Balance is important in both math and science.

Looking Ahead 6.3

Solving Equations

In science, you will learn the atmospheric pressure at sea level is about 14.1 pounds per square inch (psi). These can be represented by the following table:

Inches (n)	**Atmospheric Pressure (in $lbs.$)**
1	14.1
2	28.2
3	32.3
…	
n	?

Example 1: If the atmospheric pressure exerted on your shoulder is 176 lbs., what is th area (length $\times$ width in inches) of your shoulder?

Remember that $\frac{d}{3}$ is the same as $\frac{1d}{3}$ and $\frac{1}{3}d$ and $-\frac{3}{4}m$ is the same as $-\frac{3m}{4}$. This will help when solving equations involving fractions.

Example 2: Solve for the variable in each of the equations below. Check your solutions.

a) $4n + 18 = -10$

b) $2x - 4 = -16$

c) $\frac{1}{4}t = 10$

d) $8 = 2x^2$

Example 3: Solve for the variable in $\frac{3}{4}t = 30$. Check your solutions.

Section 6.4 Solving Equations with Variables on Both Sides

Looking Back 6.4

Last year we solved equations with one variable. The idea was to isolate the variable to find the solutions that would work for that specific equation. The balance scale represented the equation with the equal sign being the fulcrum. We would undo operations by moving all of the constants to one side of the equation and dividing by the coefficient of the variable for any coefficient greater than one and multiplying by the reciprocal for coefficients between 0 and 1. Remember that $1x$ is the same as x. We practiced these types of problems in the previous section.

Looking Ahead 6.4

Review of Rules of Exponents

- If the bases are the same and the exponents are the same, add or subtract the coefficients.

$$2x^2 + 3x^2 = \_\_\_\_\_\_\_\_\_\_$$

These are called like terms and when you combine them you keep the ____________________ and the

____________________ the same.

- If the bases or exponents are different, the monomial terms ____________________ be added or subtracted. These are not like terms.

$$2m + m^2$$

$$-3n - 4n^2$$

- To multiply terms with like bases, ____________________ the exponents and ____________________ the coefficients.

$$(3p)(-4p^2)$$

- To divide terms with like bases, ____________________ the exponents and ____________________the coefficients.

$$\frac{4q^3}{12q^3}$$

- If the larger exponent of two like bases is in the denominator, when simplified it will remain in the denominator.

$$\frac{12q^2}{4q^3}$$

- Negative exponents in the denominator are not consider simplified.

$$\frac{1}{q^{-1}}$$

- Anything to the zero power is 1: $x^0 = 1, y^0 = 1, 10^0 = 1$. That comes from the rule above.

$$\frac{q^1}{q^1} = \frac{q^{1-1}}{1} = \frac{q^0}{1} = q^0$$

$$\frac{q^1}{q^1} = 1$$

$$\therefore q^0 = 1$$

- Algebraically, the transitive property states: If $a = b$ and $b = c$, then ________ = ________; or, if $a = b$ and $a = c$, then ________ = ________. Therefore, by the transitive property, if $\frac{x^2}{x^2} = x^0$ and $\frac{x^2}{x^2} = 1$, then $x^0 = 1$.
- If two things are equal to the same thing, they are equal to each other.

Example 1:	Solve for m in the equation $3m - 4 = 2m - 8$ and check your solution.

<u>Review of Square Roots</u>

- When taking a square root, there are __________________ solutions.
- $\sqrt{2}$ is an __________________ solution
- 1.4 is the decimal ____________________ for $\sqrt{2}$
- Squares and square roots are ____________________ of one another.
- The square root of anything squared is itself.

$$(\sqrt[2]{m})^2 = m$$

$$(-\sqrt[2]{2})^2 = 2$$

The $(-\sqrt[2]{2})^2$ could also be written $(-1 \cdot \sqrt[2]{2})^2$ so the -1 gets squared also since it is in parenthesis.

Example 2:	Solve for t in the equation $-4t^2 + 20 = -2 + 7t^2$ and check your solution.

