Algebra 2 Module 2 Matrices

Section 2.1 Logic Matrices

Practice Problems 2.1

Answer Problem 1-5 using the following scenario:

Titus, Lilly, Katy, and Clara love to draw. One of the four children draw with markers, another with charcoal pencils, another with acrylic paint and the last with colored pencils.

a. Katy likes bright colors but does not like using markers to get them.

b. Katy and Clara never have paint stains on their hands, but their sibling does.

c. Titus always washes his hands once he cleans off his brushes.

d. Lilly likes bright colors too.

1. What are the two categories?

2. How many names are there? What are they? How many rows will represent this category?

3. How many drawing tools are there? What are they? How many columns will represent this category?

4. Must the number of rows and number of columns be the same? Why or why not?

5. Set up a matrix and solve the puzzle. What drawing tool does each child like?

6. Amelia, Lilian, Clarity, Nate and Amanda each have a favorite color. No two have the same favorite color. Which color is each person's favorite based on the statements below? Set up a matrix and solve.

a. Amelia's favorite color is not red.

b. Lilian does not like red or blue.

c. Green is somebody's favorite color.

d. Clarity loves purple.

e. Nate does not have anything yellow because he does not like it.

f. Amanda likes the color that Nate does not like.

7-10. Three friends live in three different style homes in three different cities. Find the type of home and the city in which each friend lives.

a. Emma's house is stucco.

b. Emma's best friend lives in a French provincial home.

c. Sage's zip code is in Ruis.

d. Emma's best friend is Reagan.

e. Emma's best friend lives in Bolian.

f. The house in Ruis is a Cape Cod.

g. Someone lives in Boldero.

7. What are the two sub-topics under the home category?

8. Write the names of the people on the left and the two categories on top. The two categories are separated by the scribbled lines.

9. Let the names of the friends be the rows. Why are there three rows?

10. Does it matter whether the type of home or city goes before or after the scribbled lines in the matrix? Why or why not?

11. Complete the matrix in Problem 8.

12. The Thiele family mark their height on the kitchen wall each year on their birthday. This past year they were all a different height. Find their order from shortest to tallest for this year.

a. Anne is a little taller than Kyle.

b. Eli is right between Allie and Cole.

c. Everyone is taller than Courtney.

d. Kyle is taller than four of his siblings.

e. Allie is a little shorter than Cole.

Section 2.2 Analyzing Data with a Matrix

Practice Problems 2.2

1. Complete the cells of the matrix below. Some are already done for you.

$A_{2,1}$	$A_{2,2}$		
$A_{4,1}$			
		$A_{5,3}$	

2. Name the dimensions of the row matrix.

$$\begin{bmatrix} -\frac{1}{2} & 2.3 & 0.\overline{6} \end{bmatrix}$$

3. Name the dimensions of the column matrix.

$$\begin{bmatrix} -0.4 \\ \frac{1}{3} \end{bmatrix}$$

4. What do you notice about the columns and rows in the square matrices?

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

5. If all the elements of a matrix are zero, it is called a "zero matrix." Make a zero matrix that has the dimensions 3 x 5.

6. Circle the pair of matrices that are equal.

$$\begin{bmatrix} 3 & 1 & -2 \\ \frac{1}{2} & 6 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -2 & \frac{1}{2} \\ 6 & 2 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 3 & 1 & -2 \\ \frac{1}{2} & 6 & 2 \end{bmatrix} = \begin{bmatrix} 3 & \frac{4}{4} & -2 \\ 0.5 & \frac{12}{2} & |-2| \end{bmatrix}$$

7. Matrices are equal if the elements in corresponding positions of each matrix are equal and they have the same dimensions. Find the values of the variables.

$$\begin{bmatrix} -3 & 2+x \\ 4y & 2 \end{bmatrix} = \begin{bmatrix} -3 & -3 \\ -12 & \frac{10}{5} \end{bmatrix}$$

8. What is the dimensions of the matrices in Problem 7? Are they square matrices?

Use the matrix Q = $\begin{bmatrix} -5 & \frac{1}{2} & 22 \\ 1 & 3 & 13 \\ 6 & 0.8 & 17 \end{bmatrix}$ to answer Problem 9-13.

9. Name the values in Row 3.

10. Name the values in Column 2.

11. Is the value in cell $Q_{1,2}$ equal to 1?

12. What is the value in $Q_{3,2}$?

13. Is this a row, column, or square matrix?

Pick from the following matrices to answer Problem 14-16 given the following information:

Apple pies regularly sell for $7.99 and pecan pie sells for $9.99. On Tuesday, which is discount day at the market, apple pies sell for $5.99 and pecan pies sell for $7.99.

a) $\begin{bmatrix} \$7.99 & \$5.99 \\ \$9.99 & \$7.99 \end{bmatrix}$ b) $\begin{bmatrix} \$7.99 & \$9.99 \\ \$7.99 & \$5.99 \end{bmatrix}$ c) $\begin{bmatrix} \$9.99 & \$7.99 \\ \$7.99 & \$5.99 \end{bmatrix}$

14. Which matrix does not accurately display the data?

15. Which matrix has the type of pie as the row heading?

16. Using Matrix C, title each row and column of the matrix appropriately.

17. How much money would be generated if nine pecan pies sold on Monday, three apple pies sold on Tuesday, and two of each sold on Wednesday?

For Problem 18-20, find the missing value of each variable. Let Q be a zero matrix.

