Module 3 Working with Probability

Section 3.1 Combinations of Two or More Items

Practice Problems 3.1

For Problem 1-10, solve the word problem.

1. A message is composed of three components: a colored circle (red, blue, or green); followed by a colored square (yellow or black); followed by a colored triangle (purple or orange). How many combinations are possible between the three components?

2. A message is composed of three components: a colored circle (red, blue, or green); followed by a colored square (red, blue, or green); followed by a colored triangle (red, blue, or green). See if you can draw a diagram to find the possible combinations.

3. In the 7^{th} grade class, Garrett, Taylor, and Jesireé are running for class president. DàMontae, Sam, and Karmen are running for vice president. How many different results are possible for the election?

4. A restaurant offers a luncheon special that includes a sandwich, a bowl of soup, and a salad. Customers have a choice from three different sandwiches, two different soups, and two different salads. How many different lunches are served? Draw a tree diagram to find all the possible combinations.

5. Ashton has four shirts, three sweaters, and two pairs of pants. How many combinations of outfits can he choose from if he wears a shirt, a sweater, and a pair of pants? It might be fun to draw these on paper and cut them out and move the pieces around for the different combinations.

6. Complete the following statement that is called the *Fundamental Principle of Counting*: If the first choice can be made r ways and the second choice can be made n ways, then the total combinations can be made __________ $\cdot$ __________ ways.

7. There are six true-false problems on a test. How many different patterns of answers are there?

8. How many four digit numbers greater than $5{,}000$ can be made using the digits 3, 7, 8, and 9 if the digits can be repeated?

9. In Loveland, the telephone numbers consist of three numbers followed by four numbers. The first number must be an 8. How many different phone numbers are possible if numbers can be repeated?

10. In Loveland, license plates consist of three letters followed by three numbers. The first number cannot be 0. How many license plates are possible?

Section 3.2 Permutations of Two or More Items

Practice Problems 3.2

For Problem 1-10, solve the word problem.

1. You have a yellow chip and a red chip. How many different arrangements can you make with these two chips?

2. In Example 1 from the Lesson Notes, you had a penny, a nickel, and a dime, and arranged them in six different ways. If you have a quarter as well, how many arrangements will be possible for you? Try it with coins and write the arrangements here.

3. How many arrangements will you get if you add a half-dollar to the coins in Problem 2?

4. How many four-digit numbers can be formed using the digits 2, 9, 3, and 4? How is that different from the total number of combinations that can be made using those four digits?

5. Six ducks walk in a line. In how many ways can these six ducks be arranged in line?

6. A little-league baseball coach picked nine players for his team. How many ways can he pick his line-up?

7. There are eight runners in the 200-meter dash. How many ways can they be placed in Lane 1 through Lane 8?

8. If the fastest of the eight runners in the 200-meter dash must be in Lane 3, but all the remaining runners can be in any of the other lanes, how many ways can the runners be lined up for the race?

9. A honey bee can fly for 6 miles at speeds of up to 15 mph. They have to be pretty fast to visit two-thousand flowers a day! A bee visits tulips, daisies, and petunias the first hour, and carnations and roses the second hour. How many different flight patterns can the bee use?

10. Morse Code consists of a dash (−), a dot (·), or a tilde (~), followed by a dash, a dot, or a tilde. Using these three symbols, how many two-component messages can be composed?

Section 3.3 Theoretical Probability

Practice Problems 3.3

For Problem 1-10, solve the word problem.

1. Write all the possible outcomes of the sums of rolling two dice.

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

2. Write the sample space for the experiment in Problem 1.

3. What is the probability of getting an even sum when rolling two dice?

$\text{P(Even Sum)} =$

4. What is the probability of getting an odd sum when rolling two dice?

$\text{P(Odd Sum)} =$

5. Outcomes are equally likely given each of the possible outcomes (which result from an experiment) have the same chance of occurring. Would you say there is an equally likely chance of rolling an even sum as an odd sum?

6. What is the probability of getting a sum of 6 when rolling two dice?

$\mathrm{P(Sum\ of\ 6)} =$

7. What is the probability of getting a sum of 18 when rolling two dice?

$\mathrm{P(Sum\ of\ 18)} =$

8. What is the probability of getting a sum greater than 12 when rolling two dice?

$\mathrm{P(Sum\ greater\ than\ 12)} =$

9. What is the probability of getting a sum between 2 and 12 not including 2 and 12 when rolling two dice?

$\mathrm{P(Sum\ between\ 2\ and\ 12\ not\ including\ 2\ and\ 12)} =$

10. What is the probability of getting a sum between 2 and 12 including 2 and 12 when rolling two dice?

$\mathrm{P(sum\ between\ 2\ and\ 12\ including\ 2\ and\ 12)} =$

Section 3.4 Experimental Probability

Practice Problems 3.4

For Problem 1, use a pair of dice to play the game and answer the questions.

1. Roll the pair of dice twenty times. If the product is an odd number, put a tally mark under Odd Product. If the product is an even number, put a tally mark under Even Product. Record each roll.

