## Module 8 Working with Algebra

## Section 8.1 Evaluating Expressions with Variables <br> Looking Back 8.1

This last module of General Math will be our introduction to Algebra. Algebra is not really a new topic for us. We have been exploring algebraic thinking whenever we have worked with variables. In this module, we will expand on this thinking and learn to solve multi-step equations involving variables. Working with expressions will lay the foundation for solving equations.

As we begin this study, we may be tempted to try things in our head and not learn the processes. In the New Testament, Matthew 7:24-27 tells us: "Therefore, he who hears these words of mine and acts upon them may be compared to a wise man who built his house on the rock. And the rain descended, and the floods came, and the winds blew and burst against that house; and yet it did not fall, for it had been founded upon a rock. And everyone who hears these words of mine and does not act upon them will be like a foolish man, who built his house upon the sand. And the rain descended, and the floods came, and the winds blew and burst against that house, and it fell, and great was its fall."

While this passage is talking about our spiritual life, and building it on God's Word, it is also true of our academic and physical life. We must have a solid foundation. For years, students have been told that the process of being able to show work and explain how to get a solution is more important than the actual solution. The process is as important as the end result. The process is what gets us to the end goal. Let us begin this section with a review of algebraic expressions.

## Looking Ahead 8.1

A variable (letter) is used to represent an unknown number. When we have a number, a variable, or the product of a number and a variable, we have a term. Examples of a term include: 4, $x, 4 x$. A number is also called a constant.

When we have one term, or more than one term connected by addition or subtraction signs, we have an expression. The terms of the expression are separated by the addition or subtraction signs.
Example 1: How many terms are in each expression below?
a)
$8 y-9 z$
b) $3 r-5 s+2 t$
c) $\quad-5 x y z$

We can substitute numbers for letters to evaluate or solve an expression.
Example 2: $\quad$ Evaluate $3 x+4 y-5$ when $x=1$ and $y=-2$.

Example 3: $\quad$ Sarah did the problem from Example 2 a bit differently than we did. The problem was to evaluate $3 x+4 y-5$ when $x=1$ and $y=-2$. She got 31 for her solution. Look at her work. What did she do wrong?

$$
\begin{gathered}
31+4-2-5 \\
38-2-5 \\
36-5
\end{gathered}
$$

Example 4: Laura did the same problem from Example 2. She got 0 for a solution. Look at her work. What did she do wrong?

$$
\begin{gathered}
3(1)+4-2-5 \\
3+4-2-5 \\
7-2-5 \\
5-5
\end{gathered}
$$

0

When substituting numbers into an equation make sure you use the operation given. Because $3 x$ means 3 times $x$, $3 x$ is 3 times 1 when $x=1$. Because $\frac{10}{x}$ means 10 divided by $x, \frac{10}{x}$ is 10 divided by 2 when $x=2$.
Example 5: $\quad$ Find the value of $7 a-4 b$ when $a=3$ and $b=2$. Substitute the values in for the variables first and then solve for the value of the expression with the given values.