18. Q = $\begin{bmatrix} a & 0 \\ c & d \end{bmatrix}$

19. Q = $\begin{bmatrix} x-3 & y+4 \\ 0 & z \end{bmatrix}$

20. P = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} x+2 & y \\ z-4 & 5 \end{bmatrix}$

Section 2.3 Matrices with Graphs and Charts

Practice Problems 2.3

The chart below represents driving distance between cities. The representatives of a company get reimbursed for travel from this information. The cities are Dayton (D), Chicago (C), Fort Wayne (F), and Pittsburgh (P).

Use the data to answer Problem 1-6.

	D	**C**	**F**	**P**
D	0	297	125	539
C	297	0	163	759
F	125	163	0	618
P	539	759	618	0

1. Why are there zeroes along the diagonal of the matrix?

2. The diagonal of the matrix is a line of reflection for the data. Why is the same data above the diagonal repeated below the diagonal?

Pick the correct answer for Problem 3-4.

3. The data in the second row matches the data in which column?

A) First B) Second C) Third D) Fourth

4. Judging from the matrix, where are the company headquarters most likely located?

A) Northeast B) Southeast C) Midwest D) Central Time Zone

5. Cell $C_{3,2}$ contains the same distance as which other cell?

6. If the data in a row is the same as the data in a column, then the data will have the same row and column

________________________.

The following is the process of a combustion reaction when a butane pocket lighter is used. The liquid butane $C_4 H_{10}$ is evaporated and then burned. Oxygen is added so that it produces carbon dioxide and water:

$C_4 H_{10} + O_2 \rightarrow CO_2 + H_2O.$

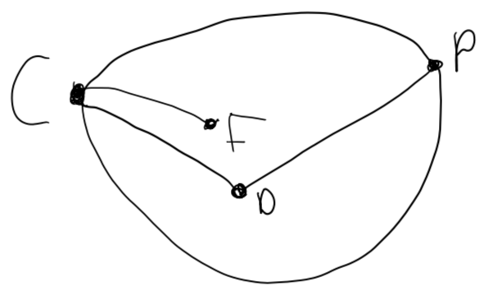
Of 50 ml. of liquid butane, 25% changes to carbon dioxide and 20% changes to water every ten minutes throughout an experiment.

Use the above information to complete Problem 7-8.

7. Draw a diagram or graph that demonstrates what is happening in the experiment.

8. Draw a matrix to show the milliliters of liquid butane converted to carbon dioxide and water and then the amount remaining after 30 minutes.

Below is a graph of flights between the cities of Dayton, Chicago, Fort Wayne and Pittsburgh. If there is a round trip between cities, they are connected by segments. Each segment represents a different round trip flight.



Use the above information to complete Problem 9-12.

9. Complete the matrix. Use a 0, 1, or 2 for the number of round trips between cities.

	D	**C**	**F**	**P**
D				
C				
F				
P				

10. What does Cell $C_{2,4}$ represent?

11. What city (or cities) would have the most available flights for travel between the cities located within the company's operations?

12. What does the data in cell $C_{4,1}$ represent?

13. Below is a matrix showing train routes between cities. Draw the matching graph.

	D	C	F	P
D	0	2	0	1
C	2	0	1	2
F	0	1	0	1
P	1	2	1	0

The decomposition $Si\ H_3F \rightarrow Si + H_2 + F_2$ is unbalanced with respect to the conservation of mass. The same number of each type of atom must be the same on the left and right side of the equation. Changing the subscripts alters the identity of the compound with the individual elements of Silicon, Hydrogen and Fluorine. The balanced decomposition reaction on the left side of the equation is $2SiH_3F$. In the problems below you will find the balanced decomposition product for the right side of the equation.

Answer Problem 14-17 to find out the balanced equation for the products side.

14. Complete the matrix for the coefficient and subscript for the balanced decomposition reactant.

Balanced Decomposition Reactant			
	Coefficient	**Subscript**	**Total**
Si			
H			
F			

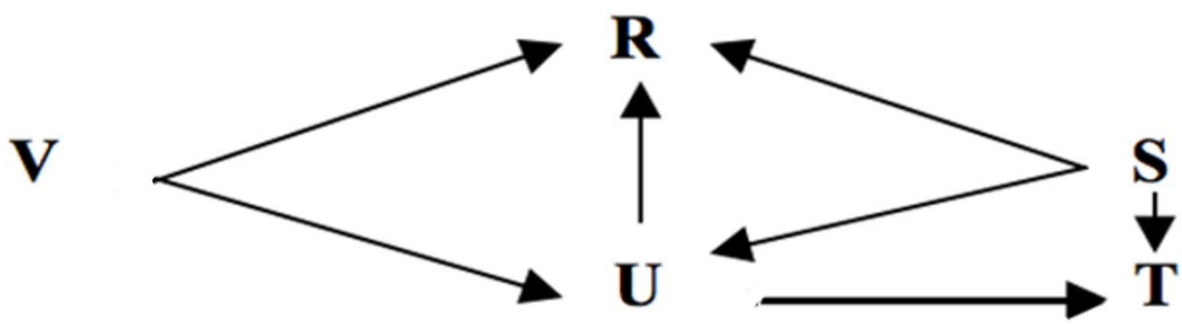
15. Complete the total in the matrix above by finding the product of the coefficient and the subscript.

16. The balanced decomposition product side of the equation has the subscripts from the unbalanced decomposition with respect to mass given above. Find the coefficient of each for the product side using the totals found in Problem 14.

