

Section 7.2 Scaling Up

Looking Back 7.2

Equal ratios are equivalent fractions. Think of the multiplication tables we learned many years ago.

The multiples of 3 are as follows: 3 6 9 12 15 18 21 24

The multiples of 5 are as follows: 5 10 15 20 25 30 35 40

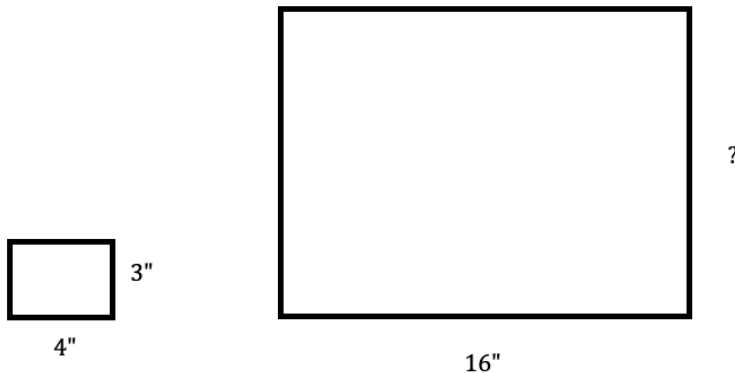
If we put the multiples of 3 in the numerator and the multiples of 5 in the denominator, we have equivalent fractions for $\frac{3}{5}$.

$$\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25} = \frac{18}{30} = \frac{21}{35} = \frac{24}{40}$$

The ratios of $\frac{3}{5}$ and $\frac{9}{15}$ are the same because $\frac{9}{15}$ simplifies to $\frac{3}{5}$. In decimal form, $\frac{3}{5} = 3 \div 5 = 0.60$, and $\frac{9}{15} = 9 \div 15 = 0.60$. When we change each fraction to a percent, we get 60%.

Looking Ahead 7.2

Example 1: Curtis looked at the rectangles on the left and right below. He was told that the length to length and width to width of the rectangles had the same ratios. The measure of width was erased on the right rectangle; Curtis wrote in 12". Karson erased the 12" and wrote in 15". Who is correct?



Scaling Up

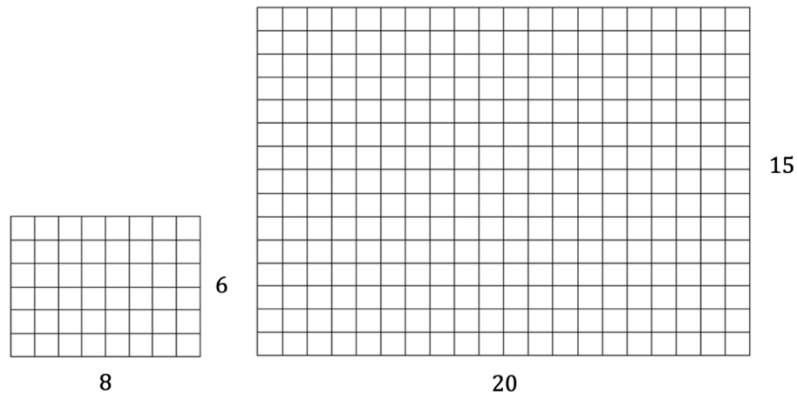
When we scale up, we multiply the numerator and denominator of the ratio by the same number.

Example 2: If 3 carnations sell for \$4.00, is that the same ratio as 9 carnations that sell for \$10.00? Complete the table to see that the ratios are not the same.

Carnations	3	6	9	12
Cost	\$4.00			

Section 7.3 Scaling DownLooking Back 7.3

All the problems we have investigated so far have increases in prices, lengths, and widths. We will now investigate problems that have decreases in prices, lengths, and widths. To find the measurement of the side of a larger rectangle, we multiply the side of a small rectangle by the scale factor.



