Module 1 Reviewing Number Systems

Section 1.1 Classifying Numbers

Looking Back 1.1

The numbers we first learn are those used for counting: 1, 2, 3, 4, etc. As we get into Pre-Algebra and higher mathematics, there are many different types of numbers. These different types of numbers allow us to solve different types of problems.

If we are talking about the number of people in a car, we would need to use the set of whole numbers. We cannot have a negative number of people and we certainly cannot have fractional or decimal parts of people. However, we could have 0 people in a car if everybody gets out when the car stops.

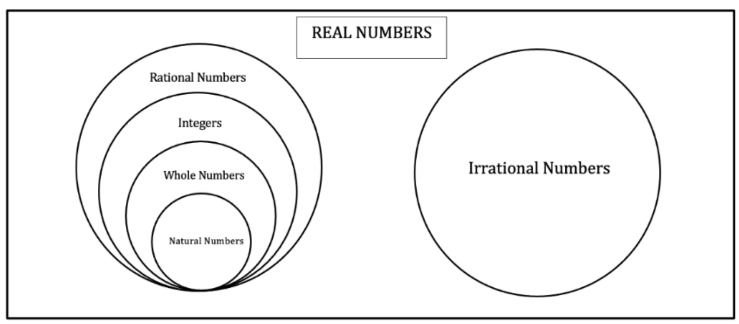
We know about natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers. In this section, we will review these different types of numbers and their relationships to one another.

Looking Ahead 1.1

Natural Numbers	__________, __________, __________, __________, … These were first called "counting numbers." They were used in bartering and trading.
Whole Numbers	__________, __________, __________, __________, … A placeholder was needed to represent no objects. Therefore, 0 was added to the set of natural numbers.
Integers	The positive integers are: __________, __________, __________, __________, … The negative integers are: __________, __________, __________, __________, … __________ is neither positive nor negative.
Rational Numbers	These include any number that can be written as a fraction $(\frac{a}{b})$ in which b does not equal 0. We cannot divide by 0, so 0 in the denominator is ____________________. Integers are rational numbers because integers can be written as fractions. The integer becomes the numerator and __________ becomes the denominator.

Irrational Numbers	These numbers cannot be written as fractions. Pi is a good example: 3.14159265… Other examples are $\sqrt{2} = 1.414213$ ... and $\sqrt{3} = 1.73205$ .... These numbers do not repeat or___________.
Real Numbers	Natural: 1, 2, 3, 4, … Whole: 0, 1, 2, 3, 4, … Integer: ... $-4, -3, -2, -1$, 0, 1, 2, 3, 4, ... Rational: Any number that can be written as a fraction (or terminating or repeating decimal). Irrational: Any number that does not terminate or repeat. (These cannot be written as fractions.) All of these sets of numbers are real numbers.

Decimal numbers can be rational or irrational. Rational numbers are decimals that come to an end such as 0.5, which is equal to $\frac{1}{2}$, or 7.0, which is equal to 7. All terminating decimals can be converted to fractions, so they are of the form $\frac{a}{b}$ in which b does not equal 0. Even mixed numbers such as $1\frac{1}{2}$ can be written as improper fractions such as $\frac{3}{2}$. Rational numbers are decimals that repeat. For example, 0.33333 or $0.\overline{3}$. The repetend bar is written over the decimal that repeats. This ($0.\overline{3}$) is the same as the fraction $\frac{1}{3}$. Repeating decimals can be converted to fractions and called rational numbers. Decimals that do not terminate or repeat cannot be converted to fractions. They are called irrational numbers.



John Venn was a mathematician and logician from England. He contributed greatly to set theory and his use of diagrams, which would go on to be named after him (Venn Diagrams), help us represent overlapping and non-overlapping sets.

However, John Venn's contributions were not limited to mathematics. Along with his father's church, John contributed greatly to the abolition of slavery and the reformation of the prison system.

Example 1: Which of the previous sets (from the Venn diagram) do the following numbers belong to?	
53	13.2
0	-22
$-1.\overline{1}$	1.61803398875…

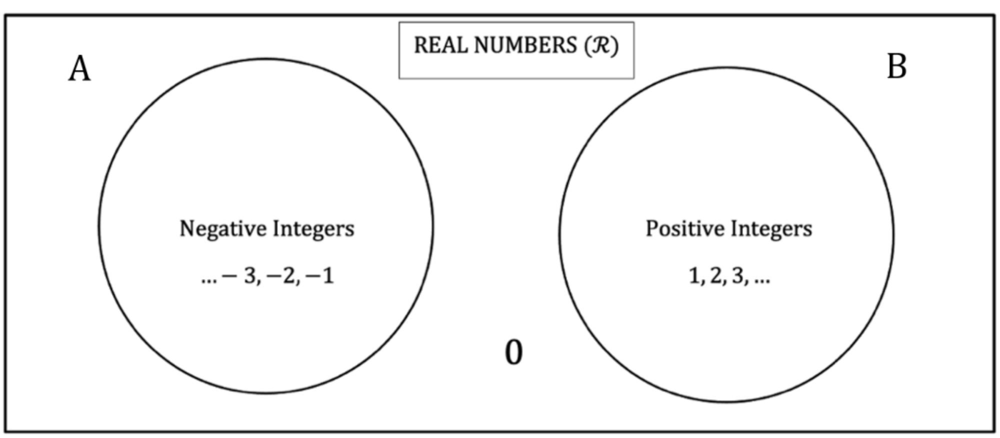
A set is any group of things. Sets are written in brackets; for example, $\{...\}$ in which we list the members. This is called the roster method. We can also write sets with a description if the number of members is very large, such as the set of real numbers $\mathcal{R}$.

Example 2: Use the roster method to list the set of odd numbers between 0 and 12.

If a number is a member of a set, it is called an element of the set. How many elements are in the set from Example 2?

