## Module 2 Working with Number Systems

Section 2.1 The Greatest Common Factor (GCF)

$$
\text { Looking Back } 2.1
$$

In this module, we will learn about factors of numbers and multiples of numbers. Relationships between the two will be investigated. The concepts in this section and the next section are very important and will prepare us to work with fractions later in this module.

A factor is an integer that can be divided into a number a certain number of times with no remainder. The greatest factor of any number is the number itself. Sometimes, this factor changes when we find the Greatest Common Factor of two or more numbers.

Example 1: List all the factors of the numbers given. Stop when the factors start repeating.
a) 22
b) 13
c) 24
d) 15

When we have two or more given numbers, the Greatest Common Factor (GCF) is the largest integer that will divide into all the given numbers a specific number of times with no remainder.

Example 2: Find the Greatest Common Factor (GCF) of 27 and 42. First, list the factors of each and then find the largest factor common to both.

## Looking Ahead 2.1

Another way to find the Greatest Common Factor is to use prime factorization. This helps with really large numbers because it takes a long time to list all of the factors. We keep breaking the number into factors until all of the factors are prime numbers and cannot be broken down any further. Think about our math facts when doing this; we can use reverse thinking to break down larger numbers: If $9 \cdot 3=27$ and $3 \cdot 9=27$, then $27 \div 3=9$ and $27 \div 9=3$. We can think of these calculations as fact families; in this case, 27 is the parent and 3 and 9 are its children. What a fun fact family!

Example 3: Find the GCF of 48 and 64 using prime factorization.

Example 4: What is the GCF of 50 and 75?

Factoring can be a bit trickier when we are dealing with variables. We know that $5 x$ means 5 multiplied by some number $x$ and $10 x^{2}$ means 10 multiplied by $x$ multiplied by $x$ again. To find the Greatest Common Factor with variables, let us break down the number into factors and the letters into factors and find the largest number and variable (letter) common to both.

Example 5: $\quad$ Find the GCF of $18 x^{2} y$ and $36 x y^{2}$.

Example 6: $\quad$ Find the GCF of $42 m n$ and $35 m^{2} n^{2}$.

Example 7: $\quad$ Find the GCF of $5 a b$ and $9 x y$.

## Section 2.2 The Least Common Multiple (LCM)

## Looking Back 2.2

We know the largest factor of two or more numbers is called the Greatest Common Factor (GCF). We also know the GCF will always be smaller than both numbers or smaller than one number and equal to the other number because we divide to find the factors.

The same is true of multiplication with variables. To find the multiples of a number means to multiply that number by 1 , then by 2 , then by 3 , then by 4 , etc. The multiplication table lists the multiples of numbers 1 through 10 across each row and down each column. We have found the first ten multiples of a number many times. The first ten multiples of 2 are $2,4,6,8,10,12,14,16,18$, and 20 . These are even numbers because we multiply each of numbers 1 through 10 by 2 to get them. They are also used to find a common denominator when adding or subtracting fractions.

## Looking Ahead 2.2

When we have two or more numbers, the Least Common Multiple (LCM) is the smallest integer that is common to both numbers. To find the LCM, multiply both numbers by 1 , then 2 , then 3 , etc. and stop multiplying when an integer they have in common is found: that common integer is the Least Common Multiple (LCM).

Example 1: $\quad$ Find the Least Common Multiple (LCM) of 12 and 18.

Example 2: Find the Least Common Multiple (LCM) of 12 and 24.

## Looking Ahead 2.2

Now we will see that the Least Common Multiple (LCM) will always be larger than both numbers or larger than one number and equal to the other number. Is there ever a time when two numbers will not have an LCM? Try two numbers such as 4 and 7. The LCM will be the product of the two numbers.

Example 3: Find the Least Common Multiple (LCM) of 20 and 45.

Sometimes, it takes a long list to find the LCM of two or more numbers. Again, it is best at those times to use prime factorization.

Example 4: Find the LCM of 4, 9, and 15 using prime factorization. After you find the prime factors, keep the larger of the common factors and all the other factors and multiply all of them together.