Balanced Decomposition Product			
	Coefficient	**Subscript**	**Total**
Si			
H			
F			

17. Write the balanced equation for the decomposition of SiH_3F.

Mr. Pool set up the computer systems so that users from Example 1 now have the capability of connecting with other computers via another user. For example, User S can connect with User T through User U and User S can see the screen of T through User U.



Use the graph to answer Problem 18-20.

18. Complete the matrix for the computers. Use a “0” if a user cannot view the screen of another. Use a “1” if the screen of the user can be viewed directly, or through another user only. Put a “2” if the screen can be viewed directly and through another user.

To User						
		R	**S**	**T**	**U**	**V**
From User	**R**					
	S					
	T					
	U					
	V					

19. If the ability to directly see the screen of User R by User S goes out, User S can still the screen of User R. How do we know that from the graph? What cell gives us that information in the matrix?

20. Using the matrix below, find the molecular mass of the compound $CHCl_3$.

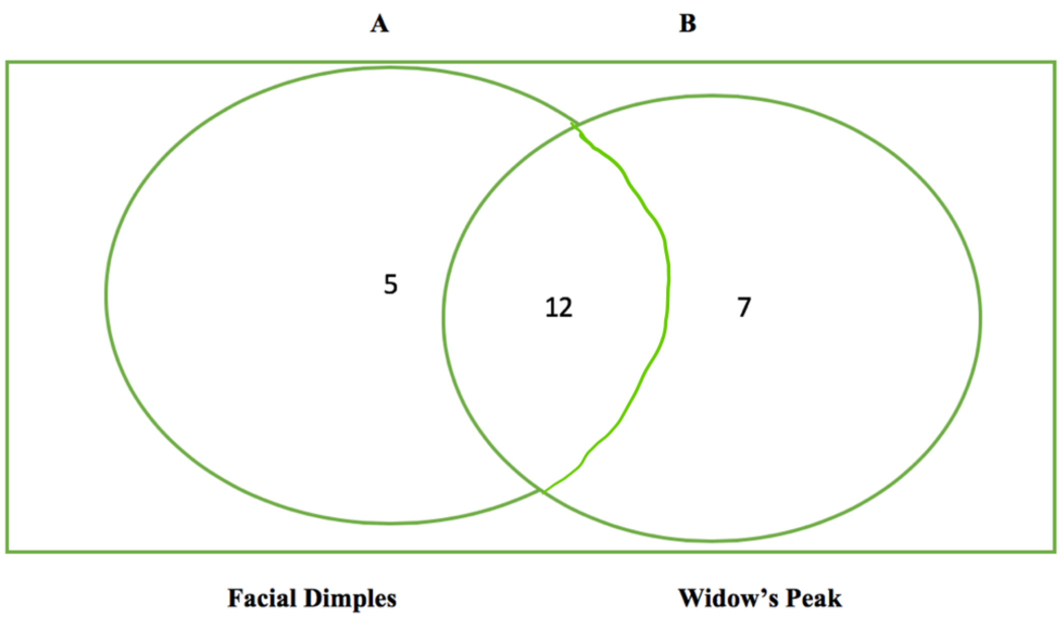
Chemical	**Amu**	**Number of Atoms**	**Molecular Mass**
C	12		
H	1.01		
Cl	35.5		

(Hint: The subscripts give the number of atoms of that specific element in the compound.)

Section 2.4 Matrices and Punnett Squares

Practice Problems 2.4

Using the Venn diagram from "Looking Back 2.4," answer Problem 1 and 2.



1. How many group members are in Set A?

2. How many group members are in Set B?

There are characteristics that may or may not be inherited traits. In a group of 27 members, only13 of them can roll their tongue, but 9 cannot cross their left thumb over the right when clasping their hand. There are 4 members of the group that can do both.

Use the above information to answer Problem 3-8.

3. Make a Venn diagram showing the characteristics of the 27 members of the group. Let *A* be the group with the ability to roll their tongues and *B* the group that places the left thumb over the right when clasping their hands.

4. Make a matrix for the data in the Venn diagram.

5. What is the probability that a member of the group will be able to roll their tongue, but not place their left thumb over their right thumb when clasping their hand?

6. What percentage of group members can roll their tongues or place their left thumb on top of the right when clasping hands, but are not able to do both?

7. What is the number of members in A'?

8. What is the number of members in B'?

A man with blood type AB marries a woman with blood type B. The woman has a recessive O allele as well. Remember from Example 4 that genotype A is either AA or AO.

Use the above information to answer Problem 9-13.

9. What is the genotype of the man?

10. What are the alleles of the woman?

11. Make a Punnett Square to show all the possible blood types for their children.

12. What are the probabilities for each type?

13. Make a matrix of the percentages for each blood type.

Eye color comes from two possibilities. The first gene is brown (B- dominant) or blue (b- recessive). If one of the two is brown, the eyes will be brown, either BB or Bb. Blue eyes come from bb. The second gene is green (G-dominant) or blue. Green eyes come from GG or Gb.

A child has blue eyes. Her father has brown eyes and a Bb genotype. Her mother has green eyes and a Gb genotype.

Use the above information for Problem 14 and 15.

14. Make a simple Punnett Square for the possible combinations for eye color.

15. Make a matrix of the possible eye colors. What is the probability the child will have blue eyes?

Eye color is much more complicated than that. The reason a brown-eyed father and a green-eyed mother can have a blue-eyed child is because there are at least two eye-color genes, which means both parents could carry the genes for blue eyes. Two genes influence eye color, and each gene comes in two versions. The first gene comes in brown (B) and blue (b). It is called OCH2. The second gene comes in green (G) and blue (b) and is called HERC2. The genotype Bb is associated with the trait of blue eyes, called the phenotype.