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Example 6: Find the value of \(t^{2}+3 t-4\) when \(t=-3\). Make sure you put -3 in parenthesis so the sign gets
``` squared as well.

Example 7: \(\quad\) Let \(x=2, y=-6\), and \(z=10\). Evaluate the expressions below.
a) \(x-y\)
b) \(y-x\)
c) \(z-y\)

Example 8: Let \(p=-4, q=-2\), and \(z=3\). Evaluate the expressions below.
a) \(\quad-8-7 p^{2}\)
b) \(\quad-5-(2 q)^{2}\)
c) \((2-3 z)+(-2 z+5)\)

\section*{Section 8.2 Writing Expressions for Word Phrases \\ Looking Back 8.2}

Word problems are exercises in which written words or verbal situations are changed into mathematical operations and algebraic expressions to solve problems. The value of the expression can then be determined by the values we substitute in for the variables, just like we did in Section 8.1. If the expression is set equal to a number, it becomes an equation and we can actually solve it. We will be working with equations such as these in Section 8.6 through the end of this module. In this section, we will learn to write expressions for word problems or verbal situations.

\section*{Looking Ahead 8.2}

A numerical expression is one term or more than one term connected by addition and/or subtraction signs.

Example 1: How many terms are in each numerical expression below?
a) \(2 \cdot(-3)+7\)
b) 10
c) \(\quad-13+8\)

An algebraic expression is one or more variables, numbers, or the product of variables and numbers connected by addition and/or subtractions signs.

\section*{Example 2: How many terms are in each variable expression below?}
a) \(5 y-11 z\)
b) \(\quad 2 x-3 y z+4 z\)
c) \(-8 y\)

It is important to carefully read the verbal expression in order to make it a mathematical expression. Some words have more than one meaning; we must think about what a word means in the given situation.

For example, 6 less than 8 could be \(6<8\) if the situation is referring to inequalities. However, 6 less than 8 could be \(8-6\) if the situation is referring to a subtraction problem.

Moreover, the word "left" could be referring to: negative numbers to the left of positive numbers; the amount of something left over; a remainder in division; what is left when we take out a certain amount or subtract.

This may seem confusing so make sure to read the verbal expression many times.
In this module we are working with the four operations of addition, subtraction, multiplication, and division: not inequalities. Therefore, 6 less 4 is \(6-4\) and 4 less than 6 is also \(6-4\).

Example 3: Highlight the operation(s) and then write a numerical expression for the phrases below and solve them.
a) 7 more than 6
b) \(\quad 2\) less than 3
c) \(\quad 11\) less than 14
d) 15 more than -80

Example 4: Highlight the operation(s) and then write an algebraic expression for the phrases below.
a) Three times a number added to five
b) \(\quad \frac{1}{4}\) of a number
c) A number decreased by 16
c) Five plus a number decreased by eleven

Example 5: Highlight the operation(s) and then write an algebraic expression for the phrases below.
a) The product of -5 and the sum of a number and 3
b) The difference between 12 and the sum of 8 and a number

Example 6: Write a word phrase for the algebraic expressions below.
a) \(n+15\)
b) \(7 x-1\)
c) \(\frac{m}{5}\)
d) \(\quad 9(y-7)\)

Example 7: Karen and Galen are building a tower on a table that is 3 feet tall ( 36 inches). Each block they add to the tower is 4 inches tall. Can you write a rule for how tall the tower will be after six levels of blocks are added? After 10 levels? After \(n\) levels?
\begin{tabular}{|c|c|}
\hline Number of Levels & Height in Inches \\
\hline 1 & \(1 \cdot 4+36\) \\
\hline 2 & \(2 \cdot 4+36\) \\
\hline 3 & \(3 \cdot 4+36\) \\
\hline 4 & \(4 \cdot 4+36\) \\
\hline 5 & \(5 \cdot 4+36\) \\
\hline 6 & \\
\hline 10 & \\
\hline\(n\) & \\
\hline
\end{tabular}

Example 8: Michael's produce farm has a bumper crop of corn this July. Therefore, when someone buys a dozen ears of corn, Michael gives them 13 ears of corn (called a "baker's dozen") rather than 12. Complete the table and make a chart for the sales of 1 to 5 dozen ears of corn. Lastly, if possible, write an algebraic expression for the sales of \(n\) dozen ears of corn.
\begin{tabular}{|c|c|}
\hline Number of Dozens & Number of Ears of Corn \\