Example 5: Find the LCM of 16, 36, and 28 using prime factorization.

In terms of $x y^{2}$ and $x^{3} y$, there are $x$ 's and $y$ 's common to both. The greatest degree of the $x$ variable is 3 . The greatest degree of the $y$ variable is 2 . The Least Common Multiple of both terms is $x^{3} y^{2}$. The first term $x y^{2}$ can be multiplied by $x^{2}$ to get $x^{3} y^{2}$, and the second term $x^{3} y$ can be multiplied by $y$ to get $x^{3} y^{2}$.

Example 6: What is the LCM of $a^{3} b^{2}, a b^{5}$, and $a^{2} b^{3}$ ?

Let us summarize what we have learned in the first two sections of this module about factors and multiples before we move on to fractions in Section 2.3 Proper Fractions.

When $m$ is a multiple of $n$ in which $m$ is the dividend and $n$ is the divisor, there will be a division with no remainder. This means $m \div n$ is equal to $k$, and $k \cdot n$ is equal to $m$ in which $0 \leq r<n$ where $r$ is the remainder. For example, if $32 \div 4$ is equal to $k$ then $k \cdot 4$ is equal to 32 and $k$ is equal to 8 , and the remainder is 0 .

If we do not know whether $m$ is a multiple of $n$ then we can check to see if there will be division with a remainder. If $q$ is the quotient and $r$ is the remainder, then $m$ is equal to $(q \cdot n)+r$ and $q \cdot n \leq m$ in which $0 \leq r<n$. For example, $32 \div 3$ is equal to $(10 \cdot 3)+2$, and $(10 \cdot 3) \leq 32$, and the remainder is 2 .

In the first example, $32 \div 4, q \cdot n$ is equal to $m$ and there is no remainder $(r=0)$. In the second example, $32 \div 3, q \cdot n$ is less than $m$ and there is a remainder $(r<n)$.

Notice that $47 \div 3$ is equal to 15 R. 2 (15 with a remainder of 2 ) and $62 \div 4$ is equal to 15 R. 2 , but they are not equal.

This means, 47 is equal to $(15 \cdot 3)+2$ and $0 \leq 2<3$. If 47 apples are divided equally into 3 groups, there are 15 apples in each group with 2 apples leftover, which do not fill a group of 3 . This means 15 is the largest multiple of 3 that is less than or equal to 47 .

List the multiples of 3 until you get close to 47 .

Starting at 0 , you will count fifteen numbers until you get to 45 , and 45 plus 2 is 47 . If you count to 48 you have gone too far because 48 is greater than 47 .

This also means, $62=(15 \cdot 4)+2$ and $0 \leq 2<4$. If 62 apples are divided into 4 groups, there are 15 apples in each group with 2 apples leftover, which do not fill a group of 4 . This means 15 is the largest multiple of 4 that is less than or equal to 62 .

List the multiples of 4 until you get close to 62 .

Starting at 0 , you will count fifteen numbers until you get to 60 , and 60 plus 2 more is 62 . If you count to 64 you have gone too far because 64 is greater than 62 .

Example 7: $\quad$ Find the LCM of $10 x^{2} y$ and $15 x y^{3}$.

## Section 2.3 Proper Fractions

## Looking Back 2.3

In the early days of our mathematics courses, the emphasis was on whole numbers. In the later years of our mathematics courses, fractions are used as much as whole numbers.

Fractions and percentages are used in everyday life. In cooking, recipes often involve fractions. In reading scripture, we are told to tithe a fraction (at least one-tenth) of earnings to our spiritual leaders for their work.

A fraction is a ratio of two things. The numerator is the top number in the fraction and the denominator is the bottom number in the fraction. The top number is the parts of the whole that represent one unit. The bottom number is the total number of parts in one whole unit. A proper fraction is a fraction in which the numerator is less than the denominator. Sometimes, the numerator is greater than the denominator; this is called an improper fraction.