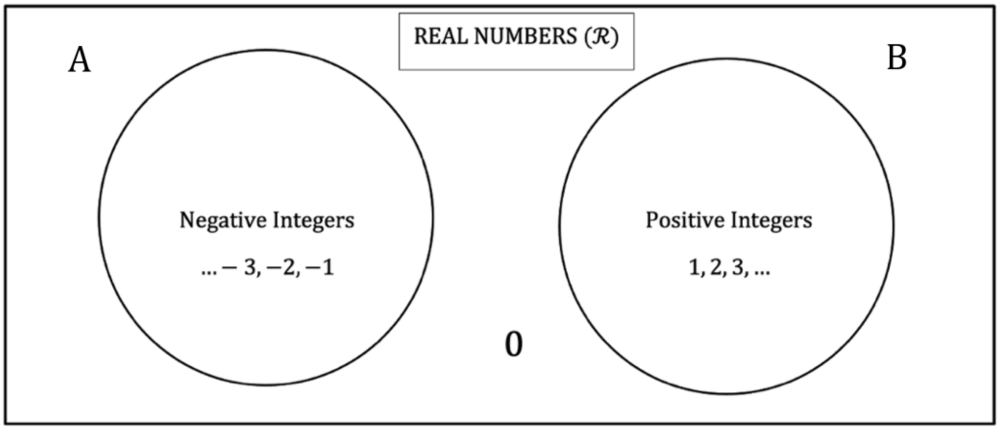
Example 3: Write the natural numbers, whole numbers, and integers as sets.

If two sets of numbers are combined together, the new set is the union of the two sets. The elements are in A or B (or both). The symbol U is used to represent union.



Set A is the set of all negative integers. Set B is the set of all positive integers.

Example 4:	Use the roster method to list A ∪ B.



Subsets are part of a larger set. The symbol ⊂ is used for a proper subset. All of the elements in a set are in another set, but the other set has more elements.

Supersets are the larger set. The symbol ⊃ is used for a proper superset, which includes the smaller sets' elements and more.

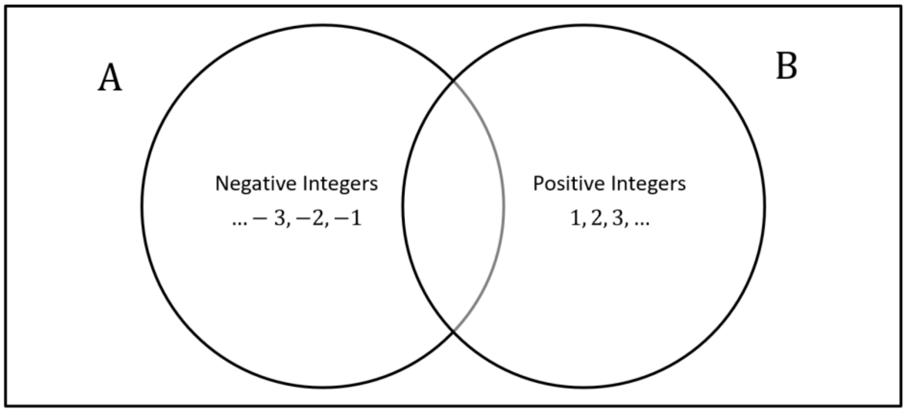
A __________ $\mathcal{R}$ and $\mathcal{R}$ __________ A

B __________ $\mathcal{R}$ and $\mathcal{R}$ __________ B

0 __________ $\mathcal{R}$ and $\mathcal{R}$ __________ 0

If the two sets overlap, the new set is the intersection of the two sets. The symbol ∩ is used for intersection. These are the elements the sets have in common. The elements are in A and B.

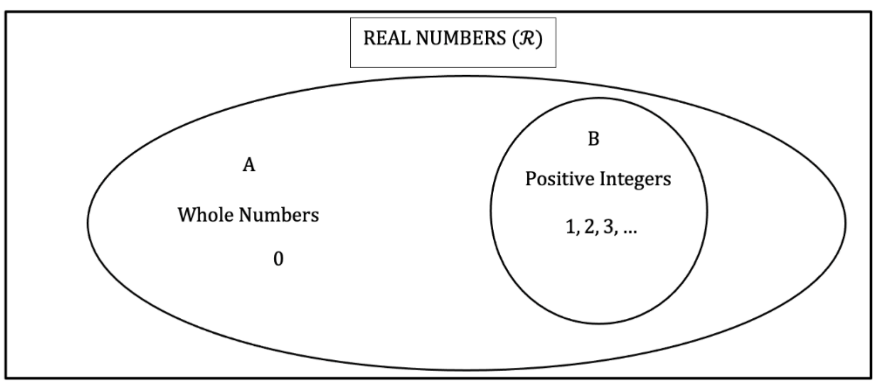
Example 5:	Complete the set notation below given the following Venn diagram.



The intersection of two sets is where they overlap. In this case, they have __________ in common, so A ∩ B is the empty set. The symbol for the empty set is ___________ . This is also called the __________ set.

Example 6:	Complete the set notation below given the Venn diagram.

Sometimes one set is completely contained within another set of numbers as shown below.



In this drawing, several things are going on.