Remember that the cube root of anything cubed is also itself; $\sqrt[3]{n^3} = n$ and $n^3 = 64$ is $\sqrt[3]{n^3} = \sqrt[3]{64}$ or $n = 4$. There is only one solution, not two solutions; there is also one solution for $\sqrt[3]{-64} = -4$ because $-4 \cdot -4 \cdot -4 = -64$.

Section 6.5 Reasoning with Ratios

Looking Back 6.5

A ratio is a comparison of two or more things. A fraction is a ratio of two things that may compare parts taken to the parts in the whole.

$$\frac{2}{3}\ \frac{\text{parts taken}}{\text{parts in the whole}}$$

Two-thirds is the same as 2 divided by 3 ($2:3$; 2 out of 3; 2 as compared to 3). Therefore, the part taken is less than 1 whole in the fraction above. The decimal approximation for $\frac{2}{3}$ is 0.67. When we divide 2 by 3 we get $0.\overline{6}$. Because $2 < 3$, the fraction $\frac{2}{3}$ is less than 1; the decimal $0.\overline{6}$ is between 0 and 1.

$$\frac{3}{3}\ \frac{\text{parts taken}}{\text{parts in the whole}} \text{ or } \frac{2}{2}\ \frac{\text{parts taken}}{\text{parts in the whole}}$$

In this example, the parts taken are equal to the parts in the whole. Therefore, $\frac{3}{3} = 1$ and $\frac{2}{2} = 1$. When we divide 3 by 3 or 2 by 2, we get 1.

$$\frac{3}{2}\ \frac{\text{parts taken}}{\text{parts in the whole}}$$

Often in mathematics, fractions are converted to a decimal for the purpose of calculation. The mathematics operation for a ratio of two things is division. In the above example, 3 divided by 2 is the same as $1\frac{1}{2}$ or 1.5; therefore, $\frac{3}{2} > 1$.

All compounds are made from mixing elements. In science, ammonia has three hydrogen atoms for every nitrogen atom. That is a 3 to 1 ratio of hydrogen to nitrogen atoms. Writing it as "$3:1$" is preferable to writing it as a fraction in this case because $\frac{3}{1}$ can be written as "3" and then the comparison of two things is not visibly present. All ammonia, under all physical conditions, has a $3:1$ ratio.

If a substance has a different ratio of hydrogen to nitrogen atoms, then it is not ammonia. Hydrazoic acid has hydrogen and nitrogen atoms as well, but with a ratio of $1:3$, or 1 hydrogen atom to 3 nitrogen atoms. Order is important here. The label that comes first is the number that is written first.

Looking Ahead 6.5

In science, we learn the composition of dry air is 78% nitrogen, 21% oxygen, and the rest is other gases.

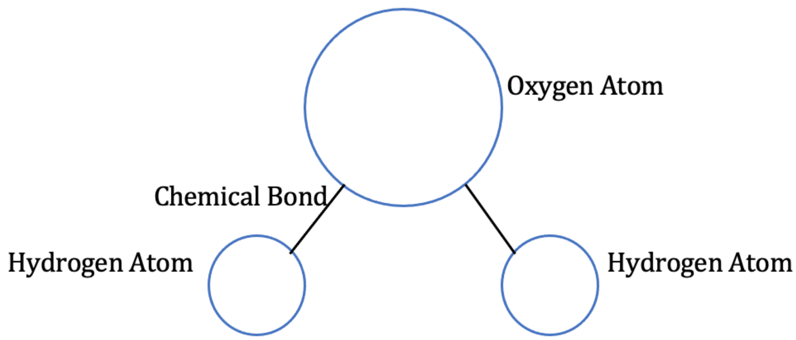
Example 1: The carbon dioxide in dry air makes up about 0.03 of the air. Carbon dioxide is a compound with one atom of carbon attached by chemical bonds to two atoms of oxygen. A subscript next to each element indicates the number of atoms that make up the element. Carbon is represented by C and oxygen is represented by O. The chemical formula for carbon is $\mathrm{CO_2}$. Answer the questions below.

a) What is the ratio of carbon to oxygen in carbon dioxide?

b) What is the ratio of oxygen to carbon in carbon dioxide?