Use the above information for Problem 16-18.

16. Make a list of all possible combinations for brown, green, or blue eyes. Here are two: BB GG and BB bb. Find the other seven.

17. If there is a B, the eyes are brown because B is dominant over both G and b. How many of the nine combinations yield brown eyes?

18. If there is a G without a B the eyes will be green. What is G dominant over?

So, you should be able to see that you do get blue eyes from one combination. It can happen, but it is rare. The reason why B is dominant over G is because eye color comes from melanin, which is a pigment in the eye. Lots of melanin makes brown eyes. Less melanin makes green eyes. The OCH2 and HERC2 work together to make the pigment melanin.

Eye Color	**OCH2**	**HERC2**	**Pigment**
Brown	Yes	Yes	Yes
Green	No	Yes	No
Blue	Yes	No	No

19. If both OCH2 and HERC2 work together, pigment is made in the front part of the eye. What color will the eye be?

20. If either OCH2 or HERC2 is not working, pigment will not be made. What color will the eyes be?

Section 2.5 Adding and Subtracting Matrices

Practice Problems 2.5

1. Lane's ice cream parlor has one-dip cones for $1.25, two-dip cones for $2.50, and three-dip cones for $3.75. Alisa's ice cream parlor has one-dip cones for $1.00, two-dip cones for $2.00, and three-dip cones for $3.00. Draw a matrix $[C]$ to represent the price of one, two, and three-dip cones at Lanes's parlor and Alisa's parlor.

2. Both parlor's offer red sprinkles for a one-dip cone for $0.65, white sprinkles for a two-dip cone for $0.85, and blue sprinkles for a three-dip cone for $1.35. Is $[T]$ (shown below) the matrix that would be added to $[C]$ to give the total price of one, two and three cones with toppings?

$$[T]$$

$$\begin{bmatrix} 0.65 \\ 0.85 \\ 1.35 \end{bmatrix}$$

3. How could Matrix $[T]$ be made with a 3 x 2 matrix that could be added to $[C]$?

4. Calculate $[C]$ + $[T]$.

5. What information does the sum matrix in Problem 4 give you?

$$[F] = \begin{bmatrix} 2 & 4 \\ 9 & 8 \\ -6 & 3 \end{bmatrix} \qquad [G] = \begin{bmatrix} 9 & 1 \\ 9 & 6 \\ 7 & -4 \end{bmatrix}$$

Use $[F]$ and $[G]$ to solve.

6. Calculate $[G] + [F]$.

7. Find $[H]$ if $[G]$ + $[H]$ = $[F]$.

8. Calculate $[F]$ - $[G]$.

9. Is $[F]$ - $[G]$ the same as $[G]$ - $[F]$? What does this tell you about matrix subtraction?

10. Find the - $[F]$.

11. Calculate the - $[F]$ + - $[G]$.

12. Calculate $[H]$ + - $[H]$.

13. What do you think $3 \cdot [F]$ is?

14. What is $-2 \cdot [G]$?

Sonya sells white T-shirts at \$3.50 for small, \$4.90 for medium, and \$5.25 for large. Colored T-shirts sell for \$0.50 more for small, \$0.75 more for medium and \$1.00 more for large.

Addison sells white T-shirts as well, but hers cost more than Sonya's. She sells them at \$4.00 for small, \$5.00 for medium, and \$6.00 for large. However, her colored T-shirts are only an additional \$0.30 more than the original cost for each size.

15. Let $[W]$ be the price of the white T-shirts that Sonya and Addison sell. Let $[C]$ be the price increase for colored T-shirts. Find $[W]$ + $[C]$. Call it $[T]$. What does $[T]$ represent?

16. Interpret the final matrix $[T]$ from $[W]$ + $[C]$. What does the value in each cell mean?

17. Fill in the blanks.

Since $[W]$ + $[C]$ = $[T]$ …

Then $[T]$ - $[C]$ =[___] or $[T]$ - [___] = $[C]$

18. Calculate $[T]$ - $[W]$. What matrix does the difference equal?

19. Find $[A]$ in the following equation:

$$\begin{bmatrix} -2 & 0 \\ 4 & 1.8 \end{bmatrix} - [A] = \begin{bmatrix} 10 & 4 \\ 0 & 1.8 \end{bmatrix}$$

20. Calculate $[A] + \begin{bmatrix} -2 & 0 \\ 4 & 1.8 \end{bmatrix}$.

Section 2.6 Scalar and Matrix Multiplication

Practice Problems 2.6

1. Let $[A] = \begin{bmatrix} 7 & 1 \\ 2 & -1 \\ 3 & 4 \end{bmatrix}$ Let $[B] = \begin{bmatrix} 1 & -3 & 1 \\ 4 & 4 & 6 \end{bmatrix}$

Find $[A] \cdot [B]$.

2. Find $[B] \cdot [A]$ using the matrices given in Problem 1.

3. Is matrix multiplication commutative?

4. Stella's T-shirts come in small, medium, and large sizes and white, red, and purple colors. Below is the inventory of a store for each of the items.

$$\begin{array}{cc} & \begin{array}{ccc} W & R & P \end{array} \\ \begin{array}{c} S \\ M \\ L \end{array} & \begin{bmatrix} 10 & 11 & 12 \\ 2 & 4 & 6 \\ 1 & 1 & 5 \end{bmatrix} \end{array}$$

Small T-shirts cost the store \$2.00 each, medium T-shirts cost the store \$2.39 each and large T-shirts cost the store \$3.08 each wholesale.