B __________ A

A __________ $\mathcal{R}$

A ∪ B = ____________________

A ∩ B = ____________________

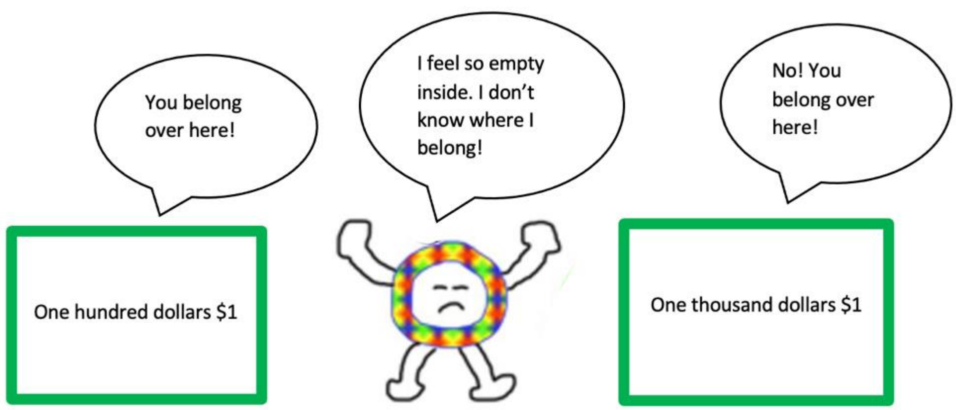
Section 1.2 The Identity Elements

Looking Back 1.2

Number systems are quite important to the whole of mathematics and are foundational to the work of mathematicians. Some historical mathematicians (such as Plato) believed the whole world rested upon the number system. We know the whole world rests upon God who created the number system, and the scientific principles and mathematics that govern the universe allow us to understand and analyze God.

It has been said that mathematics is the language of science. Given mathematics is a universal language, the numbers and letters (constants and variables) are like nouns and operations are much like verbs. Until the invention of zero, mathematics was a confusing language. The invention of zero is very important because it made computations simpler. Simpler computations helped develop Algebra and Calculus. Therefore, zero became the basis for computer code along with the number 1.

I bet you never realized zero was that important!



Looking Ahead 1.2

One is also a very important number; it is the first natural or counting number and represents initial or start values. One is the first number and represents a beginning. "In the beginning, God created the heavens and the earth. Now, the earth was formless and empty, darkness was over the surface of the deep and the Spirit of God was hovering over the waters. And God said, 'Let there be light, and there was light. God saw that the light was good, and He separated the light from the darkness. God called the light day and the darkness He called night. And there was evening and there was morning one day." -Genesis 1:1-5.

Example 1: Complete the following operations and identify which operations give you “5” for an answer.

$5 + 0 =$ ________

$5 - 0 =$ ________

$5 \cdot 0 =$ ________

$5 \div 0 =$ ________

When we add or subtract ___________ from the number we still get the ___________. This is why zero is called the

______________________ ______________________ for _______________________.

Example 2: Complete the following operations and identify which operations give you 4.

$4 + 1 =$ ________

$4 - 1 =$ ________

$4 \cdot 1 =$ ________

$4 \div 1 =$ ________

When we ______________________ or ______________________ 4 by 1, we still get 4. This is why 1 is called the

______________________ ______________________ for ______________________.

A value that does not change is a constant. Therefore, all numbers are constants. A value that does change is a variable. Therefore, all letters are variables as they represent numbers that vary or change. When a number and a variable are multiplied together, the number is called the coefficient of the expression and comes before the variable. If there is no number in front of a variable, it is assumed to be 1.

Example 3: Which term is a constant only? Which term is a variable only? In the term that is both, name the coefficient.

$-\frac{1}{4}$ t $3x$

Example 4: Underline the constants below and circle the variables.

$\frac{3}{4}$ mn $\frac{x}{y}$

Example 5: Name the coefficient in each expression below.

$-\frac{1}{3}y$ v $5.5w$

Section 1.3 Number Properties

Looking Back 1.3

As we have stated previously, number theory is the mathematics that deals with the properties of numbers. There are properties that work for real numbers that can be defined by using letters.

These properties help us solve problems. If the properties do not hold true, they cannot be used to solve problems. Since mathematics is a universal language, it is important to follow the rules and find the correct solutions. The properties that hold true for the system of real numbers are listed below.

Properties from 1.3:	For every real number a:
Reflexive Property	$a = a$
Identity Property of Addition (Additive Identity Property)	$a + 0 = a$
Inverse Property of Addition (Additive Inverse Property)	$a + -a = 0$
Multiplication Property of Zero (Multiplicative Zero Property)	$a \cdot 0 = 0$
Identity Property of Multiplication (Multiplicative Identity Property)	$a \cdot 1 = a$
Multiplication Property of -1 (Multiplicative Property of -1)	$a \cdot (-1) = -a$
	For every nonzero real number a:
Inverse Property of Multiplication (Multiplicative Inverse Property)	$a \cdot \frac{1}{a} = 1$

The Multiplication Property of -1 is used frequently when we solve for $-x$ and we get x.

$$-x = 4$$

$$(-1)(-x) = (4)(-1)$$

$$x = -4$$

Looking Ahead 1.3

Example 1: Name the property and fill in the blanks to make the statement true.

The ____________________ ____________________ means any number is equal to _____________________.

$$5 = 5$$

$$\frac{1}{3} = \frac{1}{3}$$

$$6 = 6$$

The Reflexive Property is __________=__________.

Example 2: Name the property and fill in the blanks to make the statement true.

If we add zero to any number, we get the same number. This is the ____________________ _____________________ of _____________________.

$$2 - 0 = 2$$

$$-\frac{1}{3} - 0 = -\frac{1}{3}$$

The Identity Element for addition is ___________.

The Identity Property of addition is _________+__________=__________.

Example 3: Name the property and fill in the blanks to make the statement true.

The opposite of a number is called the ____________________ of the number. The ____________________ ____________________ of _____________________ means any number plus it opposite is equal to __________.

$$4+(-4)=0$$

$$-\frac{1}{5}+\frac{1}{5}=0$$

The Inverse Property of Addition is _________+__________=__________.

Example 4: Name the property and fill in the blanks to make the statement true.

The Multiplication Property of Zero means any number multiplied by __________ is equal to ___________.