\hline 1 & \\
\hline 2 & \\
\hline 3 & \\
\hline 4 & \\
\hline 5 & \\
\hline\(n\) & \\
\hline
\end{tabular}

\section*{Section 8.3 Writing Variable Equations from Words}

Looking Back 8.3
In the previous section, we wrote algebraic expressions for verbal expressions. Now, we are going to identify key words and use them to write algebraic equations. This is very similar to the previous section so if we understand those concepts, we will find the concepts in this section quite easy.

\section*{Looking Ahead 8.3}

The difference between an expression and an equation is that an equation has an equal sign. Some key words that may represent things that are equal include: is; yields; total. Key words for our work in this module include: sum, which represents addition; difference, which represents subtraction (not contrast); product, which means multiply (not something made); quotient, which means divide (not to fill a requirement).

Example 1: Write equations for each of the verbal and numerical statements below. Highlight the key words in the phrases and let \(n\) be a number.
a) The sum of a number and 16 is \(12 \quad\) b) \(\quad\) The product of a number and 16 yields 250
c) The product of 5 and the difference between a number and 8 is 15

Example 2: Jesus had 5 loaves of bread and 2 fish. He broke each into pieces to feed 5,000 people.
a. Let \(b\) represent the number of pieces a loaf of bread is broken into. Write an equation to represent the number of pieces Jesus broke each loaf of bread into to feed the 5,000 people. Solve for \(b\).
b. Let \(f\) represent the number of pieces a fish is broken into. Write an equation to represent the number of pieces Jesus broke each fish into to feed the 5,000 people. Solve for \(f\).

Example 3: Write equations for the phrases below.
a) Twice Sallie's wages are the same as her wages minus \(\$ 90\). (Let \(w\) be wages.)
b) \(\quad\) The temperature of \(45^{\circ}\) at 7 AM is increased by \(x^{\circ}\) to equal \(65^{\circ}\) at 4 PM .

Example 4: Write an equation to represent the relationship between Hours and Fuel (Gallons) given the information and table below.

Melinda and Keva are going on their first road trip. Melinda is traveling at a rate of 60 miles per hour (MPH) and gets 15 miles per gallon (MPG) of gasoline. Find the unit rate and complete the table.
\begin{tabular}{|c|c|}
\hline Hours & Fuel (Gallons) \\
\hline 1 & \\
\hline 2 & \\
\hline 3 & \\
\hline 4 & \\
\hline\(h\) & \\
\hline
\end{tabular}

\section*{Section 8.4 Variables, Coefficients, Constants, and Terms}

\section*{Looking Back 8.4}

We have discovered the properties of real numbers, how to perform integer operations, write expressions, and write equations. As we continue, we need to review some of the terminology we need to know in order to more fully understand the "language of equations."

Solving equations takes us from concrete thinking to abstract thinking. At times, we may get frustrated. However, if we keep at it, we will find we can do it. So, do not give up! God created us with amazing minds and good reasoning skills.

\section*{Looking Ahead 8.4}

Remember, a variable is a letter used to represent a number. The most used variables are \(x, y\), and \(z\). Variables vary (change) from equation to equation. Symbols such as \(\Delta, \square\), and \(\bigcirc\) may be used to represent numbers but variables are used for unknown numbers when solving equations.

The number in front of the variable is called a coefficient. This number is multiplied by the value of the variable to result in a product. If there is no number in front of our variable, the coefficient is understood to be 1 because \(1 x=x\) and \(1 y=y\). So, we are multiplying the variable by 1 , which gives us the same value of the variable. We do not write " \(x 5\) " or " \(y 5\) " but rather " \(5 x\) " or " \(5 y\) " because of the commutative property.

The Scripture tells us there is only one God, Jehovah. We believe in one God, Jehovah. We worship one God, Jehovah. However, God has many attributes and many names:

Jehovah God is Jehovah-jireh, the God who \(\qquad\)

Jehovah Rophe, the God who heals

Jehovah-shalom, the Lord who sends \(\qquad\)

Jehovah-rohi, the Lord my \(\qquad\)