Write the matrix $[C]$ to represent the T-shirt costs for the store.

5. a) How many small red T-shirts are in the inventory?

b) How many purple T-shirts are in the inventory?

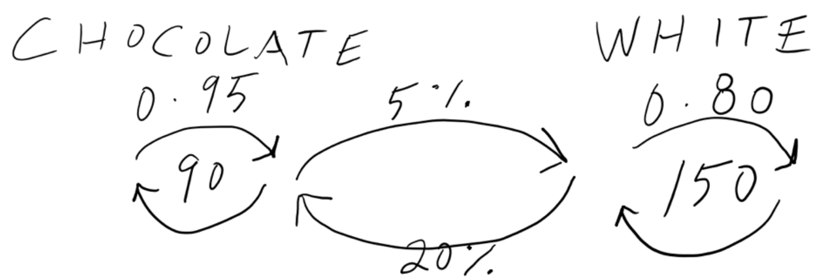
c) How many medium and large white T-shirts are in the inventory?

6. For the store's end-of-the-year inventory, what is the expenditure for the T-shirts? Use matrix multiplication to solve.

7. Bailey sells red and white T-shirts only. She buys 25 small, 14 medium, and 10 large red T-shirts for \$2.25 each. She also buys 30 small, 22 medium, and 8 large white T-shirts for \$1.80 each. What is the total expenditure for T-shirts? Is it easier to use matrix multiplication or some other method to solve this problem?

8. The school offers white or chocolate milk for lunch. The first week of school, 150 students drink white milk and 90 students drink chocolate milk. The second week, 20% of the students switch from white to chocolate milk, but only 5% switch from chocolate to white milk.

A matrix is given for the continuous graph.

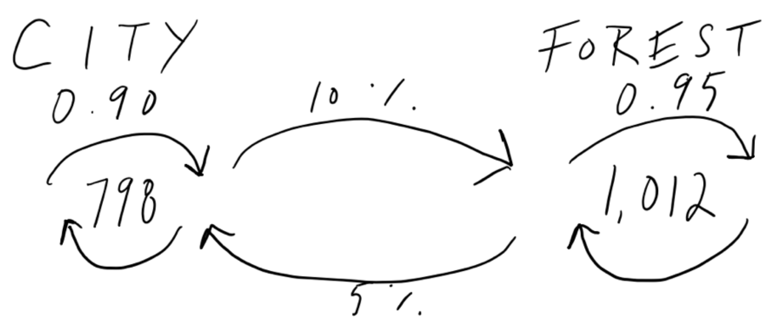


$$\begin{bmatrix} 150 & 90 \end{bmatrix} \qquad \begin{bmatrix} 0.80 & 0.20 \\ 0.05 & 0.95 \end{bmatrix}$$

How many students will choose white milk the first week and chocolate milk the second week?

9. If the cycle continues with 95% of the chocolate milk drinkers continuing to drink chocolate milk while only 5% switch to white milk, and with 80% of the white milk drinkers continuing to drink white milk while only 20 % switch to chocolate milk, how many students will drink white milk and chocolate milk the third week of school? Show the solution using matrix multiplication.

10. In Section 2.3 there was a problem of deer population relocating to cities and forests after subdivisions were built. The continuous graph is given again below.



Make matrices and use matrix multiplication to find the city and forest deer populations after the first subdivision is built.

11. Use matrices to determine how many deer populate the city and forest after a second subdivision is built.

12. How many deer populate the city and forest after a third subdivision is built? Use matrices and check your answers with Example 3 of Section 2.3 to see if they agree.

13. A triangle has the coordinates (- 5, 2), (2, 3), and (- 1, 2). Use scalar multiplication and a matrix to find the coordinates if the triangle is dilated to three times its original size.

For Problem 14-16, use the following information to solve the word problem.

A local theatre is performing *The Lion, the Witch, and the Wardrobe*. The matrices below give the ticket prices and the number of tickets sold.

$$[P] = \begin{array}{r} \\ Friday\ Evening \\ Saturday\ Afternoon \\ Saturday\ Evening \end{array} \begin{array}{c} \begin{array}{cc} Adults & Children \end{array} \\ \begin{bmatrix} 182 & 220 \\ 204 & 331 \\ 242 & 349 \end{bmatrix} \end{array} \qquad [C] = \begin{array}{r} \\ Adults \\ Children \end{array} \begin{array}{c} Prices \\ \begin{bmatrix} 9.00 \\ 5.50 \end{bmatrix} \end{array}$$

14. Find $[P] \cdot [C]$.

15. How much did the theatre make on ticket sales after the weekend performance?

16. How much did the theatre make on ticket sales for children only?

For Problem 17-19, multiply the matrices or tell if it is not possible given the dimensions of the product matrix.

17. $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \cdot \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$

18. $\begin{bmatrix} 4 & 2 & -1 \\ 6 & -1 & 8 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 \\ 2 & 3 \end{bmatrix}$

19. $\begin{bmatrix} 10 & 3 & 4 \\ 2 & 3 & -2 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 & 1 \\ 1 & 0 & 1 \\ -1 & -8 & 6 \end{bmatrix}$

20. Can a matrix be multiplied by itself? If so, what type of matrix would the product matrix be?

Section 2.7 Row Reduction Method for Solving Systems

Practice Problems 2.7

Name each property shown in Problem 1-6.