Sides that match in a figure are called corresponding sides. The bottom length of the left rectangle corresponds to the bottom length of the right rectangle above. The right-side width of the left rectangle corresponds to the right-side width of the right rectangle. To match corresponding sides, figures must be oriented in the same direction.

The rectangles are called similar figures because the corresponding angles are congruent and the corresponding sides are proportional.

What goes in the blank for $8 \times \underline{\hspace{1cm}} = 20$? What goes in the blank for $20 \div 8 = \underline{\hspace{1cm}}$? What goes in the blank for $6 \times \underline{\hspace{1cm}} = 15$? What goes in the blank for $15 \div 6 = \underline{\hspace{1cm}}$? The scale factor is 2.5 because the length and width of the small rectangle are each multiplied by 2.5 to get the length and width of the large rectangle.

Example 1: Find the scale factor of the rectangles above, which is represented by x .

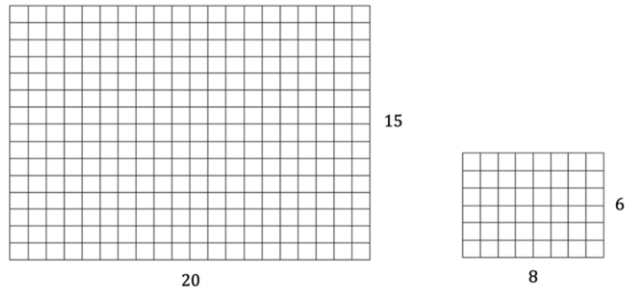
$$6x = 15$$

Example 2: Find the scale factor of the rectangles above, which is represented by x .

$$8x = 20$$

Looking Ahead 7.3

Using the rectangles from above (scaled down in proportion and shown below), try the following examples.



Example 3: As the width of the small rectangle decreases by 1, what does the width of the large rectangle decrease by?

Large Rectangle Width	15	12.5	10	7.5	5	2.5
Small Rectangle Width	6					

Example 4: As the length of the small rectangle decreases by 1, what does the length of the large rectangle decrease by?

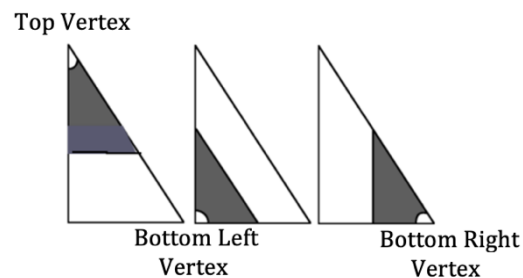
Large Rectangle Length	20	17.5	15	12.5	10	7.5	5	2.5
Small Rectangle Length	8	7	6	5	4	3	2	1

Section 7.4 Similar FiguresLooking Back 7.4

Before we begin this section, you will need an index card, ruler, and scissors. We are going to start by exploring the scale factors of right triangles.

- 1) Measure $\frac{1}{2}$ -inch from the bottom left corner of the white index card to the right and make a tick mark where it ends.
- 2) Measure 1-inch from the bottom left corner of the white index card going up and make a tick mark where it ends.
- 3) Connect the two tick marks using a ruler so that you have a straight diagonal line between them. This diagonal is now the hypotenuse of the right triangle formed from the bottom left corner of the white index card; the legs of this right triangle meet to form a T or L, an angle of 90° .
- 4) Cut out the triangle and color it black.
- 5) Turn the index card to the opposite corner from where you cut out the now black triangle.
- 6) Make a tick mark 1-inch to the right of the bottom corner and another 2-inches up from the bottom corner.
- 7) Draw the diagonal between the two tick marks from Step 6 using a ruler and then cut out the right triangle.
- 8) This triangle is white; do not color it. The white triangle should be larger than the black triangle.

Put the smaller black triangle on top of the larger white triangle and move the top vertex until it completely fits into it. Now, move the black triangle to the bottom left vertex until it completely fits into it. Finally, move the black triangle to the bottom right vertex of the white triangle until it completely fits into it. At each vertex, the angle of the black triangle and the angle of the white triangle are the same shape and size. We call these congruent angles.