Jehovah is one God. Jehovah is the coefficient. The names that come after the hyphen and follow Jehovah are like the variable. They may change. The represent a different part of God's character.
\begin{tabular}{|lll}
\hline \hline Example 1: Name the coefficient in the expressions below. \\
\hline a) \(5 x\) & b) & \(1 z\) \\
c) \(-13 y\) & d) & \(p\) \\
e) \(\frac{1}{4} b\) & f) & \(11 m\)
\end{tabular}

If an expression has a number only it is called a constant. Constants do not change; they stay constant. The value of 4 is always 4 in every equation. Like our God, who always stays the same and never changes, constants always stay the same.

In the expression \(4 x-8\), there are two terms. Terms are separated by a plus or minus sign. The first term is \(4 x\) and the second term is -8 because \(4 x-8\) may be written: " \(4 x+(-8)\)." Terms may be a variable, \(n\), a constant, 3 , or the product of a variable with a number, \(-5 n\).
Example 2: \(\quad\) Name the coefficient(s), variable(s), and constant(s) in the equations below.
a) \(2 x+3=8\)
b) \(x+4=9\)
c) \(\quad \frac{2}{3} p+\frac{1}{4}=\frac{1}{8}\)
d) \(14+3 m=-6\)
e) \(\quad-2 y-10=5\)
f) \(-4 y-5=2\)
g) \(\quad 0.5 r-1.2=3.4\)
h) \(3 x=4 y+2\)

In one expression or one equation, the variable represents the same number. That means we can add and subtract them; these are called like terms. Simply add or subtract the coefficients and keep the variable the same. Below are some examples of like terms:
\[
\begin{gathered}
3 x+5 x=8 x \\
-5 y+8 y=3 y \\
-5 x+3 y=-5 x+3 y
\end{gathered}
\]

The last one \((-5 x+3 y=-5 x+3 y)\) cannot be combined: \(-5 x+3 y \neq-2 x y\). Let \(x=3\) and \(y=4\) and check it. The base must be the same and the exponent must be the same to combine like terms.
Example 3: Underline the like terms and name the number of terms in each expression below. Next, combine the like terms to simplify them; if there are no like terms, the expression is already simplified.
a)
\(3 y+4-5 y\)
b)
\(a^{3}+a+a^{3}\)
c) \(3 x-3 y+6 z+6 x-4 t-2 z+4 y\)
d) \(3 m^{2}+2 n-5 m+6 m-n+3 n^{2}\)

\section*{Section 8.5 Open Sentences}

\section*{Looking Back 8.5}

In an expression, a variable can be replaced by any number. In an equation, there are numbers that are solutions to the equation. We will first try several values for a variable to see which makes the equation true.

Looking Ahead 8.5
Example 1: Given the equation \(3 x+5=23\), several values have been substituted for the expression \(3 x+5\) on the left side of the equation to see which ones give the solution of 23 on the right side of the equation. Complete the table below for the values of \(x \in\{3,4,5,6,7,8,9\}\) to see which ones solve the equation \(3 x+5=23\).
\begin{tabular}{|c|c|c|c|}
\hline\(x\) & \(3 x+5\) & \(=23\) & True or False \\
\hline & & & \\
\hline & & & \\
\hline & & & \\
\hline & & & \\
\hline & & & \\
\hline & & & \\
\hline & & & \\
\hline
\end{tabular}

There is only one number in the set that solves the equation. The solution is \(x=6\) for \(3 x+5=23\).
Example 2: Which of the numbers from the set \(\{1,2,3,4,5\}\) make the equation \(x^{2}-4=5\) true when substituted in for \(x\) ? Make a table of the data and show your work.
\begin{tabular}{|c|c|c|c|}
\hline \(\boldsymbol{x}\) & \(x^{2}-\mathbf{4}\) & \(=\mathbf{5}\) & True or False \\
\hline & & & \\
\hline & & & \\
\hline & & & \\
\hline & & & \\
\hline & & & \\
\hline
\end{tabular}

The only value that works for the equation is \(x=3\).

Example 3: If an equation is made up of all numbers, it can be true or false. For example, \((3)(-2)=(-1)(6)\) is true and \(3(2)+4=12-3\) is false. When an equation has a variable in it, it is called an open sentence. It may be true or false depending on the value of the variable. Are the statements below true, false, or open?
a) \(6+9=(-5)(-3)\)
b) \(5 x-8=23\)
c) \(92+3=100-5\)
d) \(2(4)-6=12+2(1)\)

\section*{Section 8.6 Balanced Equations \\ Looking Back 8.6}

When a scale is balanced, the weight on the left side of the balance is equal to the weight on the right side of the balance. When a scale is unbalanced, the weight on the left side of the balance is not equal to the weight on the right side of the balance.

\section*{Equal (Balanced)}


Unequal (Unbalanced)