$7\cdot 0\neq 2$ $7\cdot 0=0$

$-\frac{1}{4}x\cdot 0\neq\frac{1}{4}$ $-\frac{1}{4}x\cdot 0=0$

$a\cdot 0\neq a$ $a\cdot 0=0$

The Multiplication Property of Zero is __________·__________=__________.

Example 5: Name the property and fill in the blanks to make the statement true.

If we multiply a number by 1, we get the same number. This is the ____________________

____________________ of ____________________.

$$-6 \cdot 1 = -6$$

$$-\frac{1}{2} \cdot 1 = -\frac{1}{2}$$

The Identity Element for Multiplication is __________.

The Identity Property of Multiplication is __________·__________=__________.

Example 6: Name the property and fill in the blanks to make the statement true.

The Multiplication Property of -1 means any number multiplied by __________ changes the number to its opposite (positive to negative and negative to positive).

$$-\frac{1}{9} \cdot -1 = \frac{1}{9}$$

$$3 \cdot -1 = -3$$

The Multiplication Property of __________ is _______ · _______ = _______.

Example 7: Name the property and fill in the blanks to make the statement true.

The Inverse Property of Multiplication means any number multiplied by its reciprocal is equal to 1.

$$2 \cdot \frac{1}{2} = 1$$

$$-\frac{2}{3} \cdot -\frac{3}{2} = 1$$

The Inverse Property of Multiplication is _______ · _______ = _______.

Section 1.4 The Commutative and Associative Properties

Looking Back 1.4

Because variables represent numbers, any operations that can be done with numbers can also be represented by letters. This means that any property that holds true for real numbers also works with variables that represent those numbers. In this section, we will investigate the commutative and associative properties. Let's think about what that means.

Commutative comes from the word 'commute,' which means to go back and forth from place to place. Associative comes from the word 'associate,' which means to connect or can relate to the group one belongs to. So, the Commutative Property lets us change the order of the numbers and get the same answer and the Associative Property lets us change the order of the groups and get the same answer.

Looking Ahead 1.4

Example 1:	Using real numbers, does the Commutative Property hold true for addition, subtraction, or both?

The Commutative Property works for ____________________.

$$2 + 8 = 8 + 2$$

$$\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_$$

$$2 - 8 \neq 8 - 2$$

$$\_\_\_\_\_\_\_ \neq \_\_\_\_\_\_\_$$

The Commutative Property does not work for _____________________.

Example 2:	Using real numbers, does the Commutative Property hold true for multiplication or division, or both?

The Commutative Property works for ____________________.

$$8 \cdot 2 = 2 \cdot 8$$

$$\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_$$

$$8 \div 2 \neq 2 \div 8$$

$$\_\_\_\_\_\_\_ \neq \_\_\_\_\_\_\_$$

The Commutative Property does not work for ____________________.

The Commutative Property of Addition states that for any real number a and b,

_______ + _______ = _______ + _______.

The Commutative Property of Multiplication states that for any real number a and b,

_______ · _______ = _______ · _______.

Example 3:	Using real numbers, does the Associative Property hold true for addition, subtraction, or both?

The Associative Property works for ____________________.

If $a = 4$, $b = 3$, and $c = 2$, then does $(a + b) + c = a + (b + c)$?

However, if $a = 4$, $b = 3$, and $c = 2$, then $(a - b) - c \neq a - (b - c)$.

The Associative Property does not work for ____________________.

Example 4: Using real numbers, does the Associative Property hold true for multiplication, division, or both?

The Associative Property works for ____________________.

If $a = 4$, $b = 2$, and $c = 3$, then does $(a \cdot b) \cdot c = a \cdot (b \cdot c)$?

However, if $a = 4$, $b = 2$, and $c = 3$, then $(a \div b) \div c \neq a \div (b \div c)$.

The Associative Property does not work for ____________________.

The Associative Property of Addition states that for any real numbers a, b, and c:

(_______ + _______) + _______ = _______ + (_______ + _______)

The Associative Property of Multiplication states that for any real numbers a, b, and c:

(_______ · _______) · _______ = _______ · (_______ · _______)

Section 1.5 The Distributive Property

Looking Back 1.5

Just as there are laws (such as the Law of Gravity) that govern the universe God has created, there are also laws/properties that govern how we work with numbers. One of these such properties is the Distributive Property.

One way to think of the Distributive Property is as if $4 \cdot (3 + 2)$ means we have four grocery bags with $3 + 2$ in each of them. We are distributing $3 + 2$ into 4 bags.

$$4 \cdot (3 + 2) = 4 \cdot (3) + 4 \cdot (2)$$

$$4 \cdot (5) = 12 + 8$$

$$20 = 20$$

In arithmetic, we only use the Distributive Property with numbers when we want to do mental math.

Now, if we substitute x for 3 we get $4 \cdot (x + 2)$, which means we have 4 grocery bags with $x + 2$ distributed to each bag.

Distribute means to spread out or scatter. In the diagram above, 4 is being multiplied x first, then 4 is multiplied by 2 and the products are added together.

$$4 \cdot (x + 2) = 4 \cdot (x) + 4 \cdot (2)$$

$$4 \cdot (x + 2) = 4x + 8$$

A number next to parenthesis means to multiply just like a number next to a letter means to multiply.

Example 1: Use the Distributive Property to fill in the blanks.

$$3(20 + 0.4) = 3(\_\_\_\_\_\_\_\_) + 3(\_\_\_\_\_\_\_\_)$$

$$= \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_$$

$$= \_\_\_\_\_\_\_\_$$

$$9(20 - 10) = \_\_\_\_\_\_\_\_(20) - \_\_\_\_\_\_\_\_(10)$$

$$= \_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_$$

$$= 90$$

$$(12 + 10)2 = (\_\_\_\_\_\_\_\_)2 + (\_\_\_\_\_\_\_\_)2$$

$$= 24 + 20$$

$$= \_\_\_\_\_\_\_\_$$

$$(13 - 3)4 = (\_\_\_\_\_\_\_\_)\_\_\_\_\_\_\_\_ - (\_\_\_\_\_\_\_\_)\_\_\_\_\_\_\_\_$$

$$= \_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_$$

$$\_\_\_\_\_\_\_\_$$

The Distributive Property can be used for multiplication over addition:

$$a(b + c) = ab + ac \qquad (b + c)a = ba + ca$$

The Distributive Property can also be used for multiplication over subtraction:

$$a(b - c) = ab - ac \qquad (b - c)a = ba - ca$$

Example 2: Distribute $2(4 + x)$ and show your steps.