Even Product	Odd Product

Total: $\boxed{}$ + $\boxed{}$ = 20

(Even) (Odd)

What is the experimental probability that you roll an even number?

$$P(\text{Even}) = \frac{\square}{20}$$

Is it close to $\frac{3}{4}$?

What is the experimental probability that you roll an odd number?

$$P(\text{Odd}) = \frac{\square}{20}$$

Is it close to $\frac{1}{4}$?

For Problem 2-10, solve the word problem.

2. If six airlines run from Dayton to Chicago, and four airlines run from Chicago to Milwaukee, in how many ways could you fly from Dayton to Milwaukee through Chicago?

3. Jessica and Tiara go to a restaurant that offers three different meats for a main course, three different appetizers, four different salads, and five different deserts. If Jessica orders one meat, one appetizer, one salad, and one desert, how many ways could she order? If Tiara orders one from each category, but does not order desert, how many ways could she order?

4. Simon has to sign up for classes at school. He can choose from four different Math classes, three different English classes, two different Social Studies classes, three different Science classes, and two different Art or Music classes. If he has to pick one of each class and either an Art or Music course, how many different schedules can Simon make?

5. A coin collector has seven Indian-head pennies dated from 1903 through 1909.

   a) How many arrangements of the coins can be made?

   b) How many arrangements can be made if the collector sells one of the coins?

6. What is the probability that Donna will get a 1, 2, 3, 4, 5, or 6 when she rolls a die?

7. What is the probability that the number will be odd and greater than 3 when you roll a die?

8. What is the probability that the number will be even and less than 5 when you roll a die?

9. How many five-letter code words can be made if Miranda uses the letters in the word CODER? No letter in the code can be used more than once.

10. In Problem 3 from Section 3.1, we said that Garrett, Taylor, and Jesireé were running for class president, and DàMontae, Sam, and Karmen were running for vice president. What is the probability that Garrett will be elected president and Karmen will be elected vice president?

Section 3.5 Simple and Compound Events

Practice Problems 3.5

For Problem 1-3, complete the activity given to solve the problem.

1. You will need a six-sided die for this activity.

a) If you roll the die and double the number it lands on, what are all the possible outcomes? Write the sample space below:

$E = \{$____$\}$

b) If a simple event is rolling a 1 on the six-sided die and then doubling it, or $E = \{2\}$, what is the theoretical probability (or mathematical probability) of getting a 2 when the roll is doubled?

c) What is the theoretical probability for each of the possible outcomes on the six-sided die when the outcome is doubled?

d) What is the theoretical probability you will roll at least one of the possible outcomes with one roll of the six-sided die once the outcome is doubled?

2. Put each of the possible outcomes from Problem 1a) on paper. There should be six numbers on the paper. Remember, when you roll the die, you double the number.

a) Roll the die fifty times and double the number it lands on each time. Put a tally mark under the number until you have fifty tally marks in total.

3. What is the experimental probability you rolled a 1 and doubled it?

$P(E) =$

4. What is the experimental probability you rolled any of the other five numbers and doubled it?

$\mathrm{P(E)} =$

Let $P(E)$ be the probability of the event occuring and $P(\not{E})$ be the probability of an event not occuring. That means that $P(E) = 1 - P(\not{E})$ and $P(\not{E}) = 1 - P(E)$.

5. Write the probability of the event in Problem 3 not occurring.

6. Use the formula above and the information from Problem 4 to find the probability the event will occur.

7. If the probability that an event not occurring is $\frac{2}{5}$, what is the probability that it will occur?

8. If the event is rolling a 2 or 12, what is P(E)?

9. What is the probability of not rolling a 2 or 12?

10. What is the probability that a number in the sample space will occur?

Section 3.6 Permutations of Some Objects

Practice Problems 3.6

For Problem 1-10, solve the word problem.

1. A flag collector has five flags in his collection. How many ways can he arrange them on a flagpole?

2. If the flag collector from Problem 1 can only fly one flag at a time, how many arrangements are possible?

3. If the flag collector from Problem 1 can only display three flags at a time, how many arrangements are possible?

4. A boat has eight different flags. Signals are made by hoisting the flags up one above another on the boat's flagpole.

   a) How many signals can be made using 2 flags?

   b) How many signals can be made using 3 flags?

   c) How many signals can be made using 4 flags?

5. A sailor has only four flags on his boat. He can make signals by flying 4, 3, 2, or 1 flag(s).

   a) How many signals can the sailor make using all 4 flags?

   b) How many signals can the sailor make using 3 flags?

c) How many signals can the sailor make using 2 flags?

d) How many signals can the sailor make using 1 flag?

e) How many signals can the sailor make all together?

6. Suppose a woman has sixteen pictures in frames but can only display four at a time on her desk. How many ways can she arrange the pictures?

7. How many ways can six children sit around a kitchen table?

8. How many ways can six children be arranged at a table with only four chairs?

9. If there are five players on a basketball court and they each give another player a high-five before the game, how many high-fives are given? Each player can only high-five each player one time, but they must high-five every player. (Drawing a diagram will help solve this problem.)