The table on the left shows 4 out of 5 cupcakes on the plate. The table on the right shows 3 out of 5 cupcakes on the plate. Both pictures show 5 cupcakes; 5 cupcakes represent all of our cupcakes or 1 unit. Each cupcake is the same size and shape, so each cupcake is 1 out of 5 .

Let us suppose it is your birthday. You are really hungry, and you really enjoy cupcakes. Therefore, you pick the plate on the left to eat from because $\frac{4}{5}>\frac{3}{5}$.

To compare, add, or subtract fractions, we must have a common denominator. This will allow us to do all our work with only the numerators. The parts of the whole must be the same shape and size. It is easy to compare fractions when the denominator is the same: just compare the numerators as if the denominator is not even there.

The denominator tells us how many pieces are in the whole (one unit); for example, if we have one apple pie and divide it into 6 slices, then the denominator is 6 because there are 6 pieces in the whole pie. The numerator tells us how many parts of the 6 slices we are using: in this case, eating! Each slice is 1 out of 6 pieces so our numerator is 1 . Therefore, if we eat 1 piece out of the 6 pieces of pie, we have eaten $\frac{1}{6}$ of the apple pie.

If Emily and Lexie each have 1 piece of an apple pie, then together they have 2 out of the 6 pieces of pie or $\frac{2}{6}$ of the pie. If the same pie were cut into only 3 pieces, then each piece is 1 out of 3 pieces of the pie or $\frac{1}{3}$ of the pie. If we cut each of the 3 pieces in half, we get 6 pieces and there are 2 of the 6 pieces in each third, so $\frac{2}{6}$ is equal to $\frac{1}{3}$.

We know that multiple smaller pieces will fit into larger pieces; for example, 2 out of 6 pieces of apple pie fit into 1 out of 3 pieces of apple pie. The larger pieces must be cut to become the smaller pieces; for example, $\frac{1}{3}$ piece of the apple pie must be cut into $\frac{2}{6}$ pieces of apple pie. Therefore, 6 is the Least Common Multiple of 3 and 6 .

In summary, the common denominator of two fractions is the Least Common Multiple of the two denominators.

## Looking Ahead 2.3

Example 1: Find the common denominator of the two fractions given.
a) $\quad \frac{1}{2}$ and $\frac{1}{3}$
b) $\quad \frac{1}{2}$ and $\frac{2}{5}$
c) $\quad \frac{3}{5}$ and $\frac{4}{7}$

Example 2: Simplify the fractions by finding the Greatest Common Factor (GCF) of the numerator and denominator.
a) $\frac{16}{24}$
b) $\frac{75}{105}$
c) $\frac{48}{72}$

Example 3: Compare the fractions by using one of the following symbols: $<,>,=$.
a) $\frac{1}{3} \longrightarrow \frac{2}{7}$
b)
b) $\frac{2}{11}-\frac{3}{5}$
c) $\frac{1}{8} \longrightarrow \frac{1}{4}$

Example 4: List the following fractions in order from least to greatest.
$\begin{array}{lllll}\frac{1}{3} & \frac{2}{5} & \frac{3}{7} & \frac{4}{5} & \frac{1}{7}\end{array}$

## Section 2.4 Addition and Subtraction of Proper Fractions

## Looking Back 2.4

We know how to use the Least Common Multiple (LCM) to compare fractions. We also know how to find the Greatest Common Factor (GCF) to simplify fractions. We will now add and subtract proper fractions using the LCM to find the least common denominator. Any common denominator will work but the least is the easiest!

## Looking Ahead 2.4

Once we have renamed the two fractions using the Least Common Denominator, we can add the numerators (the parts of the whole). The denominators for 1 unit will remain the same because it is common to both.