Looking Ahead 7.4

The black triangle and white triangle from above are similar triangles. The angles are the same size, and the shapes are both right triangles; however, the lengths of the sides are not equal. The corresponding (matching) sides of the black triangle and white triangle are not equal, but they are proportional. Corresponding (matching) sides are bottom leg to bottom leg, or short leg to short leg, or long leg to long leg, or black hypotenuse to white hypotenuse. The triangles must be oriented in the same direction as they are in the diagram above (legs on left and bottom) in order to see the corresponding (matching) sides. Lastly, the corresponding sides in this case increase by the same scale factor.

Example 1: Fill in the blanks.

The black triangle and the white triangle from above are _____. All of the corresponding angles are _____. All of the corresponding sides are _____.

When we write the corresponding sides as two equal fractions, then we have two equal ratios. Two equal ratios are called a proportion. If one figure gets bigger to become the other, we say it is dilated. In the example above, the black triangle must be dilated to become the size of the white triangle. Put the black triangle on top of the white triangle on a table. Now, close one eye and look at the triangles. Lift the black triangle up until it completely covers the white triangle. That is a dilation.

Example 2: Below is a table with the lengths of the short legs of the black right triangle and its projected image, the white right triangle that you constructed. The table is for ten similar triangles and their projected images. Write the ten equivalent fractions from the table. If we divide the projected length by the original length, we get the scale factor.

Short Leg Original (in.)	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Short Leg Projected (in.)	1	2	3	4	5	6	7	8	9	10

This table shows a dilation. You must multiply the sides of the small triangle by a number greater than 1 to get a larger triangle.

The opposite of a dilation is a shrink. You must multiply the sides of the large triangle by a number less than 1 but greater than 0 (a proper fraction) to get a smaller triangle.

When you multiply by 2, you double the sides of the figure. When you multiply by $\frac{1}{2}$, you divide the sides of the figure by 2.

Section 7.5 Similarity Problems

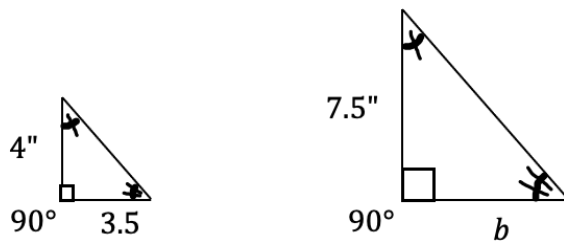
Looking Back 7.5

Now that we know about similar figures, let us use proportional reasoning to find missing side lengths in figures that are similar. Order is important. If we are comparing males to females on the left side of an equation, we must compare males to females on the right side of the equation (not females to males). Below are three equivalent ratios:

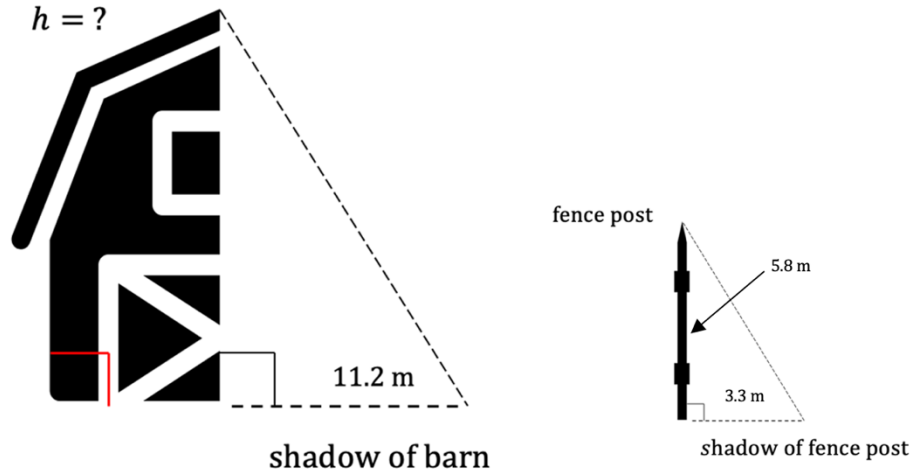
$$\frac{\text{part}}{\text{part}} = \frac{\text{part}}{\text{part}} \quad \frac{\text{whole}}{\text{whole}} = \frac{\text{whole}}{\text{whole}} \quad \frac{\text{part}}{\text{whole}} = \frac{\text{part}}{\text{whole}} \quad \text{However, this ratio is not: } \frac{\text{part}}{\text{whole}} \neq \frac{\text{whole}}{\text{part}}$$