10. There are twelve basketball players on the varsity team, but only five can start the game. How many ways can the coach arrange the starting five players?

Section 3.7 Combinations of Some Objects

Practice Problems 3.7

For Problem 1-14, solve the word problem.

1. Complete the next three rows of Pascal's Triangle.

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

_____ _____ _____ _____ _____ _____ _____

_____ _____ _____ _____ _____ _____ _____ _____

_____ _____ _____ _____ _____ _____ _____ _____ _____

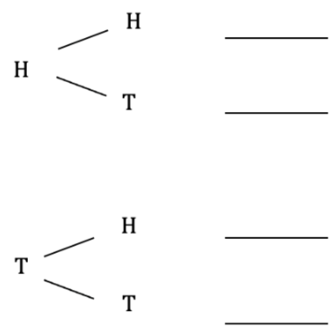
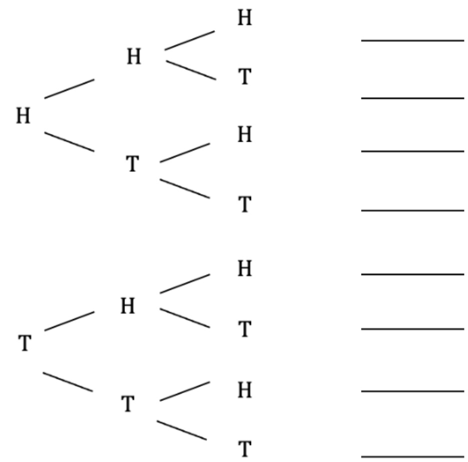
2. If there are five children and only three can fit in a ride at an amusement park, how many possible combinations are there for the children to be on the ride?

3. If there are five teachers willing to serve on a school committee, but only two positions open, how many possible combinations are there for the five teachers to be on the committee?

4. How many committees of four can be chosen from seven students?

5. If six classmates like to play games, how many different tables of three can be selected from the six classmates?

6. The left side of the diagram below represents Heads and Tails combinations when two coins are tossed. The right side of the diagram represents Heads and Tails combinations when three coins are tossed. Fill in the blanks and use Pascal's Triangle to answer the questions that follow.

1st Flip; 2nd Flip; Results	**1st Flip; 2nd Flip; 3rd Flip; Results**
	

a) If the coin is tossed twice, how many ways can you get…

… two Heads? __________ … one Head? __________ … zero Heads? __________

Which row of Pascal's Triangle is this?

b) If the coin is tossed three times, how many ways can you get…

… three Heads? __________ … two Heads? __________ … one Head? __________ … zero Heads? __________

Which row of Pascal's Triangle is this?

c) What is the probability of getting exactly one Head if you toss two coins?

d) What is the probability of getting exactly one Head if you toss three coins?

7. Look at the row of Pascal's Triangle that includes the numbers 1, 4, 6, 4, 1.

There is/are:

__________ way(s) to get four Heads and __________ Tail(s)

__________ way(s) to get three Heads and __________ Tail(s)

__________ way(s) to get two Heads and __________ Tail(s)

__________ way(s) to get one Head and __________ Tail(s)

__________ way(s) to get zero Heads and __________ Tail(s)

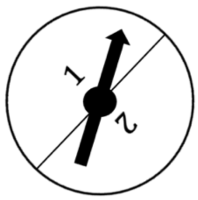
8. If four coins are tossed, what is the probability of getting one Head and three Tails?

9. If five coins are tossed, what is the probability of getting two Heads and three Tails?

10. Where can the number of successful events (getting Heads) be found on Pascal's Triangle?

11. Where does the total number of possibilities in Problem 8 and Problem 9 come from?

12. What is the probability of spinning the spinner below and getting two 1s and two 2s in four spins? Is this ${}_4C_2$ or ${}_2C_4$?



13. How many possible outcomes are there for a five-problem true-false test? Remember, these can be repeated.

14. What is the probability of getting five true answers on a five-problem true-false test?

Section 3.8 The Law of Large Numbers

Practice Problems 3.8

For Problem 1-9, solve the word problem.

1. If you roll a die…

   a) … what is the sample space for rolling a prime number?

   b) … what is the probability you will roll a prime number?

2. If you roll a die…

   a) … what is the sample space for rolling a composite number?

   b) … what is the probability you will roll a composite number?

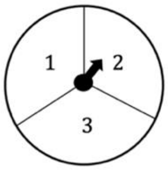
3. David and Sam are playing a game called PRIME DICE. They roll a die 100 times and count how many times they get a prime number. About how many times can they expect to roll a prime number?

4. About how many times can David and Sam expect to roll a composite number in their game of PRIME DICE?

5. David says he will play Sam in PRIME DICE if he gets 3 cents (three pennies) every time he rolls a composite number and Sam gets 1 cent (a penny) every time he rolls a prime number. Is this a fair game?