Example 1: Add the fractions below.

$$
\frac{2}{3}+\frac{1}{5}
$$

Example 2: Subtract the fractions below.

$$
\frac{5}{6}-\frac{5}{8}
$$

Example 3: Which of the solutions below is greater?

$$
\frac{1}{2}+\frac{2}{3} \text { or } \frac{4}{5}-\frac{1}{4}
$$

## Section 2.5 Multiplication of Proper Fractions

## Looking Back 2.5

Adding and/or subtracting fractions can be difficult because we must find a common denominator to do it. Multiplication of fractions is much easier because all we must do is multiply the numerator by the numerator and denominator by the denominator and then simplify the solution. Below is a multiplication of fractions problem that shows how it is done:

$$
\frac{2}{3} \cdot \frac{5}{6}=\frac{2 \cdot 5}{3 \cdot 6}=\frac{10}{18}=\frac{5}{9}
$$

Let us investigate an area model to see why $\frac{2}{3}$ multiplied by $\frac{5}{6}$ is equal to $\frac{2}{9}$.

## Looking Ahead 2.5

The multiplication problem $\frac{2}{3} \cdot \frac{5}{6}$ is really asking the question: "what is two-thirds of five-sixths?" In the model on the left below, the three columns represent thirds. The yellow represents two out of three columns, which is $\frac{2}{3}$. In the model in the middle below, the six rows represent sixths. The red represents five out of six rows, which is $\frac{5}{6}$. If we lay the left model over the middle model and align the rows and columns, we get an orange overlap, which is the model on the right below. In the right model, ten out of eighteen squares (rows by columns) are orange.


We can arrange the model to the right in order to simplify it. If we put all the double shaded (or orange) squares on the top, we have nine rows going from top to bottom and five of the rows are shaded orange. In the rectangle to the right, there are eighteen squares in total. Ten of the squares are orange; this represents ten of eighteen parts, which is $\frac{10}{18}$. There are five shaded rows out of nine total rows, which is $\frac{5}{9}$. Rearranging the rectangles is the same as renaming our fraction and $\frac{10}{18}$ in simplified
 form is $\frac{5}{9}$.

[^0]Example 2: Multiply the fractions below, but first write out all of the prime factors and then simplify the common factors in the numerator and denominator so they cancel out to become 1.

$$
\frac{8}{15} \cdot \frac{9}{20}
$$

Example 3: Multiply the fractions below, but first simplify the common factors.

$$
\frac{6}{7} \cdot \frac{2}{6} \cdot \frac{12}{14}
$$

Example 4: Multiply the fractions below, but first simplify the common factors.

$$
\frac{2}{5} \cdot \frac{25}{60} \cdot \frac{15}{28}
$$

Example 5: Multiply the fractions below, but first simplify the common factors.

$$
\frac{3}{8} \cdot \frac{4}{9} \cdot \frac{12}{16}
$$

## Section 2.6 Division of Proper Fractions

## Looking Back 2.6

Multiplication of fractions is rather straight forward. Division of fractions is much more complicated. Let us investigate how we can simplify this process. We will begin by simplifying the following expression:

$$
\frac{3}{4} \div \frac{1}{2}
$$

The yellow squares to the right represent $\frac{3}{4}$ of the stick. The fraction $\frac{3}{4}$ represents 3 out of 4 equal pieces of the whole (the whole stick in this case).

The red shade represents $\frac{1}{2}$ of the same stick. The fraction $\frac{1}{2}$ is 1 out of 2 equal pieces of the whole stick.


We can see that the expression $\frac{3}{4} \div \frac{1}{2}$ is asking the question: "How many 1 out of 2 equal pieces of a stick fit into 3 out of 4 equal pieces of the same stick?" In other words, "How many red pieces fit into 3 yellow pieces?" One red piece fits into 2 yellow pieces, which means $\frac{1}{2}$ of a red piece fits into the remaining yellow piece.

Therefore, there are $1 \frac{1}{2}$ red pieces that fit into 3 yellow pieces.

$$
\frac{3}{4} \div \frac{1}{2}=1 \frac{1}{2} \text { and } \frac{3}{4} \cdot \frac{2}{1}=\frac{6}{4}=1 \frac{2}{4}=1 \frac{1}{2}
$$

In other words, dividing a number by a fraction is the same as multiplying the number by the reciprocal of the fraction.