If we remove \(1 g\) ( 1 gram) of weight from the right side of the "Unequal (Unbalanced)" balance above, it will go up because it has less weight and will be balanced with the left side \((2 g=2 g)\). If we add \(1 g\) of weight to the left side of the balance, it will go down because it has more weight and will be balanced with the right side \((3 g=3 g)\).

The fulcrum is in the center of the balance scale. The equal sign in an equation is like a fulcrum.
For the remainder of the module, black chips will represent positive numbers (because "in the black" means a bank account has a surplus) and red chips will represent negative numbers (because "in the red" means a bank account has a deficit).

Looking Ahead 8.6
Example 1: Represent the equation below using a balance scale. Represent the variable with a box. What must be in the box to make the equation balanced? Represent the constants with circles.
\[
x+3=5
\]

\(\qquad\) \(=\) \(\qquad\)

Whenever we add weight to one side of a balance, we must add weight to the other side of the balance to keep it balanced. Whenever we remove weight from one side of a balance, we must remove weight from the other side of the balance to keep it balanced. The same is true of equations. Whenever we add or subtract from one side of an equation, we must add or subtract from the other side of the equation to keep it balanced.
Example 2: Represent the equation below using a balance scale. Remove chips from both sides of the balance until you know what number must be in the box (the variable) to keep the scale balanced.
\[
2 x+3=5
\]

Let \(-2+2=0\) be 2 red chips plus 2 black chips. They cancel one another out (become zero) because they are opposite signs.
Example 3: Represent the equation below using a balance scale. What variable must be in the box to keep the scale balanced?
\[
x-4=3
\]

Example 4: Represent the equation below using a balance scale. What variable must be in the box to keep the scale balanced?
\[
2 x-3=\frac{1}{2}
\]

\section*{Section 8.7 Undoing Addition}

\section*{Looking Back 8.7}

When we have a variable with a negative in front of it, such as \(-x\), we pronounce it "the opposite of \(x\)." If \(x\) is a positive number, the opposite of \(x\) will be a negative number. If \(x\) is a negative number, the opposite of \(x\) will be a positive number. Therefore, we will let negative variables be represented by a red box and positive variables be represented by a black box. Moreover, because one red chip is -1 and one black chip is +1 , they "zap" each other or cancel each other out because \(-1+(+1)=0\). We were introduced to this concept in the previous section. Therefore:


This is an important concept to understand and use in the next two sections.

\section*{Looking Ahead 8.7}

To find out the value of the box on a balance scale, we must start removing things from the scale so the variable is left alone on one side of the scale and the value that it represents is left alone on the other side of the scale. This value, as represented by the chips, is a number.

Remember, whatever gets removed from the left side of the balance scale also gets removed from the right side to keep it balanced and whatever gets added to the right side of the scale gets added to the left side to keep it balanced. The same concept is true for equations.

If we add or subtract something from the left side of an equation, it must be added or subtracted from the right side of the equation as well to make sure both sides are equal.

In Matthew 22:37, we are reminded of the Golden Rule: "You shall love your neighbor as yourself." In whatever way you want others to treat you, you should treat them; whatever you want them to do for you, you should do for them.

We have a Golden Rule of Algebra that is very much like the Golden Rule found in the Scripture: "Whatever you do to one side of an equation, you must do to the other side of the equation."

Example 1: Tell what must be in the box \((x)\) to keep the scale below balanced. Write the equation represented by the scale.


Example 2: Tell what must be in the box \((x)\) to keep the scale below balanced. Write the equation represented by the scale.


Example 3: Tell what must be in the box \((x)\) to make the equation balanced.


Example 4: \(\quad\) Solve for \(x\) in the equation below using operations with numbers only and reverse thinking.
\[
-9=x+3
\]