6. David says the probability he will get a composite number is $\frac{1}{3}$ so he should get three times as much money as Sam to keep the game fair. Sam says this is not fair; what reason does Sam give?

7. Bill says he made a new game in which a player has to spin a prime number on a spinner (numbered 1, 2, and 3) and roll a prime number on a six-sided die to win. What is the probability of winning this game?

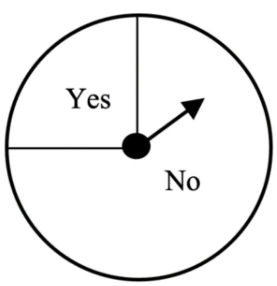


8. A family has five girls. They are thinking about having another child. What are their chances of having a boy?

9. A family has no children. They are thinking about having at least four children. What is the probability they will all be boys or all be girls?

For Problem 10-14, use the given information to solve the problem.

Quintana spins the spinner below 27 times. The results are shown below.



Yes	~~IIII~~ IIII
No	~~IIII~~ ~~IIII~~ ~~IIII~~ III

10. a) How many times did the spinner land on *Yes*?

b) How many times did the spinner land on *No*?

11. a) What is the theoretical probability the spinner will land on *Yes* in the long run?

b) What is the theoretical probability the spinner will land on *No* in the long run?

12. a) What is the experimental probability the spinner will land on *Yes*?

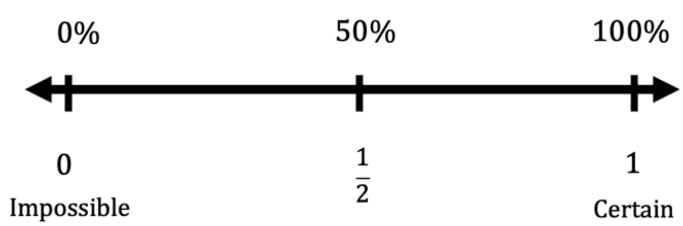
b) What is the experimental probability the spinner will land on *No*?

13. Compare the theoretical probabilities to the experimental probabilities; are they what you would expect? Why or why not?

14. Place the letter on the number line near the area the given probability would be.

a) The spinner will land on *Yes*.

b) The spinner will land on *No*.



c) You will walk 10 miles in 10 minutes.

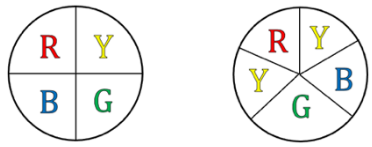
d) A family has a child and it is a girl.

e) God never changes!

Section 3.9 Area Models

Practice Problems 3.9

For Problem 1-3, use the given palettes to solve the problem.



1. Draw the tree diagrams for possible color combinations given you pick one color from the first palette and one color from the second palette. Do not diagram repeat combinations.

Complete the area modle below for the possible combinations of the first and second palette.

2.

	R	Y	B	G	Y
R					
Y					
G					
B					

Suppose you want to paint the background of your painting orange. You can pick one color from each palette to combine red and yellow for orange (red and yellow together make orange; yellow and red together make orange).

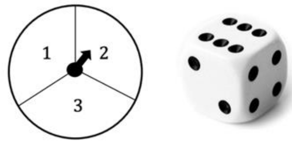
a) What is the probability of getting a red from the first palette and a yellow from the second palette? (Remember, *and* means to multiply.)

b) What is the probability of getting a yellow from the first palette and a red from the second palette?

c) What is the probability of getting a combination of red and yellow or yellow and red? (Remember, *or* means add.)

3. Looking at the table from Problem 1, what is the probability of getting some combination of red and yellow? Is this the same answer you got for c) in Problem 2?

For Problem 4 and 5, use the diagram below to solve the problem.



4. Suppose you spin the spinner and then roll the die. Make a of an area model for the possible sums.

5. a) What is the probability you will get a sum of six?

b) Which sums have a probability of $\frac{1}{18}$?

For Problem 6-12, use the given information to solve the problem.

Mariah and Keyera are playing a game. Mariah has two nickels and two dimes and must put all the possible combinations in two pouches.

6. One arrangement is to put all the coins in one pouch and none in the other pouch. Draw a diagram of nickels and dimes in the two given pouches to represent the other four possible combinations.

7. Keyera gets to pick one coin from each pouch. Given each arrangement, what is the probability she will pick a dime from each pouch? What is the probability she will pick a nickel from each pouch?

8. Which arrangement has equally likely outcomes for the following events given that only one coin may be picked form one pouch: picking a dime; picking a nickel?

9. Given Keyera picks one coin from one pouch,which arrangement will guarantee: picking a dime, picking a nickel, picking no coins?

10. Which arrangement has a 50% chance of no coins being picked given Keyera can only pick from one pouch?

11. Keyera gets to pick one coin. If she picks a dime, Mariah and Keyera both get $1.00. Which arrangement will Mariah probably give Keyera to pick from?

Section 3.10 Expected Outcomes

Practice Problems 3.10

For Problem 1 and 2, use the given information to solve the problem.