## Looking Ahead 2.6

The reciprocal of a divisor is $\frac{1}{\text { divisor }}$; the reciprocal of 5 is $\frac{1}{5}$. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.
Example 1: Find the reciprocal of each fraction in the table below.

| Fraction | Reciprocal |
| :---: | :---: |
| $\frac{2}{7}$ |  |
| $-\frac{1}{4}$ |  |
| 3 |  |
| $\frac{5}{9}$ |  |

Example 2: Divide the fractions below.

$$
\frac{5}{8} \div \frac{3}{4}
$$

Example 3: Simplify the expression below.

$$
\frac{4}{15} \div \frac{1}{3}
$$

Example 4: Divide the fraction by the whole number below.

$$
\frac{4}{5} \div 2
$$

(Notice that we made the whole number a fraction first by putting it over 1 or writing it as a 2 to 1 ratio because $\frac{2}{1}=2 \div 1=2$. Any whole number may be made into a fraction by dividing by 1 .)

## Section 2.7 Improper Fractions and Mixed Numbers

## Looking Back

A proper fraction is when the |numerator| is less than the |denominator|. Proper fractions are greater than 0 , but less than 1.

When the |numerator| is greater than or equal to the |denominator|, the fraction is called an improper fraction. For example, $\frac{11}{4}$ and $\frac{13}{5}$ are improper fractions. An improper fraction is always equal to 1 or greater than 1. An improper fraction is 1 if the |numerator| is equal to the |denominator|. An improper fraction is greater than 1 if the |numerator| is greater than the |denominator|.

An improper fraction can be changed to a mixed number that is made up of a whole number and a proper fraction. A mixed number can also be changed to an improper fraction. We will do both in this section.

Example 1: $\quad$ Convert the improper fraction $\frac{19}{5}$ to a mixed number.


Example 2: $\quad$ Convert the mixed number $5 \frac{3}{4}$ to a proper fraction.


Looking Ahead 2.7
Example 3: Convert the mixed number $3 \frac{3}{8}$ to an improper fraction.

| Example 4: Convert the mixed number $9 \frac{1}{3}$ to an improper fraction. |
| :--- | :--- |

Example 5: Convert the improper fraction $\frac{42}{11}$ to a mixed number. Check your answer by converting it back to an improper fraction.

Example 6: Which is greater, $\frac{9}{7}$ or $\frac{4}{3}$ ?

## Section 2.8 Adding Mixed Numbers

## Looking Back 2.8

We have reviewed proper fractions and we have performed operations with proper fractions. We have also reviewed improper fractions and converted them to mixed numbers. We will now perform operations with improper fractions and mixed numbers.

We can find a common denominator and then add the numerators of improper fractions. This might result in really large numbers. At the end, convert the improper fraction back to a mixed number.

Example 1: Convert the mixed numbers $2 \frac{3}{5}$ and $6 \frac{5}{6}$ to improper fractions and add them together. Then convert the improper fraction in the solution back to a mixed number.

## Looking Ahead 2.8

As we can see, there are quite a few steps to perform in using this method. It might be easier to add the whole numbers to whole numbers and the fractions to fractions. We may get another improper fraction that will need to be converted to another mixed number and added again. Your final answer will be a whole number or a mixed number.

Example 2: Add the fractions below by adding the whole numbers and adding the proper fractions.

$$
2 \frac{3}{5}+6 \frac{5}{6}
$$

Example 3: Add the terms below by adding whole numbers to whole numbers and fractions to fractions.

$$
8 \frac{3}{4}+5+2 \frac{2}{3}
$$

Example 4: Add the terms below by changing the whole numbers and mixed numbers to improper fractions first. Compare your solution to Example 3. It should be the same.

$$
8 \frac{3}{4}+5+2 \frac{2}{3}
$$

## Section 2.9 Subtracting Mixed Numbers

## Looking Back 2.9

When subtracting mixed numbers, we subtract the proper fractions and then subtract the whole numbers (given there is a common denominator). If we have to find a common denominator, we may have to do some borrowing prior to it just as when we subtract whole numbers.