Example 3: Distribute $4(3 - x)$ and show your steps.

<u>Summary of Properties from Sections 1.4 & 1.5</u>

For every real number a and b:

$a + b = b + a$ Commutative Property of Addition

$a \cdot b = b \cdot a$ Commutative Property of Multiplication

For every real number a, b, and c:

$(a + b) + c = a + (b + c)$ Associative Property of Addition

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$ Associative Property of Multiplication

$a(b + c) = ab + ac$ Distributive Property of Multiplication over Addition

$(b + c)a = ba + ca$ Distributive Property of Multiplication over Addition

$a(b - c) = ab - ac$ Distributive Property of Multiplication over Subtraction

$(b - c)a = ba - ca$ Distributive Property of Multiplication over Subtraction

Section 1.6 Opposites, Inverses, and Reciprocals

Looking Back 1.6

Let us talk more about opposites, inverses, and reciprocals so there is no confusion about how they function. The Inverse Property of Addition states that $a + -a$ is equal to 0 for every real number. Therefore, if a is equal to 3, then $-a$ is equal to -3 and $3 + -3$ is equal to 0. In this case, $-a$ is the opposite of a or the opposite of 3, which is -3. The opposite also happens to be the additive inverse because when added together they result in the identity element of addition, which is 0. Remember, if we add 0 to any number, the number does not change its identity. Therefore, $-a$ is the additive inverse of a, or the opposite of a.

The Inverse Property of Multiplication states that $a \cdot \frac{1}{a}$ is equal to 1 for every real number a. However, a cannot equal 0. Therefore, if a is equal to 2 then $\frac{1}{a}$ is equal to $\frac{1}{2}$ and $2 \cdot \frac{1}{2}$ is equal to 1 because 2 is equal to $\frac{2}{1}$ and $\frac{2}{1} \cdot \frac{1}{2}$ is equal to 1. In this case, when $a = 2$, then $\frac{1}{a}$ is the reciprocal of 2; it is $\frac{2}{1}$ flipped over. Therefore, the denominator becomes the numerator and the numerator becomes the denominator, which is the reciprocal of 2, $\frac{1}{2}$. The reciprocal happens to be the multiplicative inverse because when multiplied together they result in the identity element of multiplication, which is 1. Remember, if we multiply 1 by any number, the number does not change its identity.

Looking Ahead 1.6

Example 1: Fill in the blanks.

The opposite of -4 is __________.

The opposite of 3 is __________.

The additive inverse of 0.5 is __________.

The additive inverse of -1.82 is __________.

The opposite of $4x$ is __________.

Therefore, $2x + -2x =$___________.

The additive inverse of $-\frac{1}{3}x$ is __________.

Therefore, $-\frac{1}{3}x + \frac{1}{3}x =$__________.

This is true for ____________________.

As we can see, ____________________ and ____________________ ____________________ are the same for addition.

Example 2: Fill in the blanks.

The reciprocal of $-\frac{1}{3}$ is ___________.

The multiplicative inverse of -7 is ___________.

The reciprocal of $\frac{2}{3}$ is ___________.

The multiplicative inverse of $-\frac{4}{5}$ is ___________.

The reciprocal of $5x$ is ___________.

The multiplicative inverse of $-\frac{1}{2x}$ is ___________.

This is true for ______________________.

As we can see, ______________________ and ______________________ ______________________are the same for multiplication.

Unlike opposites, reciprocals have the ______________________ ______________________.

<u>Section 1.7 Absolute Value and the Number Line</u>

<u>Looking Back 1.7</u>

Activity 1.7:

Supplies:

- Tracing paper
- Ruler
- Red & blue colored pencils/markers

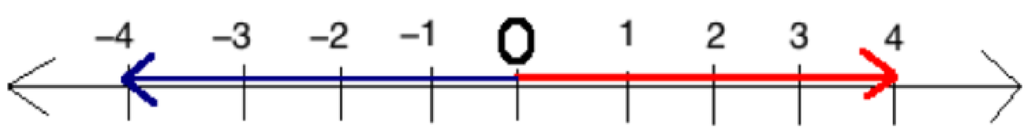
Introduction:

This activity will help us use the number line to represent opposites (additive inverses).

Procedure:

1. Fold a piece of tracing paper in half.
2. Mark the fold with a dark line from top to bottom.
3. Turn the paper so the dark line across the fold is going from left to right.
4. Put the arrows at each end from left to right so it looks like a number line.
5. Fold the right arrow onto the left arrow so the line is on the top of itself and make a fold going up and down.
6. When we open it up, it looks like a cross. Mark “0” where the two lines meet.
7. Put the ruler at “0” and mark off every inch to the right with a “1” at 1 inch, a “2” at 2 inches, etc.
8. Put the ruler at “0” and mark off every inch to the left with a “1” at 1 inch, a “2” at 2 inches, etc.
9. Mark the left side with a negative sign in front of the numbers.

Start at 0 and draw an arrow above the line in red pencil/marker that points 4 units to the right. Start at 0 and draw an arrow above the line in blue pencil/marker that points 4 units to the left. These numbers are opposites because they are the same distance from 0 and are on opposite sides of the 0 on the number line. We call this distance from 0 the absolute value. We write the absolute value of 4 as $|4|$ and the absolute value of -4 as $|4|$. Both are 4 units from 0. Therefore, $|4| = 4$ and $|4| = -4$.