Because we are solving for the variable (box/x), it is always best to remove the number (chips) from the side that contains the variable (box/ \(x\) ). In that way, the variable (box/ \(x\) ) is on one side of the equation (scale) by itself and the numbers (chips) are combined on the other side of the equation (scale). We call this isolating the variable: we are getting it on a side alone, or by itself, so we know what it equals. The value of the number is the value of the variable.

\section*{Section 8.8 Undoing Subtraction}

\section*{Looking Back 8.8}

Before this section, we used red chips (negative) to "zap" black chips (positive) and get zero. We used subtraction to "undo" addition. In this section, we will use addition to "undo" subtraction.

We already know that subtraction is the same as adding a negative and the opposite of subtraction is addition. This helps in the reverse thinking process. If the problem includes a variable with subtraction, we will have to use addition to work backwards and solve for the "unknown" variable. This is called "undoing subtraction." We undo subtraction with addition. Think of it like filling a glass with water. If we want to get back to the original glass, we have to empty out the water. "Pouring out" is the opposite of "Filling Up." We have to reverse the process to get back to where we started.

Looking Ahead 8.8
Because \(x-4=12\) is the same as \(12+4=x\), then \(x=16\). We can check this using the following method:
\[
\begin{gathered}
x-4=12 \text { when } x=16 \\
16-4=12 \\
12=12 \\
\text { This checks! }
\end{gathered}
\]

This is how we did these problems earlier in this module; we used reverse thinking. However, now that we have investigated algebra problems with the balance scale in mind, we can use the Golden Rule of Algebra and solve them algebraically (using algebra).

Example 1: Solve for \(y\) and check your solution: "undo" subtraction with addition.
\[
y-9=0
\]

Example 2: \(\quad\) Solve for what is in the box \((x)\) in the scale below.


Example 3: \(\quad\) Solve the problem from Example 2 another way.


Example 4: Use algebra to solve for the variable in the given equation.
\[
x-13=11
\]

Example 5: \(\quad\) Use algebra to solve for the variable in the given equation.
\[
-11=x-13
\]

\section*{Section 8.9 Undoing Multiplication \\ Looking Back 8.9}

We have been learning about "undoing" operations when we want to find the value of an unknown variable; there is some number that will solve the equation and we want to know what it is. Earlier, we learned that equations with \(x\) to the second power can have two solutions. For example, if \(x^{2}=4\), then \(x=2\) or \(x=-2\) because \((2)^{2}=4\) or \((-2)^{2}=4\).

Addition and subtraction are called inverse operations. The inverse of multiplication is division, so when using reverse thinking to solve a multiplication problem, use division. Because they are inverses, when working backwards to solve a division problem, use multiplication. We will work backwards to solve division problems using multiplication in the next section.

\section*{Looking Ahead 8.9}

If \(2 x=12\), then we know that double some number results in 12 . That number must be 6 because 6 is half of 12 . We divided 12 by 2 to get back to where we started from. If \(3 x=9\), then we know that some number tripled results in 9 . That number must be 3 because \(9 \div 3=x\) and \(x=3\).
\[
\text { Example 1: } \quad \text { Complete the balance scale below for } 2 x=12 \text { and solve for } x .
\]


Example 2: \(\quad\) Complete the balance scale below for \(3 x=9\) and solve for \(x\).


Remember, \(25 \div 25=1\) or \(\frac{25}{25}=1\). Any number divided by itself results in the identity element for multiplication and any number times 1 is itself. Therefore, any variable times 1 is also itself:
\[
1 \times m=m \quad 1 \times y=y \quad x \cdot 1=x \text { and } 1 x=x
\]

This is how we isolate the variable using division; we divide by the number that is being multiplied by the variable; we divide by the coefficient of the variable to isolate the variable.

If there is a fraction that is the coefficient of the denominator and the numerator is 1 , multiply by the denominator on both sides to clear the denominator. We can always multiply the fraction by its reciprocal to change it to 1 because 1 times any variable is just the variable. Make sure to multiply by the reciprocal on either side of the equation as well in respect to the Golden Rule of Algebra.