Looking Ahead 7.5

Example 1: The lengths of the smaller figure are given. The height for the projected image is given, but the length for the base is not. Use internal and external ratios to find the missing base length.



Example 2: Use shadows from a fence to find the height of the barn.



A triangle can be made from the top of the barn out to the shadow's end.

A triangle can be made from the top of the fence post out to the shadow's end.

The triangles formed are similar. Set up the internal ratios and solve for the height of the barn. This compares one side of a figure to another side of the same figure.

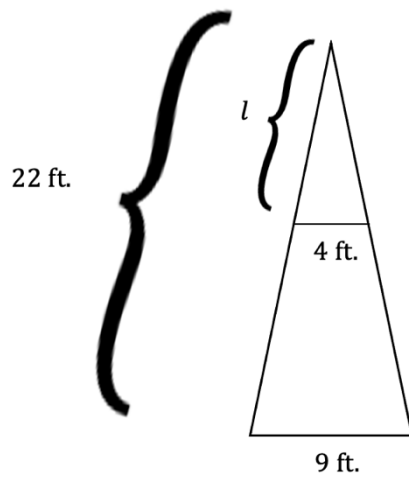
$$\frac{\text{barn height}}{\text{barn shadow}} = \frac{\text{fence post height}}{\text{fence post shadow}}$$

Example 3: Using the information from Example 2, set up the external ratios and solve for the height of the barn. This compares one side of one figure to the corresponding side of another figure.

$$\frac{\text{barn height}}{\text{fence post height}} = \frac{\text{barn shadow}}{\text{fence post shadow}}$$

The internal and external ratios in similar figures are the same.

Example 4: Some similar triangles are inside other triangles and share corresponding sides and angles. Find the missing length in the triangle below.



Section 7.6 Unit RateLooking Back 7.6

In order to compare two things in terms of price (to see which is a better buy), or room size (to see which has more space per person), we must find one unit of cost or one unit of space for each item and then compare the items. If one can of tomatoes (Brand A) costs \$0.69 and another can of tomatoes (Brand B) costs \$0.89, we know Brand A is a better buy (assuming both cans are the same size). However, if the cans are different sizes, how do we know which is a better buy?

To figure out which can is a better buy, we can calculate the cost per ounce that represents one unit of measure. Then we can compare one unit of one item to one unit of the other item and see which can has less cost per unit. In this case, the unit of measure is ounces. When no measurement is given, as in some of the previous examples or practice problems in this module, simply use a unit. That unit is one unit of weight or one unit of distance or one unit of cost, etc. It could be one gram, one pound, one mile, one kilometer, one cent, one dollar, etc. Simply use one unit!

Looking Ahead 7.6

Example 1: Let us revisit our problem in which 3 carnations cost \$4.00. Below is the complete table. Which is a better buy, 3 carnations for \$4.00 or 9 carnations for \$10.00?

Carnations	3	6	9	12
Cost	\$4.00	\$8.00	\$12.00	\$16.00

Remember, mathematics is not always exact in human finite terms, so we often must round or approximate numbers. Only God knows the exact number of grains of sand on the seashore. At best, man could gather a sample of grains of sand and make an approximate estimation of the number on the seashore. Matthew 10:30 says: “But the hairs on your head are all numbered.” His care toward us is unending; nothing is hidden from God. He knows the exact number of hairs on your head!