It may be easier to convert the mixed numbers to improper fractions, find a common denominator, and then convert the improper fractions back to a mixed number after the subtraction. We will try a few examples and you can decide which method is best for you. Make sure to always simplify your answer when possible.

Example 1: Subtract the mixed numbers below that have a common denominator.

$$
5 \frac{2}{3}-4 \frac{1}{3}
$$

Example 2: $\quad$ Subtract the mixed numbers below by finding a common denominator (subtract the fractions and then the whole numbers).

$$
5 \frac{2}{3}-4 \frac{1}{5}
$$

Example 3: Subtract the mixed numbers below by finding a common denominator and borrowing.

$$
5 \frac{1}{5}-4 \frac{2}{3}
$$

Example 4: Subtract the mixed numbers below by changing them to improper fractions and finding a common denominator. Then convert the improper fraction in the solution to a mixed number.

$$
8 \frac{2}{5}-6 \frac{5}{6}
$$

## Section 2.10 Multiplying Mixed Numbers

## Looking Back 2.10

One way to multiply mixed numbers is to change each mixed number to an improper fraction and then simply multiply the numerators, multiply the denominators, and simplify the product. We cannot multiply the whole numbers by the whole numbers and multiply the fractions by the fractions. Let us look at some examples to see why this is the case.

Example 1: Multiply the mixed numbers below.

$$
3 \frac{1}{3} \cdot 2 \frac{5}{8}
$$

Example 2: Multiply the mixed numbers below.

$$
5 \frac{1}{8} \cdot 3 \frac{1}{6}
$$

Example 3: Multiply the fractions below.

$$
\frac{1}{4} \cdot 7 \frac{1}{4}
$$

Example 4: Multiply the fractions below.

$$
1 \frac{1}{3} \cdot 5
$$

## Section 2.11 Dividing Mixed Numbers

## Looking Back 2.11

Multiplying by the reciprocal of the divisor is the same as dividing fractions. This process is also the same process used to divide mixed numbers, but mixed numbers must first be converted to improper fractions. Then, again we will multiply the numerators and multiply the denominators and simplify the product, just as we did in the previous section. You may also try and simplify before multiplying so you are working with smaller numbers.

Example 1: Divide the mixed numbers below.

$$
3 \frac{1}{3} \div 2 \frac{2}{3}
$$

Example 2: Demonstrate why the solution from Example 1 is so.

Example 3: Divide the mixed numbers below.

$$
2 \frac{2}{3} \div 3 \frac{1}{3}
$$

Example 4: Divide the fractions below.

$$
\frac{1}{4} \div 2 \frac{1}{5}
$$

Example 5: Divide the fractions below.

$$
5 \frac{2}{7} \div \frac{3}{4}
$$

## Section 2．12 Operating with Decimals

## Looking Back 2.12

Proper fractions represent parts of one whole．Proper fractions are greater than 0 but less than 1 ．Decimals can demonstrate parts of a whole number as well．The fraction $\frac{1}{2}$ is also the decimal $\frac{5}{10}$ ，and $\frac{50}{100}$ ，and $\frac{500}{1,000}$ ，etc．

Below is a place value chart．To the left of the ones place is the tens place，and to the left of that is the hundreds place．There is a pattern in the place value chart to the left and right of the ones place．

We can keep multiply by 10 to get the values of the numbers to the left of the ones place on the chart．We can divide by 10 to get the values of the numbers to the right of the ones place on the chart．

Any whole number is understood to have a decimal to the far right．For example，＂342＂is＂342．＂or＂342．0＂or ＂ 342.00 ＂，etc．

|  |  | 弟关 |  |  |  |  |  |  |  | $\mid \underset{\sim}{\underset{H}{e n}}$ | 菏 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 1 | 0 | 9 | 8 | 7 ， | 6 | 5 | 4 | 3 | 2 | 1. | 2 | 3 | 4 | 5 | 6 |

Just like fractions，decimal numbers may be added，subtracted，multiplied，and divided．
Example 1：Aailiyah goes to the store to buy some milk，bread，and cereal．Milk is $\$ 2.99$ ，bread is $\$ 2.85$ ，and a box of cereal is $\$ 3.89$ ．She has $\$ 8.00$ ．Does she have enough money to buy these three items？

Example 2: Barry has to paint three rooms before his kids come home from college. The first room is 10.5 feet by 12.75 feet; the second room is 18.25 feet by 12 feet; the third room is 8.5 feet by 8.5 feet. Estimate how many square feet Barry has to paint and then find the actual area being painted.