Mosley had a 70% free-throw average his senior year, but only had a 60% free-throw average his junior year.

1. How would this have changed the outcome of the game from Example 1 in the Lesson Notes? Is it still more likely he would tie the game and send his team into overtime?

2. Draw an area model to represent the situation.

For Problem 3, solve the word problem.

3. Is tossing a coin twelve times and counting the number of times it lands on Heads a binomial experiment? If the answer to the each of the following questions is *yes*, it is a binomial experiment:

a) Does it consist of repeated trials? (Tell what they are.)

b) Does each trial have two possible outcomes? (Tell what a success is and what is a failure.)

c) Is the probability of success the same for each successive trial? (Tell what it is.)

d) Are the trials independent? (Explain.)

For Problem 4-8, use the given information to solve the problem.

Byron and Jared are shooting free-throws. They both make twelve out of twenty shots. Jared takes ten additional shots and makes three of them. Byron takes twenty additional shots and makes eight of them. Both end up with a 50% average.

4. How can they both shoot the same percentage when Byron made more total shots?

5. How many trials of the experiment did Jared perform? How many trials of the experiment did Byron perform?

6. What are the two events of this basketball experiment?

7. What is a success in this experiment? What is a failure in this experiment?

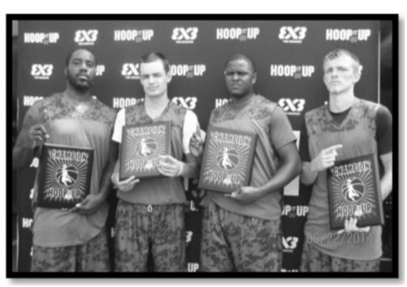
For Problem 8-10, solve the word problem.

8. Suppose your parents have friends that have two children, and you know that at least one of them is a girl. What is the likelihood the other child is a girl?

9. Suppose you are on a game show and there are ten suitcases to choose from. You think the money is in *Suitcase Number 9*, so you pick it. The host of the show opens all the suitcases except for *3* and *9*; they are empty. He then asks you if you want to trade *9* for *3*, or keep *Suitcase Number 9*. What should you do to give yourself the highest probability to pick the suitcase with the money?

10. There are seventeen children on a bus. What is the probability that two of them were born in the same month?

Mosley made the shot and the "Frontline Soldiers for Christ" won the Hoop-it-up 3-on-3 World Championship!



Pictured left to right: Antonio, Mosley, Joseph, Byron

Section 3.11 Expected Value

Practice Problems 3.11

For Problem 1 and 2, use the given information to solve the problem.

During his junior basketball season, Mosley went to the foul line 100 times while his team was in the double bonus period. Mosley had a free-throw average of 60% this season. We expected a 0-point score 16% of the time, a 1-point score 48% of the time, and a 2-point score about 36% of the time.

1. What is the average points per trip to the foul line Mosley would earn for his team given these expected values? How many times would you expect Mosley to get a 0-point score, a 1-point score, or a 2-point score? What are the average points per trip to the foul line Mosley earned at the foul line during his junior season?

2. One of Mosley's teammates, Eric, had a 70% free-throw average during Mosley's junior season (the same as Mosley's senior year average). What is the average percent per trip to the foul line Eric earned this season?

For Problem 3-9, use the given information to solve the problem.

Suppose you have a very generous friend. He decides to use the $20 he earned mowing lawns to play "Match Up" at the fundraiser carnival from the Lesson Notes because he knows it will go to charity either way! Your friend plays twenty rounds of "Match Up" at $1 a game.

3. Complete the table to find his expected winnings per game.

Experimental Results for Twenty Rounds of "Match Up"				
Event	**Prize**	**Number of Wins**	**Probability of a Win**	**Expected Winnings ($)**
0 matches	$0	15		
1 match	$1	4		
2 matches	$2	1		
	Total	20		

4. What is your friend's expected winnings from the table?

5. Another way to calculate the expected value is to multiply each prize by the number of wins and add them all together. Calculate this expected value and compare it to your solution in Problem 4.

6. You know your friend is giving his money to charity either way, but under normal gambling circumstances, would this be a fair game to him?

7. If your friend won $0.50 each time he played "Match Up," would this be a fair game? (The carnival would win $0.50 each time he loses.)

8. If the ticket for "Match Up" costs $2, what would be your friend's expected earnings per game to make it a fair game?

9. If your friend runs a booth at the carnival and knows the expected winnings per game are $0.40, what would they charge for a ticket so the fundraiser makes money? If he sells three tickets at a time for a discounted price, what would he have to make the three-pack price so the fundraiser still makes money?

For Problem 10, use the given diagram to solve the problem.

10.

Player	Free-Throw Total	Free-Throw Makes
Eric	30	26
Byron	20	14
Mosley	38	25
Jared	44	28

a) What is the free-throw probability for a make for each player?

b) Which player has the greatest chance of making the next free-throw?