Psalms 193:17-18a says: “How precious also are thy thoughts to me, O God! How vast is the sum of them! If I should count them, they would outnumber the grains of sands.” How long do you think that problem would take to solve? It would take an infinite amount of time as it is constantly changing. But our God knows that number at any instant in time. We do know an Almighty God!

Example 2: For a)-c), given the scenario described, find the cost of the unit in question.
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- a) If 12 cans of beans sell for \$6.96, what is the cost of 1 can of beans?
- b) If 4 pairs of socks sell for \$5.99, what is the cost of 1 pair of socks?
- c) If 8 pieces of candy sell for \$1.00, what is the cost of 1 piece of candy?

If you buy 8 pieces of candy at a store, they will round the $12\frac{1}{2}$ cents up to 13 cents so they will always make a little more profit on approximate values. Therefore, you would pay $\$0.13$ (per piece) \times 8 pieces of candy, which is equal to \$1.04. However, $12\frac{1}{2}$ cents \times 8 pieces is equal to \$1.00. Stores usually have the unit price marked next to the cost of each item so you can compare it to similar items on the shelf. Sometimes, people just prefer a certain brand no matter what the unit cost.

Section 7.7 Unit ConversionLooking Back 7.7

Many problems require conversions from one unit to a different unit; sometimes (when we are comparing items of different measures), it takes many steps to make these conversions. The problem may be stated in miles per hour but the answer may be required in feet per second. In the Mars Orbiter example from Section 6.1 (the previous module), simple conversions from Metric units to English units would have spared a catastrophe!

Metric units are widely used around the world, but English units are still commonly used in the United States. The Mars Orbiter was programmed in one unit of measure (Metric units) and commanded in another unit of measure (English units), which caused it to get too close to Mars and explode.

When we change measurements from one unit of measure to another unit of measure, we are converting units. An example would be converting meters per second to feet per second. The first (meters per second) is the Metric unit and the second (feet per second) is the English unit. We have learned about the Metric System and what unit of measure each prefix refers to. Let us review the Metric System before we begin making conversions.

Looking Ahead 7.7

Example 1: Complete the chart below. Refer to Section 2.13 if you need help with the Metric System.

Prefix	Numerical Meaning
milli (m)	
meter (m) / liter (l) / gram (g)	

Example 2: Write out the two fractions that make up the ratio for 1 meter = 1,000 millimeters.

Example 3: Convert 300 meters to millimeters. Write down what you start with and what you want to end with. Let us call this the “cancel-keep” method. Write meters in the denominator so that it cancels with the meters in the numerator. Write millimeters in the numerator because that is what you want to keep in the numerator.

Example 4: Convert 3 millimeters to meters. Put millimeters in the denominator because you want to cancel the millimeters out. Put meters in the denominator because meters are what you want to end with.

Example 5: Suppose you are making curtains for a friend. Your friend tells you his windows are 50×60 , but neglects to tell you that he measured in centimeters. You measure in inches and when you take the curtains over to your friend’s house, they are too long for his windows. What should have been the length and width of the curtains in inches? The basic unit of conversion is $2.54 \text{ cm.} = 1 \text{ in.}$ Let us first convert 50 cm. to inches and then 60 cm. to inches.

Example 6: If a cheetah runs 55 MPH (as fast as a car), how fast does the cheetah run in meters per second? There are two types of conversions needed; miles to meters for distance (there is 1,609.34 meters in one mile). Another type of conversion is time, but there are two of those since there are 60 minutes in an hour and 60 seconds in a minute. This is a multi-step conversion problem.