Looking Ahead 1.7

Absolute value is how many units a number is from 0 on the number line.

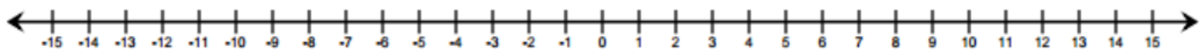
Example 1:	Find the absolute value of each number.

a) $|-2.2|$

b) $\left|-\frac{1}{8}\right|$

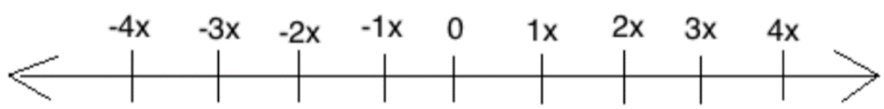
c) $|0|$

d) $|10|$

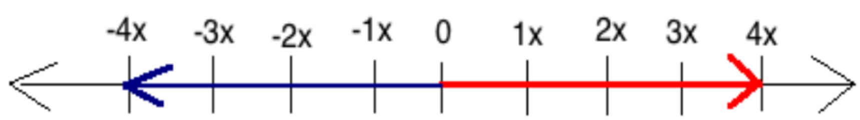


Example 2:	Mark "X" next to each number on the number line and answer the questions that follow.

What is the opposite of $3x$?



What is the opposite of $-2x$?



Example 3: Solve the absolute value problems below.

a) $|3-2|$

b) $|2-3|$

c) $|a-a|$

d) $|7-3|$

Example 4: Solve the absolute value problems below.

a) $|1+6|-2$

b) $|2-3|+|5-5|$

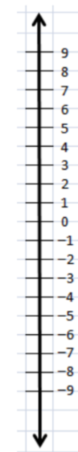
c) $|1.1+2.8|-2.1$

d) $|4+2|\cdot|-6|$

Section 1.8 Walk the Number Line

Looking Back 1.8

Walk outside and draw a number line on the pavement with chalk. Have it oriented like a thermometer so the positive numbers are above 0 and the negative numbers are below 0.



We will be walking the number line to review the rules for adding and subtracting positive and negative integers.

Follow the given rules:

1. Always start at 0 facing the positive direction.
2. Slide forward for positive numbers while still facing forward.
3. Slide backward for negative numbers while still facing forward.
4. If it is an addition problem, stay facing the same direction.
5. If it is a subtraction problem, turn around and face the opposite direction because subtraction is the opposite of addition.

Looking Ahead 1.8

Let's try four examples using addition.

Example 1: Both Positive: $3 + 2$

Start at ___________. Move forward ___________, then forward ____________. You should still be facing forward because it is________________________. The answer is ___________.

Example 2: Both Negative: $-3 + -2$

Start at __________. Move backward __________ spaces to -3. You should still be facing _____________________ because it is addition. Move _____________________ 2. The answer is ___________.

Example 3: Negative and Positive: $-3 + 2$

Start at 0. Move backward 3 spaces to -3. You should still be facing _____________________ because it is addition. Move _____________________ 2. The answer is ___________.

Example 4: Positive and Negative: $3 + (-2)$

Start at ___________. Move forward ___________ spaces. You should still be facing forward because it is addition. Move backward ___________ spaces. The answer is ___________.

Now let's try four more examples using subtraction.

Example 5: Both Positive: $3-2$

Start at 0. Move forward 3 spaces to 3. Subtraction means do the opposite; therefore, ____________________ ______________________ so you are facing the ______________________ ______________________. Move forward 2 spaces. The answer is __________.

Example 6: Both Negative: $-3-(-2)$

Start at 0. Move backward ____________ spaces to -3. Subtraction means do the opposite; therefore, _____________________ _____________________ so you are facing the ____________________ direction. Move backward 2 spaces. The answer is __________.

Example 7: Negative and Positive: $-3-2$

Start at __________. Move backward ___________ spaces, then ____________________ ____________________ for subtraction. Now move _____________________ 2 spaces. The answer is ___________.

Example 8: Positive and Negative: $3-(-2)$

Start at __________. Move forward __________ spaces, then ____________________ ____________________ for subtraction. Now move ____________________ 2 spaces. The answer is __________.

Note: Walking the number line works for two numbers being added or subtracted. If there are three numbers, then get an answer for the first two and start at 0 again to add or subtract the third number.

Section 1.9 Adding with Variables

Looking Back 1.9

Now that we have reviewed adding integers, we are going to work on addition with variables. When we add 2 x's to 4 x's, we get 6 x's. In $2x$, the 2 in front is called the coefficient and the x is the variable. The x being multiplied by the 2 is the same number being multiplied by the 4. This is why the Distributive Property works:

$$2x + 4x$$

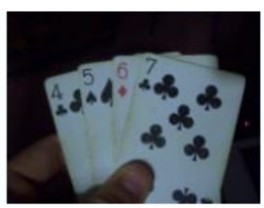
$$(2 + 4)x$$

$$6x$$

(Both numbers are being multiplied by the same number.)

This is called combining like terms. The $2x$ and $4x$ are called monomials (one term). The two together are separated by an addition sign and make a binomial (two terms).

We will be playing a couple of games to reinforce this concept. To begin, we need a deck of playing cards and a partner, and paper and pencil to tally the score.



Looking Ahead 1.9

Activity 1.9

Supplies:

- Deck of cards
- Paper and pencil (for tallying scores)

Introduction:

This activity will help us understand that when we add common monomial terms for those with like variables, we simply add the coefficients and keep the variables the same.