Example 2: Ammonia has an atomic ratio of $1:3$ for nitrogen to hydrogen. If N represents nitrogen and H represents hydrogen, what is the chemical formula for ammonia?

Example 3: Water is a molecule that has hydrogen atoms and oxygen atoms linked together. The diagram below demonstrates a water molecule.



What is the chemical formula for water?

Section 6.6 Unit Rates and Conversions

Looking Back 6.6

A ratio is a comparison of two numbers that can be divided and that can be written x to y;x: y; $\frac{x}{y}$. If $y = 1$ then the ratio is a unit rate. For example, if one can of corn is \$0.99, then the unit rate is given by the ratio $\frac{\$0.99}{1 \text{ can}}$. If a driver's rate is 55 miles per hour, then the unit rate is given by the ratio $\frac{55 \text{ miles}}{1 \text{ hour}}$. "Per" means one unit of whatever the measurement may be. If a can of corn is \$0.99 for 12 ounces, then to find the unit rate (cost) for one ounce, set up a ratio and divide:

$$\frac{\$0.99}{12 \text{ ozs.}} \approx \$0.08 \text{ or } \frac{\$0.99}{12 \text{ ozs.}} = \frac{\$0.08}{1 \text{ oz.}}$$

Therefore, 1 ounce of corn is approximately \$0.08. That is a ratio written in the simplest form. When two ratios are equal to one another, that is called a proportion. Do not worry about proportions now: they will be reviewed later in this module.

Sometimes, unit rates must be converted from one measurement to another. Just as like terms must have a common base to be combined, the ratios must have like labels in order to manipulate them mathematically using the four operations.

Looking Ahead 6.6

We will use metric units to review conversions from one measurement to another. Let us review metric conversions first.

Units of Measure	=	**Measurement**	**Grams, Liters, and Meters**
Kilo	=	1,000	kg, kL, km
Hecto	=	100	hg, hL, hm
Deca	=	10	dag, daL, dam
Meter	=	1	g, L, m
Deci	=	0.1	dg, dL, dm
Centi	=	0.01	cg, cL, cm
Milli	=	0.001	mg, mL, mm

Divide numbers by 10 if you are getting smaller (moving down from a unit)

Multiply numbers by 10 if you are getting bigger (moving down from a unit)

Example 1: The mass of a rock is 14.351 kg. What is the mass of the rock in grams?

Example 2: The length of a tabletop is 96.012 cm. How many inches is that? Use the conversion factor $2.54 \text{ cm} = 1 \text{ in}$. This can be written as the unit rate $\frac{2.54 \text{ cm}}{1 \text{ in}}$ or the ratio $\frac{1 \text{ in}}{2.54 \text{ cm}}$.

Example 3: Now take your solution from Example 2 and convert it form inches to centimeters.

Example 4: A container of lemonade is 3 Liters . How many centiliters is that?

Example 5: How many decagrams is 200 milligrams?

Example 6: A bowl has 100 mg of marbles that will be poured into plant jars which hold 5 cg. of marbles. How many plant jars can be filled with the marbles?

Section 6.7 Properties of Proportions

Looking Back 6.7

A proportion is an equation in which two ratios are equal. The unit rate of corn or price per ounce of corn is the price of one can of corn divided by the total ounces of corn in one can, $\frac{\$0.99}{12 \text{ ozs}}$. The ratio yields a unit rate that can also be written as a ratio, $\$0.08 = \frac{\$0.08}{1 \text{ oz.}}$. The ratio after division is the same as the original ratio but the ratio after division is in simplest terms, $\frac{\$0.99}{12 \text{ ozs}} = \frac{\$0.08}{1 \text{ oz}}$. This equation is a proportion. Therefore, when one buys a 12-ounce can of corn for $\$0.99$, they are paying $\$0.08$ per ounce. This is the same as paying $\$0.16$ for 2 ounces or $\$0.24$ for 3 ounces of corn. There are an infinite number of equivalent fractions or ratios that represent the same thing:

$$\frac{\$0.99}{12 \text{ ozs.}} = \frac{\$0.24}{3 \text{ ozs.}} = \frac{\$0.16}{2 \text{ ozs.}} = \frac{\$0.08}{1 \text{ oz.}}$$