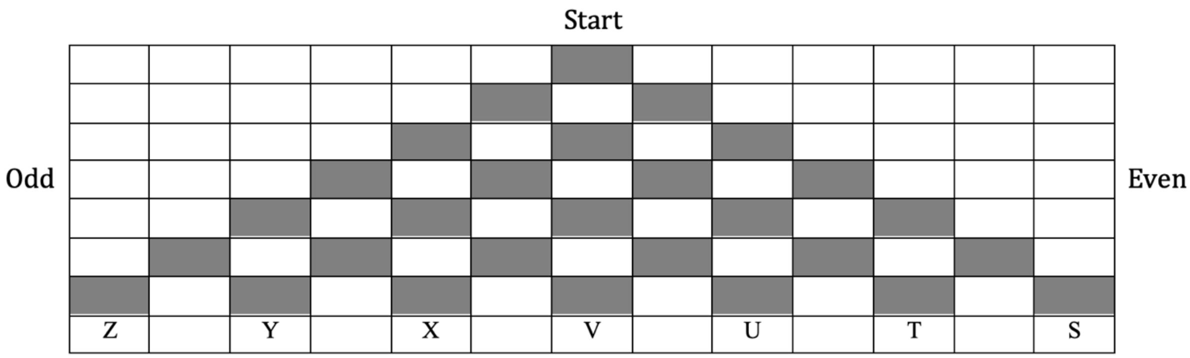
Jared makes a pass to Lorenzo; too bad he is not shooting a free-throw!

c) Why is this not a fair comparison?

d) What would make this a fair comparison in deciding which player has the greatest chance of making the next free-throw?

Section 3.12 "Dice Sums" Game

Practice Problems 3.12



For Problem 1-5, solve the problem before playing the "Dice Sums" game.

1. Pick the letter you think you will end on the most. Explain why.

2. Which two letters do you think you will end on the least? Explain why.

3. Are there letters you think you will have an equal probability of ending on?

4. How could you find the theoretical probability of ending on each of the letters? Find the theoretical probability for the following terms given the event is finding the possible routes to get from the "Start" to each of the letters: P(Z), P(Y), P(X), P(V), P(U), P(T), P(S). (Hint: There are sixty-four possible routes to all the letters.)

5. How do you think this "Dice Sums" Game relates to Pascal's Triangle?

For Problem 6-10, solve the problem after playing the "Dice Sums" Game.

6. Find the experimental probabilities for the following terms given the event is ending on each of the letters; the denominator will be 10 because you played ten rounds of the game; the numerator will be the total number of tally marks below each letter: P(Z), P(Y), P(X), P(V), P(U), P(T), P(S).

7. If you add all of the experimental probabilities together, what is the total?

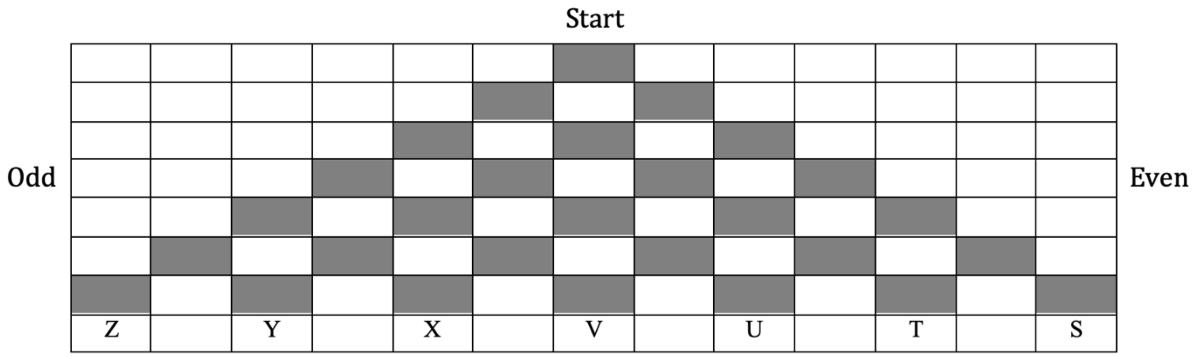
8. What is the probability of ending on letter Z or letter S?

9. Did you land the most on the letter you thought you would (from Problem 1)? If not, explain why.

10. Did you land the least on the letters you thought you would (from Problem 2)? Were there any other equal experimental probabilities?

Section 3.13 "Dice Products" Game

Practice Problems 3.13



For Problem 1-5, solve the problem before playing the "Dice Products" game.

1. Pick the letter you think you will land on the most. Explain why.

2. Pick the letters you think will land on the least. Explain why. How are these values similar to or different from those of the "Dice Sums" game you thought you would land on the least?

3. How is this game similar to or different from the "Dice Sums" game?

4. Are there the same number of paths to get to each letter or a different number of paths to get to each letter? (Hint: Use Pascal's Triangle to solve this problem.)

5. Is it equally as likely to get an even product as an odd product?

For Problem 6-10, solve the problem after playing the "Dice Products" game.

6. Which letter would you end on if you rolled the following combination of products: EEEOEO? What is the probability of following that one path to the letter O?