Procedure:

1. Remove the face cards, except for the aces, from the deck of cards.
2. The value of cards 2 through 10 will be the number on the card. The number “2” is the integer 2, “3” is 3, and so on. Aces represent 1.
3. In banking, “in the red” means one is in debt and owes money; “in the black” means one has money in the bank. So, red cards represent negative integers and black cards represent positive integers.
4. Only like cards can be added or combined. So, 3 of spades (3 spades) can be added to a 2 of spades (2 spades) to give us a 5 of spades (5 spades).
5. A 6 of hearts (6 hearts) is a negative 6. A 4 of hearts is a negative 4 (4 hearts). So, a 6 of hearts and a 4 of hearts gives us a sum of a 10 of hearts (-10 hearts).
6. Remember, aces represent the number 1, so when we add an ace of clubs (A clubs) with a 3 of clubs (3 clubs), we get a sum of a 4 of clubs (4 clubs).

Before we begin:

Find a partner to play the following two games with. Preferably, play with somebody who knows how to add and subtract integers. Read the rules before beginning and make sure you can explain them to your partner. You may also want to play against yourself by making two piles of cards, placing one stack on the right and one on the left. When the right pile wins, discard right. When the left pile wins, discard left. In essence, whether you have a partner or not, the right side is playing against the left side to win.

Game 1

Shuffle all the number cards and aces in a deck of cards. Put them in a pile. Draw 5 cards from the pile. Give 5 to your partner. Using cards of the same suit, lay down the suit that has the greatest sum. You must lay down at least 2 cards, but you can lay down more if you have more than two of the same suit. The greatest sum wins all the cards for a discard pile. (Remember, red cards are negative.)

During the next round, each player picks up enough cards from the draw pile so each player has 5 cards again. You keep playing until neither player has any matches to lay down. If there are not enough cards from the draw pile for both players to have 5 cards in their hand, then split the cards in the draw pile. If there are an odd number of cards in the draw pile, split the cards evenly and discard the one extra card. After the discard pile is divided evenly, play one more round.

After the last round, each player counts the number of cards in their discard pile, subtracting from the number of cards leftover in their hand. The player with a greater number of cards wins.

So, if you have an Ace of clubs, a 3 of clubs, and a 5 of clubs, you have a 9 of clubs (you must lay down all the clubs in your hand). If your partner lays down a 2 of diamonds and a 7 of diamonds, they have a 9 of diamonds (they must lay down all the diamonds in their hand). Because your number is greater, you would win all these cards. You replace the 3 cards from the draw pile and your partner replaces their 2 cards from the draw pile.

Game 2

This is a simple variation of Game 1. Play exactly the same way but this time the player with the smallest number of cards wins. Therefore, going back to the example above in which you lay down an Ace of clubs, 3 of clubs, and 5 of clubs, and your partner lays down a 2 of diamonds, and 7 of diamonds, this time your partner would win because 9 of diamonds is negative 9, which is smaller than your 9 of clubs, which is positive 9.

Section 1.10 Subtracting with Variables

Looking Back 1.10

You may be beginning to understand that the rules that apply to operations with real numbers also apply to operations with variables. In this section, we will continue practicing with combining like terms that involve variables.

Example 1:	Let x be 2 to demonstrate that $4x + 3x$ is equal to $7x$.

Example 2:	Let x be 2 and y be 5 to demonstrate that $4x + 3y$ is not equal to $7xy$.

Looking Ahead 1.10

Example 3:	Combine like terms and use the Distributive Property to verify the equation.

$$3z + 5z = 8z$$

Example 4: Combine like terms and use the Distribute Property to simplify the expressions.

$-3m - 7m$ $\qquad$ $-5t - (-3t)$

<u>Rules for Addition with the Same Sign</u>

When adding integers or monomial terms with the same sign: Add the absolute value of the numbers or coefficients of the variables and keep the sign in the answer the same as those terms being combined. If there are variables, keep the variables the same. The variables must be the same in order to add them, and the exponents must be the same as well. They cannot be combined using addition if they are not like terms.

$$3m + 2m$$

<u>Rules for Addition with Different Signs</u>

When adding integers or monomial terms with different signs: Subtract the smaller absolute value number or the coefficient of the variable from the larger absolute value number or coefficient of the variable and keep the sign of the number with the larger absolute value from the number or coefficient of the variable. If there are variables, keep the variables the same. The variables must be the same in order to add them, and the exponents must be the same as well. They cannot be added if they are not like terms. They cannot be combined using addition if they are not like terms.

$$3x^2 + 3y^2$$

Cannot be simplified as there are no like terms. The exponents are the same, but the variables are different.

<u>Rules for Subtraction with the Same or Different Signs</u>

When subtracting integers or monomial terms with the same sign or different signs: Change subtraction to addition and change the sign of the number or coefficient following the subtraction sign to its opposite. Then follow the rules above for addition.

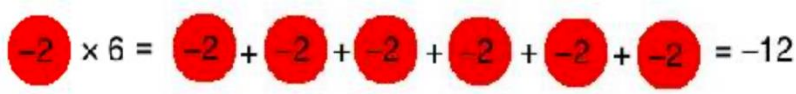
$$3y^2 - 2y^2$$

Section 1.11 Multiplying with Variables

Looking Back 1.11

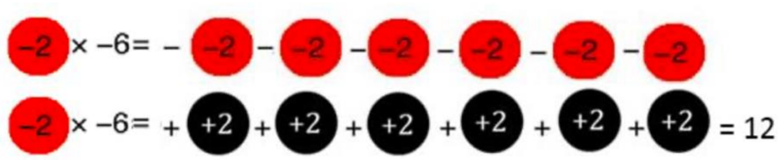
Remember, multiplication is repeated addition. If we have a positive 2 chip and multiply it by 6, we are essentially adding $+2$ six times.

Therefore, if we have a negative 2 chip and we multiply it by 6, we are adding -2 six times.