Any two equivalent ratios set equal to each other form a proportion.

$$\frac{\$0.99}{12 \text{ ozs.}} = \frac{\$0.24}{3 \text{ ozs.}}$$

(This is one proportion.)

$$\frac{\$0.16}{2 \text{ ozs.}} = \frac{\$0.08}{1 \text{ oz.}}$$

(This is another proportion.)

Looking Ahead 6.7

There is a scale factor that can be used to rename one proportion and change it into an equivalent proportion. In General Math, we used multiple boards to find equivalent fractions. With fractions, we can scale up or scale down by multiplying or dividing by a clever form of 1. Multiplying or dividing by 1 does not change a ratio, it simply renames it.

If two ratios are equivalent, they are a proportion. The easy way to verify whether or not two ratios are proportions is to see if their cross-products are equal. If the cross-products are equal the two ratios are a proportion.

In General Math, we learned that in the proportion, we have means and extremes. In $\frac{a}{b} = \frac{c}{d}$ the variables a and d are called the extremes and b and c are called the means. This proportion reads as: “a is to b as c is to d.” It could be written as: “$a\!:\!b = c\!:\!d$”

When we use the cross-products formula, we get $ad = bc$ in which $b \neq 0$ and $d \neq 0$. The reason this works is due to the Multiplication Property of Equality.

$$\frac{a}{b} = \frac{c}{d}$$

$d \cdot \frac{a}{b} = \frac{c}{d^1} \cdot d^1$ (Multiply both sides by d to eliminate the denominator on the right side.)

$$\frac{da}{b} = c$$

$b^1 \cdot \frac{da}{b^1} = c \cdot b$ (Multiply both sides by b to eliminate the denominator on the left side.)

$$da = cb$$

$ad = bc$ (Commutative Property of Multiplication)

The Multiplication Property of Equality tells us we may multiply one side of an equation by a constant or a variable as long as we multiply both sides by the same thing.

A common denominator is $b \cdot d$ because b is one denominator and d is the other. Multiplying the two results in a product bd in which both b and d are factors. Therefore, both denominators could be eliminated in one step by multiplying both sides of the equation by a common denominator.

$\frac{a}{b} = \frac{c}{d}$

$b_1 d \cdot \frac{a}{b_1} = \frac{c}{d_1} \cdot b d_1$ (Multiplication Property of Equality)

$da = cb$ (Division of Common Factors)

$ad = bc$ (Commutative Property of Multiplication)

In the above proportion ad and bc are called cross-products.

Example 1: Use the Multiplication Property of Equality to solve for m in the proportion below.

$$\frac{2}{3} = \frac{m}{9}$$

Example 2: If $m = 6$, then $\frac{2}{3} = \frac{6}{9}$. Use the cross-product property to show that the solution in Example 1 is true.

$$\frac{2}{3} = \frac{6}{9}$$

Example 3: Use the cross-product property, which states that the means of a proportion equals the extremes of a proportion, to solve for n below.

$$\frac{n}{5} = \frac{4}{7}$$

We could find the scale factor used to change 5 to 7 and then use it to solve for n: $\frac{n}{5} = \frac{4}{7}$. Because $7 \div 5 = \frac{7}{5}$, then

$5 \cdot \frac{7}{5} = 7 \therefore n \cdot \frac{7}{5} = 4$ $\quad \frac{7}{5}n = 4$ and $n = \frac{20}{7}$