Section 7.8 ProportionsLooking Back 7.8

We have been working with equivalent ratios. When we have two ratios that are equal to one another, we have a proportion. Similar figures are proportional. We often have to solve problems in everyday life using proportional reasoning. The scale factor is a constant of proportionality. It is the magnitude of increase or decrease. The formal definition of a proportion is an equation stating that two ratios are equal.

$$\frac{4}{5} = \frac{36}{45}$$

The second ratio does not reduce to the first ratio. It is just renamed. They are the same: 4 out of 5 is the same as 36 out of 45 when we scale up. Mathematically speaking, $\frac{a}{b} = \frac{c}{d}$ is a proportion when $b \neq 0$ and $d \neq 0$. If we know the value of a , b , and c , we can find d . If we know the value of b , c , and d , then we can find a . If we know the value of any three of the variables, we can find the value of the fourth one.

Looking Ahead 7.8

Example 1: Find the value of d in the proportion below.

$$\frac{12}{13} = \frac{24}{d}$$

Not all proportions are so simple to solve. There may not be an integer that divides into both ratios so that the remainder of each is zero as we have previously seen. It may be a fraction or a decimal number.

Example 2: Use reverse thinking to find d in the proportion below.

$$\frac{5}{6} = \frac{61}{d}$$

Example 3: Solve for d in the proportion below by clearing the denominator first. This is done by multiplying the denominator on both sides of the equation.

$$\frac{5}{6} = \frac{61}{d}$$

Example 4: Clear the denominators in $\frac{a}{b} = \frac{c}{d}$ to show that cross-products are equal.

Because $\frac{a}{b} = \frac{c}{d}$ represents two ratios, it can be written: “ $a : b = c : d$ ” (read: “a is to b as c is to d”). When we write a proportion in this form, b and c are called the means and a and d are called the extremes.

$$\begin{array}{c} \text{Extremes} \\ \text{┌───────────┐} \\ a : b = c : d \\ \text{└───────────┘} \\ \text{Means} \end{array}$$

We can set the product of the means equal to the product of the extremes and divide by the number next to the variable we are solving for.

This is called the cross-product property because in a proportion, clearing the denominators on both sides of equivalent fractions give the following formation:

$$\frac{a}{b} = \frac{c}{d}$$

Which is:

$$ad = bc$$

Example 5: Use the cross-product property to find a if $b = 122$, $c = 65$, and $d = 1,586$.

Example 6: If 16 yards of material sell for \$144.00, how much does 18 yards of material sell for? Let the selling price of the material be s and set up a proportion first.

Example 7: Why would the proportion below be incorrect for Example 6?

$$\frac{16}{\$144} = \frac{s}{18}$$

Section 7.9 PercentLooking Back 7.9

We have been working with percents throughout this module. Percent means per one hundred. One unit of anything is 1 whole unit or 100%.

Given \$1.00, consider the following information and follow the instructions given:

One (1) cent of \$1.00 is 1 cent out of 100 cents ($\frac{1}{100}$); $\frac{1}{100}$ is 1% because we have 1 per hundred. We can also write " $\frac{1}{100}$ " (a fraction) as "0.01" (a decimal). Divide 1 by 100 on your calculator and see what you get.

Ten (10) cents of \$1.00 is 10 cents out of 100 cents ($\frac{10}{100}$); $\frac{10}{100}$ is 10% because we have 10 per hundred. We can also write " $\frac{10}{100}$ " (a fraction) as "0.1" (a decimal). Divide 10 by 100 on your calculator and see what you get.

Looking Ahead 7.9

For Example 1, 2, and 3, fill in the blanks.

Example 1:

Twenty-five (25) cents of \$1.00 is _____ cents out of _____ cents or _____; $\frac{25}{100}$ is _____% because we have _____ per hundred. We can also write " $\frac{25}{100}$ " (a fraction) as _____ (a decimal), which is the same as _____ cents.

Example 2:

Fifty (50) cents of \$1.00 is _____ cents out of _____ cents or _____; $\frac{50}{100}$ is _____% because we have _____ per hundred. We can also write " $\frac{50}{100}$ " (a fraction) as _____ (a decimal), which is the same as _____ cents.