7. What are the possible paths to the letter Y? (Use Es for even numbers and Os for odd numbers.)

8. There are four combinations of even or odd followed by an even or odd. What is the probability of taking each path? For example, the probability of an Even followed by an odd is $P(EO) = \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{16}$. List the other three.

9. Make an area model for rolling an even number firstly followed by an odd number secondly, and the other three combinations from Problem 8.

10. Did you land the most and least number of times on the letters you expected to? What were your experimental probabilities for each letter?

Section 3.14 Module Review

For Problem 1-20, use the instructions given to solve the problem.

1. A restaurant offers five different sandwiches, four different salads, and four different soups. How many possible lunches are there consisting of a sandwich, a salad, and a soup?

2. On a test consisting of eight true-false problems, how many different arrangements of answers are there?

3. In Trotwood, the telephone numbers consist of seven digits. The first digit is 8, the second digit is 3, and the third digit can be any of the other eight digits. The remaining four digits can be any of numbers 0 through 9, and can be repeated. How many different telephone numbers are possible in Trotwood?

4. A little-league baseball coach has only nine players on his team and has nine positions to fill. How many different batting lineups are possible for this team?

5. There are six runners in a 200-meter dash. How many possible orders of a first through sixth finish are there?

6. How many permutations are there for seven objects?

7. Using four chips (red, yellow, blue, and green), list all the different ways four objects can be arranged…

a) … two at a time.

b) … three at a time.

8. List all the arrangements in Problem 7 for a) and b) using the letter of the chip (R for red; Y for yellow; B for Blue; G for green).

9. Suppose you have a red chip, a yellow chip, a blue chip, and a green chip in one cup. Suppose you have a red chip, a yellow chip, a blue chip, and a green chip in another cup. If you pick out two chips (one from each cup), what are the possible combinations for chip color? Use an area model to show these possibilities.

10. What is the probability you would pick a red chip from one cup and a yellow chip from the other cup given you pick two chips (one from each cup)?

11. Use a tree diagram to show all the possible outcomes for tossing four coins (Heads and Tails).

12. What is the probability of getting all Heads or all Tails when tossing three coins?

13. Suppose you are writing a song and want to use two half-notes; you also want to use two pitches: A and B. How many melodies could you write? List them.

14. Write all the possible melody arrangements given you want to use two half-notes, and also you want to use three pitches: A, B, and C in writing a song. How many melody arrangements are there in total?

15. Tell how many arrangements there are for each of the following melody arrangements (use exponential notation):

Two half-notes and four pitches __________

Two half-notes and five pitches __________

Two half-notes and six pitches __________

Two half-notes and ten pitches __________

16. Use your reasoning from Problem 15 to tell how many arrangements there are for the following melody arrangements (use exponential notation):

Three half-notes and two pitches __________

Three half-notes and three pitches __________

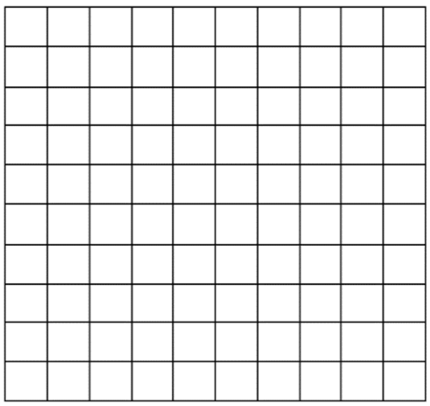
Three half-notes and four pitches __________

Three half-notes and ten pitches __________

17. Using your logic from Problem 15 and Problem 16, how many different melodies could be written using seven half-notes, and also seven different pitches?

18. A throat culture test for strep throat is more accurate than a rapid strep throat test but takes longer to get results. The results for a throat culture test for strep throat are accurate 90% of the time but are inaccurate 10% of the time. Therefore, 10% of the time this test will indicate one does not have strep throat when they actually do. Dr. Robb Snider is aware of these statistics, so he orders two tests to be run on each patient. This assures more accuracy. What are all the possible outcomes and the probability of each for two throat culture tests for strep throat?

19. Draw an area model to show the probabilities of the possible outcomes for Problem 18.



20. What is the probability that at least one test from Problem 18 will say the patient does not have strep throat when they actually do?

Section 3.15 Module Test

For Problem 1-20, use the instructions given to solve the problem.

1. Marielle has four blouses, five pairs of shorts, and three pairs of shoes. How many outfits can she make consisting of a blouse, a pair of shorts, and a pair of shoes?

2. On a ten-question multiple-choice test consisting of only a) and b) choices for each answer, how many different arrangements of possible answers could there be?

3. Area codes in a small remote town in Canada consist of three digits. The first digit is a number from 2 through 9; the second digit is either 0 or 1; the third digit is a number from 1 through 9.

a) How many different area codes are there in this small remote town?

b) How many different area codes can end with the digit 0?

c) How many different area codes can begin with the digit 3?