Section 6.8 Reasoning Mathematically with Proportions

Looking Back 6.8

There are many strategies used to solve problems in mathematics, several of which were used in Section 2 of this module:

- Drawing a diagram or picture
- Making a chart or table
- Working backwards or using reverse thinking
- Using the process of elimination
- Making it a simpler problem first

A problem-solving strategy used in the previous few sections of this module is to set up a proportion to solve a problem. Proportions and the cross-product property are two of the most used strategies in algebra and all mathematics to solve pricing problems, unit/rate conversion problems, missing parts of similar figures problems, and percent and rate problems. Some of these strategies will be further investigated in the next few sections. It makes sense to devote some time to this one strategy for solving mathematics problems, reasoning mathematically with proportions, because it is so widely used and has multiple applications. It is easier to find unknowns when they can be compared to something known.

When converting from one unit of measure to another, the unit of measurement must be known; for example, $1 \text{ kilogram} = 1{,}000 \text{ grams}$. Once the "unit" measured is known, the smaller and larger measures can be compared and converted.

Looking Ahead 6.8

Example 1: In Section 5 of this module, the chemical formula for methane was given as CH_4. How many of each atom are in one molecule of methane?

1 carbon and 4 hydrogen

How many atoms of hydrogen must combine with 240 atoms of carbon to form methane molecules?

Example 2: Howard earns $\$65{,}000$ annually (per year) at his place of employment. He pays 28% to the government for local, state, and federal taxes.

a) How much does Howard pay in taxes?

After studying these scriptures: Leviticus 27:30; Numbers 18:20-32; Deuteronomy 12:17 and 14:22-23, concerning tithing a tenth to priests and such, Howard decides to tithe 23% of his annual salary to pastors, ministry work, and worldwide missions.

b) How much money does Howard give back to God each year?

Howard totals the bills and expenses needed to provide for his family and realizes that 41% of his income will meet this need.

c) How much money does Howard use to pay his annual bills?

Howard puts the rest of his money in the family savings account.

d) How much money does Howard put in the family savings account?

Howard and his family are big givers. Malachi 3:10 reads: *" 'Bring the whole* ____________________ *into the* storehouse, *so that there may be food in My house, and* _______________ _______________ *now in this,' Says the Lord of Hosts, 'If I will not open for you the* ____________________ *of heaven, and pour out for you a* ______________________ *until it overflows.'"* This is the only place in scripture God allows us to test Him. You cannot out-give God!

John Wesley was an Anglican minister in England born in the early 1700s who lived to be 88 years old. When the new world opened up (America!), Wesley traveled extensively to share the Gospel of Christ. The world was his parish: he traveled over 4,000 miles each year and preached over 40,000 sermons in his lifetime. Wesley did not mean to start a denomination, but was a brilliant organizer, and he and his followers were dubbed "Methodists."

One of the remarkable things about John Wesley was his belief in giving all we can to whomever we can. Each year, his income continued to increase, but each year, he lived off the same amount; this means that each year Wesley gave away a larger and larger proportion of his earnings. His first year, he earned 30 pounds, lived off 28 and gave away 2 pounds. In the end, John Wesley earned 1,400 pounds, lived off 30 and gave away 1,370 pounds.

Section 6.9 Proportions and Similar Figures

Looking Back 6.9

Proportions have many applications in solving problems in the field of mathematics, particularly in problems that involve similar figures. These problems were first introduced in General Math, but it is helpful to review those concepts here.

Similar figures have the same shape and congruent angles in each polygon, but they do not have the same size. If they had the same shape and size, they would be congruent. Congruent means they are equal: If $m\angle \text{A} = 10°$, and $m\angle \text{B} = 10°$, they are congruent (≅) and we write "∠A ≅ ∠B."

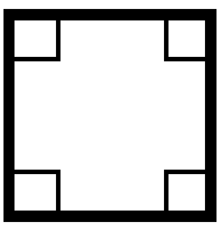
In similar figures, the sides of the figures are proportional, which means that each of the sides increase or decrease by the same scale factor. A scale factor happens when we zoom in or zoom out on a picture. The shape stays the same but the size increases or decreases.