Example 3:

Eighty (80) cents of \$1.00 is _____ cents out of _____ cents or _____; $\frac{80}{100}$ is _____% because we have _____ per hundred. We can also write " $\frac{80}{100}$ " (a fraction) as _____ (a decimal), which is the same as _____ cents.

One-hundred (100) cents is 100 cents out of 100 cents ($\frac{100}{100}$); $\frac{100}{100}$ is 100% because we have 100 per hundred. We can write " $\frac{100}{100}$ " as 1 whole, which is \$1.00. If $1 = 100\%$, then $2 = 200\%$, $3 = 300\%$, etc.

Example 4: Convert the decimal 0.75 to a fraction and a percent.

Example 5: Convert the decimal 2.33 to a fraction and a percent.

Example 6: Convert the percent 22% to a fraction and a decimal.

Example 7: Convert the fraction $\frac{2}{25}$ to a decimal and a percent.

Example 8: Convert the fraction $\frac{1}{8}$ to a decimal and a percent.

Example 9: A laboratory had 8 rats for testing at the beginning of the year. By the end of the year, the number of rats had increased by 275%. How many rats did the laboratory have at the end of the year?

Section 7.10 Finding the Amount in a Percent ProblemLooking Back 7.10

There are many problems involving percents that can be solved using proportions (given direct multiplication cannot be done). Once equations are set up for percent problems, the following algorithm can be used to determine the solution: Percent of Total Number = Amount. The “Amount” could be an increase or a decrease.

Finding the amount is the most direct method to solving a problem that involves percents because the percent can be converted to a decimal and then multiplied (because “of” means multiply) directly by the total number. This is a one-step multiplication problem.

$$\% \text{ of Total Number} = \text{Amount}$$

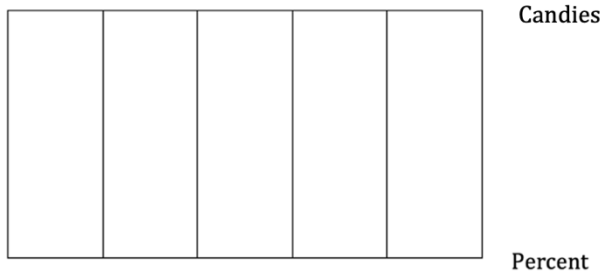
Convert the percent to a decimal number

$$\text{Decimal Number} \times \text{Total Number} = \text{Amount}$$

Looking Ahead 7.10

Example 1: About 20% of the candies in a bag of Rainbow Candy are the color blue. If there are 60 candies in a small bag, approximately how many will be blue?

Example 2: Complete the graphical representation of the information about Rainbow Candy in Example 1.



Example 3: There are 150 students in the 8th-grade class at a school. Of those 150, 22% stayed at school and did not go on the Washington D.C. class trip. How many students went on the class trip?

Example 4: Use a proportion to solve Example 3.

Section 7.11 Finding the Total Number in a Percent Problem

Looking Back 7.11

For the next few sections, we will be using the algorithm below to solve percent problems:

$$\% \text{ of Total} = \text{Amount}$$

The “Amount” is easiest to find because the percent can be converted to a decimal and multiplied directly by the total number next to it.

The total number and percent can be found using different methods. The two methods we will investigate are setting up proportions and solving by division.

In this section, we will explore “Finding the Total Number.”

Looking Ahead 7.11

Example 1: About 25% of entering freshmen at an online college choose to take a multi-media web design class their first semester. If 20 students are enrolled in the class, how many entering freshmen are there at the online college?

				Students
				Percent

Example 2: Solve the problem in Example 1 using proportions.

Example 3: Suppose 85% of the students in multi-media classes play an instrument as well. If 244 students play an instrument, how many students take multi-media classes? Use division and proportions to find your answer.

It is important to think about what percent of your money to save and what percent of your money to spend. However, even more important may be to think about this: Give to God first, save second, and spend last.