1. $-3 \cdot \begin{bmatrix} 1 & 10 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 10 \\ 2 & 4 \end{bmatrix} \cdot -3$

2. $\begin{bmatrix} 4 & 2 & 3 \\ -1 & 6 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 5 \\ 2 & 1 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 2 & 3 \\ -1 & 6 & 2 \end{bmatrix}$

3. $1 \cdot [M] = [M]$

4. $0 \cdot \begin{bmatrix} 1 & 4 \\ -7 & 2 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

5. $\left(\begin{bmatrix} 1 & 9 \\ 7 & -4 \end{bmatrix} + \begin{bmatrix} 3 & 8 \\ 3 & 8 \end{bmatrix}\right) \cdot (-2) = (-2) \cdot \begin{bmatrix} 1 & 9 \\ 7 & -4 \end{bmatrix} + (-2) \cdot \begin{bmatrix} 3 & 8 \\ 3 & 8 \end{bmatrix}$

6. $\begin{bmatrix} 1 \\ 6 \\ -2 \end{bmatrix} + \left(-\begin{bmatrix} 1 \\ 6 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Use the following matrix to solve Problem 7-10:

$$[Q] = \begin{bmatrix} 1 & -6 \\ 3 & 0 \\ 2 & 4 \end{bmatrix}$$

7. Find $[Q] \cdot -4$

8. Find $m \cdot [Q]$

9. Let $[R] = \begin{bmatrix} 2 & -6 \\ 4 & -3 \\ 1 & 0 \end{bmatrix}$. Then find $[Q] + 2 \cdot [R]$.

10. Find $[Q]$ - $[R]$.

For Problem 11-13, tell whether each of the Properties for Matrix Multiplication are true or false. Demonstrate by using the matrices given below.

$$[A] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad [B] = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix} \qquad [C] = \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix}$$

11. Distributive Property for Matrix Multiplication:

    $[A]([B] + [C]) = [A] \cdot [B] + [A] \cdot [C]$

12. Commutative Property for Matrix Multiplication:

    $[A][B] = [B][A]$

13. Associative Property for Matrix Multiplication:

    $[A]([B] \cdot [C]) = ([A] \cdot [B])[C]$

14. If the identity matrix $[I]$ is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, what is $[A] \cdot [I]$?

15. Without using matrix multiplication, what is $[B] \cdot [I]$?

16. Fill in the blank: $[C] \cdot [\_\_] = [C]$.

17. Let $[O]$ be the zero matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. What is $[A] \cdot [O]$? What is $[B] \cdot [O]$?

18. Solomon buys two of each pencils, pens and sharpeners. Elijah buys double the number of pencils as Solomon, triple the number of pens, and half the number of sharpeners. Ezra does not buy any pencils or pens but buys the same number of sharpeners as Solomon. Solomon spends \$5.18 all together for the supplies, Elijah spends \$5.09 and Ezra spends \$3.98.

Write three equations in three variables to represent the situation. Let x = price per pencil, y = price per pen, and z = price per sharpener.

19. Write the augmented matrix for the three equations in three variables that are the costs of school supplies for Solomon, Elijah, and Ezra.

20. Use elimination and the row operations to solve the school supply problem using the row reduction method. What are the costs of each pencil, each pen, and each sharpener?

Section 2.8 The Identity Matrix

Practice Problems 2.8

1. Let $[A]$ be the identity matrix. Complete the 5 x 5 matrix with elements.

$$[A] = \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

2. What is the element in $C_{3,1}$ of the identity matrix $[A]$?

3. What is the element in $C_{5,5}$ of the identity matrix $[A]$?

4. Find the matrix $[B]$ to make the following equation true.

$$\begin{bmatrix} 1.2 & 6.0 \\ 3.4 & 2.9 \end{bmatrix} + [B] = \begin{bmatrix} 4.0 & 2.0 \\ 1.1 & 2.1 \end{bmatrix}$$

5. Find the matrix $[A]$ to make the following equation true:

$$[A] - \begin{bmatrix} 13 & -1 \\ 10 & -11 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ 3 & 9 \end{bmatrix}$$

Chris' Cookies are delicious. She sells butter cookies for \$0.79 each, chocolate chip cookies for \$1.00 each, and macadamia nut cookies for \$0.85 each.

The array represents how many of each cookie was delivered to three locations on Monday.

	Butter	Chocolate Chip	Macadamia Nut
Lansing Store	159	170	80
Traverse City Store	303	209	105
Cincinnati Store	212	316	93

6. Set up a matrix multiplication problem to find the total amount collected for the cookies delivered on Monday. What are the dimensions of each matrix?

7. Find the total amount collected for all the cookies delivered in Monday. Use matrix multiplication.

8. The cost to make each cookie is $0.35 for butter cookies, $0.50 for chocolate chip cookies and $0.42 for macadamia nut cookies. Use matrix multiplication to find the cost of making the cookies to be delivered to the stores.

9. Which store brings in the largest profit for cookies?

10. What was the total profit made from all the cookies delivered on Monday?

11. Find $[B]$ that makes the following equation true:

$$\begin{bmatrix} 2 & 1.8 \\ -16.4 & 9.2 \end{bmatrix} \cdot [B] = \begin{bmatrix} 2 & 1.8 \\ -16.4 & 9.2 \end{bmatrix}$$

12. Find $[A]$ that makes the following equation true:

$$[A] \cdot \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

13. Find $[C]$ that makes the following equation true:

$$\begin{bmatrix} -33 & 46 \\ 18 & -54 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [C]$$

14. Write the 3 x 4 identity matrix.

15. Write the 3 x 3 identity matrix.

A farmer raises only chickens and cows. There are 49 livestock animals on his farm and 178 legs running them around all together.