In a drawing, the scale is the ratio of the distance in the scale model (drawing) to the actual distance. Therefore, a scale drawing is an enlarged or reduced drawing similar to a place (on a map) or object (such as a room or building). This enlargement or reduction is the scale factor.

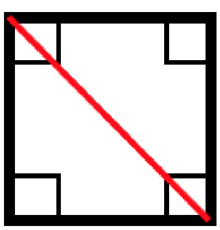
After the enlargement or reduction, we can set up a proportion to find the missing side lengths if we know that two shapes are proportional.

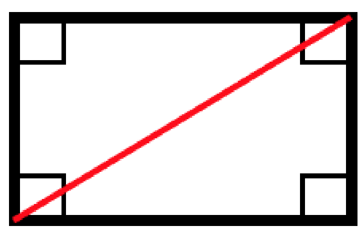
Looking Ahead 6.9

The sum of the degrees of a quadrilateral (square, rectangle, etc.) add up to $360°$ ($4 \cdot 90° = 360°$).



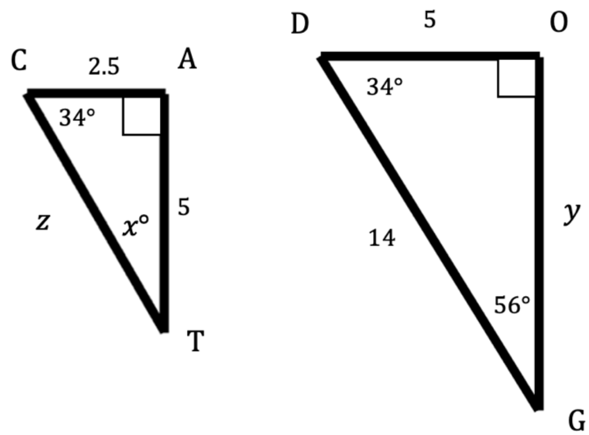
A triangle is one-half of a triangle (square, rectangle, etc.), so the sum of the angles of a triangle is $180°$.



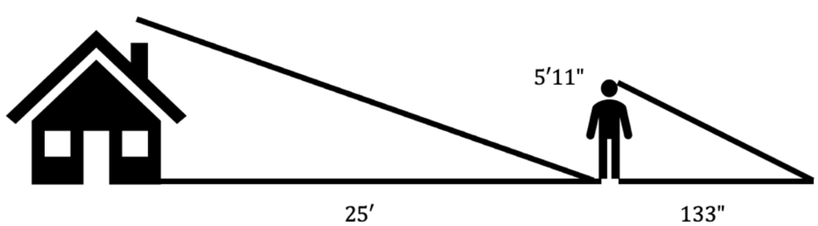


The two figures below are similar. Their relation can be written as "$\Delta CAT \sim \Delta DOG$" (read: "triangle c-a-t is similar to triangle d-o-g"). Notice that the order in which the similar figures are named is important. The congruent angles and proportional sides are listed in the same positions.

Example 1: Fill in the missing angle and find the missing side lengths in the shapes below.



Example 2: Blair is $5'11"$ tall and his shadow is $133"$ long. He is standing at the start of the shadow of his house, which is $25'$ long. How high is the top of the house? Some conversions must be made so that inches can be compared to inches.



Section 6.10 Thinking Proportionally to Problem-Solve

Looking Back 6.10

Through the many and varied examples, it should be evident that reasoning proportionally is helpful in many mathematical situations. Reasoning proportionally to solve real-world problems becomes helpful when two things are being compared: perhaps it is the age of persons, the rate of investments, or the price for various quantities of a product. Usually, the information is given so a comparison can be made.