God will always bless our giving because we are giving back to Him that He has given to us. This is one test in Scripture the Lord encourages us in: He challenges us to give more!

In Malachi 3:10, we find the following message:

“ ‘Bring ye all the tithes into the storehouse, that there may be meat in mine house, and prove me now herewith,’ saith the LORD of hosts, ‘If I will not open you the windows of heaven, and pour you out a blessing, that there shall not be room enough to receive it.’ ”

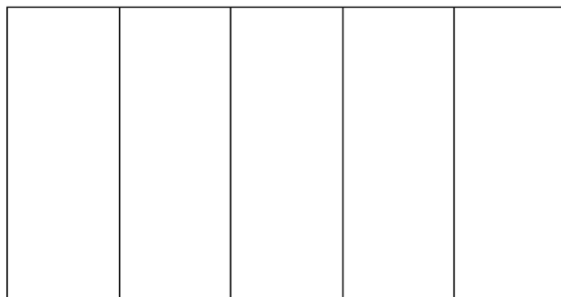
Example 4: Emelio gives 22% of what he makes each week to the church. If he gives \$55 to the church each week, how much does Emelio make in a week?

Section 7.12 Finding the Percent in a Percent ProblemLooking Back 7.12

We have now solved percent problems in which the amount is missing or the total number is missing. The final thing to find is the percent. Again, we can make it a division problem or a proportion. The only difference is that when we use a division problem, the final answer is a decimal and must be converted to a percent. When we solve for a percent using a proportion, the final answer is a percent and does not need to be converted to a decimal or fraction.

Looking Ahead 7.12

Example 1: Twenty-five (25) students entered a spelling bee. Five (5) of the students made it to the final round. The local newspaper reported the percent of participants that made it to the final round; what percent was reported?



Students

Percent

Example 2: Use proportions to solve the problem in Example 1. The answer will be a percent (per one-hundred).

Example 3: There are 32 people living in a condominium community. There are 12 people in the community who are over the age of 65. What percent of the condominium members are considered senior citizens (over 65)?

Section 7.13 Estimating Taxes and TipsLooking Back 7.13

We have been solving problems that are very practical and are useful in everyday life. When we are shopping, these methods can be used with mental mathematics to find the sale price of items or the amount of tax on a purchase. Rounding and estimating help us use mental mathematics to determine how much money to leave as a tip when we eat at a restaurant. Ratios and proportions are probably used more than any other mathematical device to solve every day problems.

In Module 1, we learned that when we multiply by 10, we move the decimal point one place to the right. In this section, we will multiply by 10%, which is $\frac{10}{100}$ simplified to $\frac{1}{10}$ and 0.1 in decimal form. Multiplying by 10% moves the decimal place one place to the left.

$$10\% \text{ of } \$100.00 = \$10.00 \text{ (because } 0.1 \times \$100.00 = \$10.00\text{)}$$

$$10\% \text{ of } 10 = \$1.00 \text{ (because } 0.1 \times \$10.00 = \$1.00\text{)}$$

$$10\% \text{ of } \$1.00 = \$0.10 \text{ (because } 0.1 \times \$1.00 = \$0.10\text{)}$$

Putting these together makes figuring sales tax, discounts, or tips easy to figure out in one's head.

Looking Ahead 7.13

Example 1: A blouse costs \$9.98. It is on sale for 20% off. Approximately how much does the blouse cost? Round \$9.98 to \$10.00.

Example 2: How much will the blouse from Example 1 cost given sales tax is 5%?

Example 3: A pair of shorts cost \$11.98. They are on sale for 15% off and tax is 7%. What is the total cost of the shorts?

Example 4: A meal at a restaurant costs \$46.21. It is proper to leave an 18% tip. How much is the total bill given one leaves the proper 18% tip?

Example 5: If a starting teacher earns a salary of \$36,000.00 but 25% is taken out for taxes, how much will the starting teacher bring home in one year?