Addition and multiplication are both commutative: $-2 \cdot 6 = 6 \cdot -2$. When we multiply a negative number by a positive number, we will always get a negative number, regardless of the order of the numbers. If the signs of the two numbers are different, the product will be negative.

If we have a -2 chip and multiply it -6 times, then we are subtracting -2 negative six times. So, instead of adding the -2 chip 6 times, think of it as doing the opposite and subtracting the -2 chip six times.



When the two numbers being multiplied are positive the product is positive. When we multiply a negative number by a negative number, we will always get a positive number. Therefore, if the signs are the same when multiplying two numbers, the product is positive.

If we have more than two numbers, the sign will be positive or negative depending on the number of negative numbers we are multiplying. Look at the diagram below.

$-1 =$__________

$-1 \cdot -1 =$__________

$-1 \cdot -1 \cdot -1 =$__________

$-1 \cdot -1 \cdot -1 \cdot -1 =$__________

$-1 \cdot -1 \cdot -1 \cdot -1 \cdot -1 =$__________

$-1 \cdot -1 \cdot -1 \cdot -1 \cdot -1 \cdot -1 =$__________

$-1 \cdot -1 \cdot -1 \cdot -1 \cdot -1 \cdot -1 \cdot -1 =$__________

Referring to the pattern above, when we multiply an odd number of negative numbers, our answer is negative. When we multiply an even number of negative numbers, our answer is positive.

In Algebra, $a \cdot b$ can also be written "$a \times b$," "$a(b)$," "$(a)b$," "$(a)(b)$," or "ab." We will use the form $a \cdot b$ for the examples that follow.

Looking Ahead 1.11

Example 1:	Multiply:

$4 \cdot (2y)$ $4 \cdot (-2y)$

Example 2:	Multiply:

$(3) \cdot 6y$

Example 3:	Multiply:

$$(2) \cdot (-5y)$$

Example 4:	Multiply:

$$(-4)2z$$

Example 5:	Multiply:

$$-9(-7)$$

<u>Some Things to Remember when using Multiplication with Variables:</u>

The rules of constants do apply to variables. All the variables we are using are to the first power.

- If there is no sign in front of a number, it is positive: $3 = +3$
- If there is no decimal point shown in a number, it is at the end of the number: $3 = 3.0$
- If there is no coefficient in front of a variable, the coefficient is 1: $1x = x$
- If there is no exponent shown with a variable, the exponent is 1: $x = x^1$ and x^1 is x

Section 1.12 Dividing with Variables

Looking Back 1.12

Because division is the inverse of multiplication, the rules for multiplying numbers apply to dividing variables as well. Let us investigate four different cases.

Case 1:

$$2 \cdot 8 = 16$$

$$\therefore 16 \div 8 = 2$$

Case 2:

$$3 \cdot -4 = -12$$

$$\therefore -12 \div -4 = 3$$

When the signs are the same for the dividend and divisor, the sign of the quotient is positive.

Case 3:

$$-4 \cdot 5 = -20$$

$$\therefore -20 \div 5 = -4$$

Case 4:

$$-16 \cdot -2 = 32$$

$$\therefore 32 \div -2 = -16$$

When the signs are opposite for the dividend and the divisor, the sign of the quotient is negative.

Looking Ahead 1.12

In Algebra, $a \div b$ can also be written $\frac{a}{b}$ when b is not zero. We will use this form in regards to our problems. Let us try these rules with variables.

Example 1:	Simplify:

$$\frac{8mn}{2}$$

Example 2: Simplify:

$$\frac{-22x}{-11}$$

Let a and b represent numbers or variables.

$\frac{+a}{+b} =$__________ $\frac{-a}{+b} =$__________

$\frac{+a}{-b} =$__________ $\frac{-a}{-b} =$__________

<u>Summary of Rules for Multiplication and Division with Variables</u>

The product or quotient of two positive numbers (or variables) is positive. The product or quotient of two negative numbers (or variables) is positive.

$a \times b = a \cdot b = ab$ $a \div b = \frac{a}{b}$ in which $b \neq 0$

$-a \times -b = -a \cdot -b = ab$ $-a \div -b = \frac{a}{b}$ in which $b \neq 0$

When the signs of two numbers or variables are the same, the product or quotient is positive.

The product or quotient of a positive number (or variable) and negative number (or variable) is negative. The product or quotient of a negative number (or variable) and a positive number (or variable) is negative.

$(-a) \times b = (-a) \cdot b = -ab$ $(-a) \div b = -\frac{a}{b}$ in which $b \neq 0$

$a \times (-b) = a \cdot (-b) = -ab$ $a \div (-b) = -\frac{a}{b}$ in which $b \neq 0$

When the signs of two numbers or variables are the opposite, the product or quotient is negative.

Section 1.13 The Distributive Property Revisited

Looking Back 1.13

The Distributive Property is used often for combining like terms because it helps us understand the properties of real numbers and apply them to our calculations. The Distributive Property states that for all real numbers a, b, and c…

$$a(b+c) = ab + ac$$

$$ab + ac = a(b+c)$$

Looking Ahead 1.13

When there is a negative sign outside of the parenthesis, we distribute it among each term inside the parenthesis. Terms are separated by addition or subtraction signs. Distributing the negative sign outside will change each term on the inside to its opposite:

$-a(b+c) = -ab + (-ac)$ $-a(b+c) = -ab - ac$

Example 1: Use the Distributive Property to simplify $3(x-2)$.

Example 2: Use the Distributive Property to simplify $-4(x-5)$.

Example 3: Use the Distributive Property to simplify $-(4x+3)$.

Example 4:	Use the Distributive Property to simplify $-5(-2x+3)$.

Factoring

When we take the common divisor of two or more terms, we call it factoring. Some think of it as "un-distributing."

Example 5:	Factor $4x+12$.

Example 6:	Factor $5x-15$.