For example, there are chain rings in the center of each wheel for a ten-speed bicycle. The larger chain ring is on one wheel and the smaller chain ring is on the other. The setting of these dictates the speed at which the bike moves: first gear; second gear; third gear; etc.; up to the tenth gear. The larger chain is the circle with teeth connected to the pedal crank. On the back wheel is a cog that has those teeth on it. Counting the teeth on cogs of the front wheel and back wheel can be written as a proportion:

$$\frac{\text{number of teeth on front cog}}{\text{number of teeth on back cog}} = \text{gear ratio}$$

The gear ratio determines the speed of the bicycle.

On a ten-speed bicycle you can change the gear settings while riding. For example, the tenth gear ratio could have 44 teeth on the front cog and 11 teeth on the back cog. This creates a 4-to-1 gear ratio. If you have a bike, check out your cog ratio and try it for different gears.

Gear ratios are also used in building machinery. The ratios for smaller machines would stay the same for larger machines but the number of teeth on the gear would increase. Proportions help determine the number of teeth needed for large machinery.

Looking Ahead 6.10

Below is an activity to try:

1. Measure your height.

2. Measure the length of your foot.

3. Write the ratio of height:length as $\frac{\text{height}}{\text{length}}$.

4. Determine how big your foot will be if you grow 3 inches. Use the ratio above and set up a proportion. Your new height for the second ratio will be 3 inches added to your present height. The length of your foot is the variable you are trying to in the second ratio. Make sure that you have all measurements in inches; you may have to convert feet to inches for this.

Example 1: A photo is 4 inches wide by 5 inches long. If an enlarged poster is 12 inches wide how long will the poster be?

Example 2: There are 4 tablespoons of milk in 11 peanut butter cookies. How many tablespoons of milk are in 18 peanut butter cookies?

Section 6.11 Using Proportions to Solve Percent Problems

Looking Back 6.11

In General Mathematics, we used proportions to solve problems involving percentages. Last year, in Module 7: Section 7.10-7.12, we looked at three different types of problems: first, when the amount was unknown; second, when the total number was unknown; third, when the percent was unknown. You can review these by looking back at Section 7.10-7.12 of Module 7 of General Mathematics. The problems are represented geometrically with diagrams; therefore, you know exactly what you are looking for: the amount, the total number, or the percent.

Let us review the process described above now. The algorithm we used was
$n\%$ of Total Number $=$ Amount.

Looking Ahead 6.11

Example 1: If 250 students were hired to work at an amusement park for the summer and the total number of employees is 3,475, what percent of employees at the amusement park are international students?

Example 2: The following year, new attractions were added to the parks so that approximately 12% of all employees hired were international students. The total number of international students that year was 576. How many total employees worked at the amusement park that year?

Example 3: The next year, the park will add two new attractions, a new reofstaurant, and another gift shop. They will need 4,500 employees. If they want to hire international students to make up 18% of their employees, approximately how many employees will be international students?

Some of the Practice Problems will use proportions to solve problems about the 2010 earthquake in Haiti. Because of the natural disasters and level of poverty in Haiti, many Christians have traveled on mission teams to assist in building homes, supplying sanitary water stations, and providing food and medical relief. All of this fulfills the commission of Christ when He said in Matthew 25:34-36: "*Then the King will say to those on his right, 'Come, you who are blessed by my Father; take your inheritance, the kingdom prepared for you since the creation of the world. For I was hungry, and you gave me something to eat, I was thirsty, and you gave me something to drink, I was a stranger and you invited me in, I needed clothes, and you clothed me, I was sick, and you looked after me, I was in prison and you came to visit me.'*"

Section 6.12 Interest Rate Problems

Looking Back 6.12

Proportions are useful for finding sales tax and the cost of items on sale as well. The total amount of an item is 100%. If the item is on sale, you are paying less than 100%. If the item is on sale for 25% off, instead of paying 100%, you are paying 25% less, or $100\% - 25\% = 75\%$. Therefore, finding 75% of the cost of the item will directly result in the price of the item on sale.

Indirectly, you could find 25% of the cost of the item and subtract that amount from the full price (100%) to get the cost of the item.