Example 3: Solve for the variable below using the Golden Rule of Algebra and check your solution.
\[
25=25 m
\]

Example 4: \(\quad\) Solve for the variable below using division.
\[
6 m=36
\]

Example 5: "Undo" multiplication using division to solve for the variable below and check your solution.
\[
6 m=22
\]

Example 6: \(\quad\) Solve for the variable below and check your solution using the reciprocal of \(\frac{1}{3}\).
\[
\frac{1}{3} t=21
\]

Example 7: Solve for the variable below by undoing multiplication using division.
\[
0.3 n=27
\]

Example 8: The decimal from Example 7 has been converted to a fraction. Solve for the variable below. Do you get the same solution as in Example 7?
\[
\frac{3}{10} n=27
\]

\section*{Section 8.10 Undoing Division}

\section*{Looking Back 8.10}

We have seen how addition problems can be written as subtraction problems and subtraction problems can be written as addition problems. Also, multiplication problems can be written as division problems and division problems can be written as multiplication problems. Addition "undoes" subtraction and subtraction "undoes" addition; multiplication "undoes" division and division "undoes" multiplication. These are called inverse relationships. In the previous section, we learned how to solve for the variable if it is part of a multiplication problem. In this section, we will learn how to solve for the variable if it is part of a division problem.

\section*{Looking Ahead 8.10}

Remember, \(m \div 4=-6\) is the same as \(-6 \times 4=m\). This means \(m=-24\), which is true because \(-24 \div 4=6\). A division problem may be written using a fraction bar because \(12 \div 4\) is 12 divided into 4 parts, which is \(\frac{12}{4}\) (which is equal to 3 ). This means \(m \div 4\) may be written " \(\frac{m}{4}\) " and \(m \div 4=-6\) may be written " \(\frac{m}{4}=-6\)."
Example 1: \(\quad\) Solve for the variable in the equation below.
\[
\frac{m}{4}=-6
\]

Example 2: \(\quad\) Clear the denominator to solve for the variable in the equation below.
\[
\frac{x}{-4}=-1.2
\]

Example 3: Use reciprocals to solve for the variable in the equation below.
\[
\frac{x}{-4}=-1.2
\]

Example 4: Use multiplication to undo division to solve for the variable in the equation below.
\[
\frac{y}{1.2}=6
\]

\section*{Section 8.11 Two-Step Equations}

\section*{Looking Back 8.11}

Some problems are very straightforward. We can see when a problem is a multiplication problem. We can see that a number is being multiplied by an unknown number to get another number. We know we can use division to "undo" multiplication.

Given a subtraction problem, we can see that a number is being subtracted from an unknown number to get another number. We know we can use addition to "undo" subtraction. However, sometimes these problems look a little different and we need to take a different approach.

Looking Ahead 8.11
Example 1: Solve the equation below and check your solution.
\[
n-2.1=68.1
\]

Example 2: \(\quad\) Show the steps to solve \(2.1-n=68.1\) and check your solution.

Remember, if there is no number in front a variable, the coefficient is understood to be 1 because \(1 \cdot n=n\). This means \(-n\) is understood to be \(-1 n\) because \(-1 \cdot n=-n\).
Example 3: \(\quad\) Show the steps to solve \(3.4=14-n\) and check your solution.

When we have a problem with a variable in the numerator that has a coefficient of 1 , then we can clear the denominator by multiplying by the denominator on both sides of the equation. In one step, we have isolated the variable and have a solution.

Example 4: Solve the equation below and check your solution.
\[
\frac{x}{20}=5
\]

When the variable has a coefficient of 1 but is in the denominator, we have to multiply by the variable to clear the denominator. This does not isolate the variable, but multiplies it by the number on the other side of the equation. Then we must divide by the new coefficient and this makes a two-step equation.
Example 5: \(\quad\) Solve the equation below and check your solution.
\[
\frac{20}{x}=5
\]

Example 6: Solve the equation below and check your solution.
\[
13=\frac{10}{y}
\]