Example 3: Add the decimal numbers below.
$7.5+3.25+9.75$

Example 4: Use the graph below to demonstrate the sum found in Example 3.


Example 5: $\quad$ Subtract the decimal numbers below.

$$
10.4-3.2
$$

Example 6: Multiply the decimal numbers below.

$$
8.26 \cdot 5.3
$$

Example 7: Divide the decimal numbers below.

$$
43.778 \div 5.3
$$

Example 8: Divide the decimal numbers below and check your answer using multiplication.
$232.5 \div 0.31$

## Section 2.13 Conversions in the Metric System

## Looking Back 2.13

Fractions and decimals are used in metric conversions. The United States still uses the US Customary System rather than the Metric System because much of its large factory equipment was manufactured according to these standards.

However, most of the world uses the metric system so we will now find equivalent measurements between the Metric System and US Customary System on a number of products. Even though it is in the Metric System, most North Americans still know what a 2-liter bottle of soft drink looks like. Metrics are used in science, automechanics, computer science, engineering, and many other fields as well. It is essential to be able to convert between the two systems of measurement as well as within both systems.

In this section, we will look at conversions within the metric system. We explore this topic after working with fractions and decimals because conversions from one of these units to the other are really decimal conversions.

## Looking Ahead 2.13

> Meter, Liter, and Gram are the standard units of measure in the metric system.

Meter is used to measure length.
Liter is used to measure liquids.
Gram is used to measure mass.

## Example 1: Which standard unit of measure would you use for each of the following descriptions?

a) the distance across a room
c) the amount of water in a sink
e) the amount of water in a bottle
b) the weight of a paperclip
d) the distance you can roll a ball
f) the weight of an apple

Knowing prefixes helps us understand how the Metric System works. Meter, Liter, and Gram are the ones unit; "Deka" means tens; "Hecto" means hundreds; "Kilo" means thousands. These are to the left of the ones unit.

| Kilometer (km.) | Hectometer (hm.) | Dekameter (dam.) |
| :---: | :---: | :---: |
| Kiloliter (kl.) | Hectoliter (hl.) | Dekaliter (dal.) |
| Kilogram (kg.) | Hectogram (hg.) | Dekagram (dag.) |

"Deci" means tenths; "Centi" means hundredths; "Milli" means thousandths; these are to the right of the ones unit.


As we move left from one unit to the next, we divide by 10 or move the decimal point one place to the left for each metric prefix.

As we move right from one unit to the next, we multiply by 10 or move the decimal point one place to the right for each metric prefix.

This means 1 kilometer is equal to 10 hectometers are equal to 100 dekameters is equal to 1,000 meters, or 0.1 dekameters is equal to 0.01 hectometers is equal to 0.001 kilometers. Because it takes 1,000 meter sticks to equal 1 kilometer, one meter stick is equal to 0.001 of a kilometer.

[^1]Example 3: Convert 2.5 centimeters ( cm .) to meters (m.).

Example 4: Convert 65 liters (1.) to kiloliters (kl.).

Example 5: $\quad$ Convert 5.35 grams (g.) to kilograms (kg.).

Example 6: $\quad$ Convert 7.25 kilograms (kg.) to centigrams (cg.).


[^0]:    Example 1: $\quad$ Multiply $\frac{2}{5}$ by $\frac{12}{13}$ and simplify the product if possible.

[^1]:    Example 2: Convert 5 meters (m.) to millimeters (mm.).