Let us suppose you put money in a bank account that has an interest rate in which your amount deposited increases. If the interest rate is 4% annually, then you will have the full amount of your deposit plus 4% more, or $100\% + 4\% = 104\%$ at the end of a year. Therefore, if you find 104% of the deposited amount, this will directly result in the amount you have in the account now including the interest.

However, indirectly, you could find 4% of the amount of the money you have in the account and add that to the amount you have (the 100%) to get the total amount at the end of a year.

Looking Ahead 6.12

Example 1: A tote bag regularly costs $19.97 but is on sale for 15% off. What is the sale price? Find the amount both directly and indirectly.

Example 2: Write each new percent below as an increase or decrease of 100%. Then write the percent as a decimal number.

a) Sales tax is 5%

b) A tote bag is marked down 33%

c) The interest rate on a car loan is 4%

d) Jewelry is marked up 100%

Example 3: Peggy wants to open a savings account for her four years of college in order to buy a car when she graduates. She is going to deposit $6,500.00 in the account, which she received in gifts at her high school graduation party. One bank offers a return of $275.00 each year for four years. The other offers an interest rate of 4% each year. Which will give Peggy more money at the end of the four years?

Section 6.13 Solving for One Variable in Terms of Another

Looking Back 6.13

In solving equations, the variable represents the "unknown." This unknown may be a constant. That is the case when there is one specific number that solves an equation, such as the problems in Section 3 of this module. Let us look at the example $3m + 2 = -9$. There is one numerical value for m that makes the equation true.

$$3m + 2 = -9$$
$$-2 \quad -2$$
$$3m + 0 = -11$$
$$3m = -11$$
$$\frac{3m}{3} = \frac{-11}{3}$$
$$m = \frac{-11}{3}$$

If indeed the correct value for m is $\frac{-11}{3}$ then we should get $-9 = -9$ when we check the problem.

Check:

$$3m + 2 \overset{?}{=} -9$$
$$3\left(\frac{-11}{3}\right) + 2 \overset{?}{=} -9$$
$$-11 + 2 \overset{?}{=} -9$$
$$-9 = -9 \checkmark$$

In Section 4 of this module, there were variables on both sides of the equation, so we moved all the variables to the same side to find the solution. Let us look at the example below:

$$-2n - 6 = 1 + 2n$$
$$-2n \qquad -2n$$
$$-4n = 7$$
$$\frac{-4n}{-4} = \frac{7}{-4}$$
$$n = -\frac{7}{4}$$

In the above example, the $2n$ from the right side of the equation was moved to the left side of the equation to combine with $-2n$. Moving the $-2n$ on the left side to the right side first and moving all of the constants to the left side of the equation will result in the same solution.

$$-2n - 6 = 1 + 2n$$
$$+6 \quad +6$$

$$-2n = 7 + 2n$$
$$-2n \quad -2n$$

$$\frac{-4n}{-4} = \frac{7}{-4}$$

$$n = -\frac{7}{4}$$

Looking Ahead 6.13

In the next module, we will be solving two-variable equations. It is not possible to solve for two unknowns at the same time so we must get one variable on a side by itself in terms of the other variable. The *terms* may be that the other variable is multiplied, divided, added, or subtracted from something. Once this is done, the process of substitution may be used to solve for one variable in terms of the other. This is a process often used in algebra when we want to know specific parameters for unique equations. Sometimes, this is called "solving literal equations." This process is used to solve linear equations for y in terms of x; in such cases, a term would be substituted for x to solve for y.

Example 1:	Solve for a in terms of b in the equation $2a + 3b = -4$. (This means to isolate the variable a.)

Example 2:	Solve for m in terms of n in the equation below.

$$2m - 1 = 3m + 4n$$

Example 3:	Solve for the height in each equation below.

a) $V = l \cdot w \cdot h$

b) $A = \frac{1}{2} \cdot b \cdot h$

c) $A = \frac{1}{2}(b_1 + b_2)h$

d) $A = b \cdot h$