\section*{Section 8.12 Multi-Step Equations}

\section*{Looking Back 8.12}

Balance is very important in solving equations and in solving real-world problems. God wants us to have balance in our lives. He wants to be the center of our lives, the fulcrum that keeps things balanced. He will supply us with wisdom and knowledge to keep our friends, family, work, service, and leisure all in balance.

James 1:5-8 says: "But if any of you lacks wisdom, let him ask of God, who gives to all men generously and without reproach, and it will be given to him. But let him ask in faith without doubting, for the one who doubts is like the surf of the sea driven and tossed by the wind. For let not that man expect that he will receive anything from God being a double-minded man, unstable in all his ways."

God does not want the scale of our lives to be going up and down. If He is the center of our lives, no matter the circumstances, He will keep us balanced and guide us in perfect peace.

Isaiah 26:3 says: "God will keep him in perfect peace whose mind is stayed on Thee."

\section*{Looking Ahead 8.12}

When we want to use reverse thinking to solve for a variable, we should think about the steps we would take to solve for the problem if we knew the number represented by the variable. Think about the operations we can use and the order of those operations. Reverse thinking uses inverse operations and the reverse order.

For example, in the equation \(2 x-5=13\) (given we know the value of the variable is \(x=9\) ), we will multiply by 2 firstly and subtract 5 secondly. The reverse order and operations are to add 5 firstly and divide by 2 secondly (given we do not know the value of the variable).

Example 1: Complete the steps to "do" and "undo" the equation below and check your solution for \(x\).
\begin{tabular}{|l|l|}
\hline \multicolumn{1}{|c|}{ "Do" } & \multicolumn{1}{c|}{ "Undo" } \\
\hline 1. Multiply by 2 & 2. Divide by 2 \\
\hline 2. Subtract 5 & 1. Add 5 \\
\hline
\end{tabular}

Example 2: Complete the steps to "do" and "undo" the equation below and check your solution for \(x\).
\begin{tabular}{|l|l|}
\hline \multicolumn{1}{|c|}{ "Do" } & \multicolumn{1}{c|}{ "Undo" } \\
\hline 1. Multiply by 3 & 2. Divide by 3 \\
\hline 2. Add 4 & 1. Subtract 4 \\
\hline
\end{tabular}
\[
3 x+4=25
\]

Example 3: Complete the steps to "do" and "undo" the equation below and check your solution for \(x\).
\begin{tabular}{|l|l|}
\hline \multicolumn{1}{|c|}{ "Do" } & \multicolumn{1}{c|}{ "Undo" } \\
\hline \begin{tabular}{l} 
1. Multiply by \(\frac{1}{3}\) or \\
divide by 3
\end{tabular} & \begin{tabular}{l} 
2. Divide by \(\frac{1}{3}\) or \\
multiply by 3
\end{tabular} \\
\hline 2. Subtract 2 & 1. Add 2 \\
\hline
\end{tabular}
\[
11=\frac{1}{3} x-2
\]

Example 4: Solve for \(x\) in the equation below and check your solution.
\[
60-2 x=40
\]

\section*{Section 8.13 Solving Equations with Fractions and Decimals}

\section*{Looking Back 8.13}

We started the first module of General Math by exploring fractions and decimals. We have investigated solving equations. We have investigated equations in which the coefficient of the variable is a fraction. These equations will include more than one fraction or decimal, which may be the coefficient of the variable or the constants or both. We will end this module where we started by using the Greatest Common Factor to add and subtract fractions.

In this section, the last section before the module review and test, we will look at and solve equations with fractions and decimals. We will also use what we know about reciprocals to multiply and divide fractions.

\section*{Looking Ahead 8.13}

To solve equations with fractions, find a common denominator or convert the fraction to a decimal if it is a rational number that ends (terminates).

Example 1: Combine like terms first to solve the equation below and check your solution.
\[
\frac{2}{3} x-\frac{3}{4} x=5
\]

Example 2: Clear the denominators first to solve the equation below because all of the denominators are 5.
\[
\frac{1}{5} x+\frac{1}{5}=\frac{2}{5}
\]

Example 3: \(\quad\) Using three different methods, solve the proportion below for \(x\). \(\frac{3}{x}=\frac{7}{6}\)

Example 4: Using three different methods, solve the proportion below for \(x\).
\[
\frac{x-3}{4}=\frac{1}{2}
\]```