Use the above information to answer Problem 16-19.

16. Write a system of two equations that represents the number of livestock and legs.

17. Use the elimination or substitution method to find out how many chickens and cows the farmer has.

18. Set up an augmented matrix for the system of equations in Problem 16.

19. Use the row reduction method to solve the system and find out how many cows and chickens the farmer has.

20. How is the row reduction method similar to the method you used in Problem 17?

Section 2.9 Inverse Matrices

Practice Problems 2.9

Let $M = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ and $N = \begin{bmatrix} 3 & 3 \\ 5 & 7 \end{bmatrix}$

1. Find $[M]\ [N]$.

2. Find $[N][M]$.

Is the answer for Problem 2 the same as the answer for Problem 1? Why or why not?

3. Find $[M]^{-1}$.

4. Find $[N]^{-1}$.

5. Find $[M]\cdot [M]^{-1}$. What do you think the answer should be?

6. Find $[N]\cdot [N]^{-1}$. What do you think the answer should be?

7. Find $[M]\cdot [N]^{-1}$.

8. Find $[M]^{-1} \cdot [N]$.

9. Find $[M]^{-1} \cdot [N]^{-1}$.

10. Find $[N]^{-1}\cdot [M]^{-1}$

11. Find $([M] \cdot\ [N] \cdot)^{-1}$.

12. What can you conclude from Problem 9 through Problem 11?

Answer the remaining questions using the system of equations.

$$2x + 3y = 11$$

$$5x + 6y = 26$$

13. The equations rewritten as $\left[\begin{array}{cc|c} 2 & 3 & 11 \\ 5 & 6 & 26 \end{array}\right]$ would be called a(n) _________________ matrix.

14. The equations could also be written as a matrix another way. Fill in the blanks.

$$\begin{bmatrix} 2 & \_\_ \\ \_\_ & 6 \end{bmatrix}\begin{bmatrix} x \\ \_\_ \end{bmatrix} = \begin{bmatrix} \_\_ \\ 26 \end{bmatrix}.$$

15. Find $\begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}^{-1}$

16. How can you check to make sure this is the inverse matrix? Check it.

17. Multiply $\begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}^{-1}$ by the left side of the equation in Problem 14. What do you get on the left side of the equation?

18. Multiply $\begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}^{-1}$ on the right side of the equation in Problem 14. What you get on the right side of the equation?

19. Set the two sides of the equation equal to each other using the solutions to Problem 17 and 18. What is the solution for *x* and *y* to the system of equations?

20. Why did this method help you find the solution?

Section 2.10 Determinants and Inverse Matrices

Practice Problems 2.10

Explain why each matrix in Problem 1 and 2 does not have an inverse matrix.

1. $\begin{bmatrix} 19 & -3 & 6 \\ 4 & 1 & 8 \end{bmatrix}$

2. $\begin{bmatrix} 3 & 6 \\ -2 & -4 \end{bmatrix}$

Tell whether each matrix in Problem 3-6 does or does not have an inverse.

3. $\begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$

4. $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

5. $\begin{vmatrix} -3 & 8 \\ 1 & 5 \end{vmatrix}$

6. $\begin{vmatrix} 0.2 & 3.5 \\ 6.1 & -0.3 \end{vmatrix}$

Given the following system of equations, answer Problem 7-15:

$$3x - 2y = -17$$
$$x + 4y = 13$$

7. Write the equation using matrices.

$\begin{bmatrix} \quad & \quad \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$

8. Find the determinant.

9. Does an inverse exist?

10. Use the theorem at the end of this section in Example 5 to determine the inverse.

11. If A^{-1} exists, then the following theorem is true.

$$AX = B$$

$$A^{-1}(AX) = (B)A^{-1}$$

$$(A^{-1}A)X = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$

In our problem, the matrix X is $\begin{bmatrix} x \\ y \end{bmatrix}$.

If the matrix A is $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$, then what is the matrix B?

12. Let the equation $AX = B$ be the matrices in Problem 7. Now let A^{-1} be the inverse you found in Problem 10 and substitute them in for the following equation:

$$X = A^{-1}B$$

13. What is the solution for X using $A^{-1}B$ from Problem 12?

14. Are the two lines formed by the two equations parallel or do they intersect in a point?

15. Does the solution set have infinite solutions or one unique solution?

Evaluate each determinant in Problem 16-19.

16. $\begin{vmatrix} 4 & -1 \\ -1 & 4 \end{vmatrix}$

17. $\begin{vmatrix} 22 & -11 \\ 1 & \frac{1}{2} \end{vmatrix}$

18. $\begin{vmatrix} 1 & 4 & -2 \\ 2 & 1 & 1 \\ -3 & 6 & 0 \end{vmatrix}$

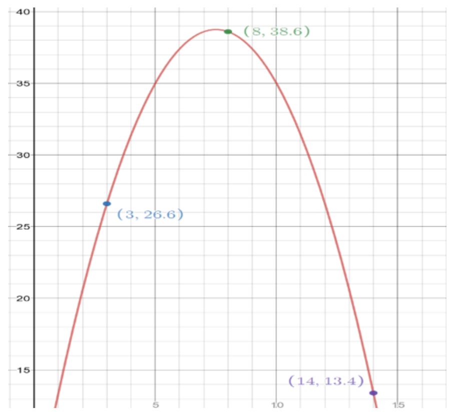
19. $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

20. What is the inverse of the matrix in Problem 19?

Section 2.11 Using Technology with Matrices

Practice Problems 2.11

The graph shows three points on the parabola. We will use our knowledge of quadratic equations, matrices, and the graphing calculator to find the quadratic equation that models this parabola.