4. A basketball lineup consists of five positions: a point guard, a swing guard, a center, a power forward, and a small forward. On a team of ten players, there are three players that are point guards, three swing guards, two centers, one power forward, and one small forward. How many possible lineups could the coach make with one player in each position given each player stays in their position?

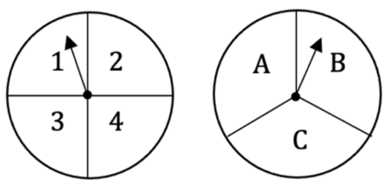
5. Suppose you have four cards with a number 7 on them. One is a heart, one is a spade, one is a diamond, and one is a club. How many permutations can be made with the four cards?

6. How many permutations can be made from five objects?

7. Suppose there are ten runners in the 100-meter dash but it is on an eight-lane track. How many possible arrangements are there for eight runners to be on the track?

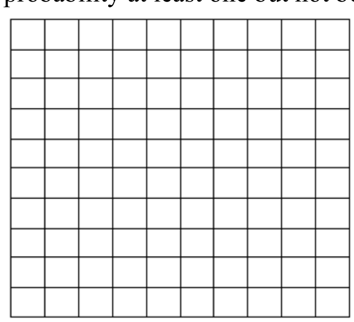
8. There are eight flags for different signals that can be displayed in a security building in Cincinnati. Only three can be flown vertically on a flagpole at one time. How many different signals can be made?

9. Suppose you have two spinners (shown below). Use an area model to show all the possible combinations when you spin the first spinner (on the left below) followed by the second spinner (on the right below).



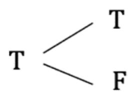
10. What is the probability you will get a B when you spin each spinner once in Problem 9?

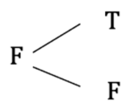
11. Mr. Brown took his son to Dr. Fedrizzi to be tested for strep throat. Her test is accurate 80% of the time. It is inaccurate 20% of the time which means the test shows he does not have strep throat when he does. Dr. Fedrizzi runs two tests on Mr. Brown's son to assure more accurate results. Make an area model for the outcomes. If Mr. Brown's son has strep throat, what is the probability at least one but not both tests will indicate this?



12. If Mr. Brown's son from Problem 11 has strep throat, what is the probability that both tests will indicate he does not have strep throat?

13. Suppose you take a pop-quiz that consists of four true-false problems. Make two tree diagrams to list all of the possible outcomes for the pop-quiz. They are started for you.





14. If you mark all four problems on the pop-quiz from Problem 13 "true," what is the probability you are correct on all four of them?

15. On the four-problem true-false test from Problem 13, let T (for true) indicate a correct answer, and F (for false) indicate an incorrect answer. If you guess on every problem, how many times will you get…

… four correct answers? __________

… three correct answers? __________

… two correct answers? __________

… one correct answer? __________

… zero correct answers? __________

What is the name of the famous triangle in which this pattern can be found?

16. On the four-problem true-false test from Problem 13, what is your probability of getting…

… four correct answers? __________

… three correct answers? __________

… two correct answers? __________

… one correct answer? __________

… zero correct answers? __________

17. Pascal's Triangle can be used to analyze binomial experiments. Complete the last two rows in the triangle shown below.

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

_____ _____ _____ _____ _____ _____

_____ _____ _____ _____ _____ _____ _____

18. Fill in the blanks for the definition of a *binomial experiment*.

a) A binomial ____________________ consists of ____________________ trials.

b) Each trial has ____________________ possible outcomes that consist of a(n) ____________________ or __________________________.

c) The _____________________ of success is the same for each successive ____________________.

d) The trials are _____________________ of one another.

19. Circle the examples below that represent a binomial experiment. If they do not represent a binomial experiment, explain why.

a) You flip a coin ten times and count the number of times it lands on Heads.

b) You test forty random circuits on an assembly line and count the number of defective units.

c) You count the penalty shots on a goal and calculate if the shooter improves their average.

d) You provide a property on Tampa Beach with flood insurance and count the number of floods in the last ten years.

e) You draw ten marbles out of a bag that consists of ten green marbles and ten blue marbles. You do not replace the marbles after they are drawn out of the bag. You count the number of blue ones drawn out.

20. You and your friend are playing a game called “Slider” on the game board shown below. Play the game and answer the questions given.

<u>“Slider” Rules</u>

First, read the rules and then you and your partner will pick the letter you think you will land on at the end of the game and initial below it.

Put a chip at A to begin. You and your partner will take turns tossing a coin for a total of four times. If the coin lands on a Head, move the chip right one space and if it the chip lands on a Tail, move the chip left one space. See where the chip is after four flips. If it is on the letter with your initial, then you win. If it is on the letter with your partners initial, then your partner wins. If it is on neither then play the game again or pick another letter and play again.

<u>“Slider” Board</u>

D	C	B		A		B	C	D

Suppose your partner says they will give you two letters to place chips on: C and D. They will take only one letter: B. Your partner says neither of you gets A because it is not possible to end up on A given you started on A. Do you accept your partner’s rules? Are they correct in their thinking?