The x values are 3, 8, and 14. The y values are 26.6, 38.6, and 13.4.

A quadratic equation is of the form $f(x) = ax^2 + bx + c$. Since we know x, we can multiply it by itself to get x^2. One equation using the first set of points, (3, 26.6), is $26.6 = a(3)^2 + b(3) + c$. The number 26.6 was substituted for $f(x)$ (or y), and the number 3 was substituted for x. It is best to put the coefficients in front of the variable as shown here: $9a + 3b + c = 26.6$

1. Can you find two more equations using the next two ordered pairs?

2. Write the three equations in matrix form.

$9a + 3b + c = 26.6$

$64a + 8b + c = 38.6$

$196a + 14b + c = 13.4$

3. Let $A = \begin{bmatrix} 9 & 3 & 1 \\ 64 & 8 & 1 \\ 196 & 14 & 1 \end{bmatrix}$ Let $x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ Let $B = \begin{bmatrix} 26.6 \\ 38.6 \\ 13.4 \end{bmatrix}$

We know that if $AX = B$ then $X = A^{-1}B$. Use the calculator to find $A^{-1}B$ and write it here:

Make sure you use the 3x3 matrix template, not the 2x2. It will ask you how many rows and columns you want. Press the up (↑) or down (↓) on the Navpad® so that both numbers are 3 and "Tab" down to "Ok" when you are done and click "Enter."

After you type the -1 in the exponent, click the right arrow on the Navpad to come out of exponent (superscript) mode to click the multiplication sign.

For Matrix B, the number of rows is 3 and the number of columns is 1. "Tab" from rows to columns and change the number by clicking the down arrow (↓). Then "Tab" to "Ok" and insert the values in the 3x1 matrix on the calculator page. What are the solutions for a, b, and c?

$$\begin{bmatrix} 9 & 3 & 1 \\ 64 & 8 & 1 \\ 176 & 14 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 26.6 \\ 38.6 \\ 13.4 \end{bmatrix} \qquad \begin{bmatrix} -0.6 \\ 9 \\ 5 \end{bmatrix}$$

4. Use the general form of the quadratic equation and substitute -0.6 for a, 9 for b, and 5 for c:

$$f(x) = ax^2 + bx + c$$

$$f(x) = -0.6x^2 + 9x + 5$$

Is the lead coefficient negative or positive? Is the parabola opening upward or downward?

5. Now that we know the equation for the parabola, let us see if the calculator agrees. We can have the calculator draw the data and analyze it with a quadratic regression.

First press “Ctrl” and “N.” This means “Control New.” We want to get rid of any data the calculator has previously stored so there is no confusion. When it asks, “Do you want to save ‘Unsaved Document?’” And you see “Yes,” “No,” and “Cancel,” “Tab” to “No.” It will be highlighted, then press “Enter.”

Use your Navpad® to scroll down to 5: “Add Lists and Spreadsheets,” then press “Enter” and a list and spreadsheet will appear. Scroll up to the top of Column A. The heading goes here. Type in “X” as we will store the x values here. Scroll down to Cell 1 and type in 3 and click “Enter.” Scroll down to Cell 2 and type in 8 and click “Enter.” Scroll down to Cell 3 and type in 14 and click “Enter.”

Navpad® right, then up to the top of Column B. Type in Y to store the y-values here. Scroll down to 1 and type in the three values for y in Cell 1, 2, and 3.

		A x	B y
	♦		
1		3	26.6
2		8	38.6
3		14	13.4

The row with the diamond is to enter formulas (♦). We are just entering our ordered pairs as data so the calculator can store, retrieve and graph them when instructed. This row will not include equations in this section.

Go back to the “Home” screen and scroll over or tab to the Data and Statistics page. This is where the graph will be drawn using the stored x and y values.

Move to the bottom of the screen to where it says “Click to Add Variables” and $\ldots x$ and $\ldots y$ will appear when you hit “Enter.” This is because those are the names (column headings) where our data is stored. Highlight the $\ldots x$ and press “Enter” since you are on the horizontal x-axis. Now scroll to the upper left side of the screen where the y-axis is. Press “+” and “Click to Add Variable” will appear. Press “Enter” and $\ldots x$ and $\ldots y$ will be displayed again.

This time, highlight $\ldots y$ and press “Enter.” The data will be drawn on the screen. To draw a connected graph, press the “Menu” button. This allows you to work on the screen or page you are on. Scroll down to “#4 Analyze” and click “Enter.” Then scroll to “#6 Regression” and click “Enter.” Scroll to “#4: Show Quadratic” since this a quadratic equation.

The data is connected with a parabola when you click “Enter” and the quadratic equation appears on the screen. It is the same as the one we derived:

$$y = -0.6x^2 + 9x + 5$$

Section 2.12 Deciphering Codes Using Inverse Matrices

Practice Problems 2.12

1. You are sending a message to a receiver. The message is "MATH IS IT." Use the encoding matrix $\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$ since it has a determinant of -1. What is the message you will send? Check it to make sure it works.

2. Your receiver sends you back a message that reads "ZEOGBSUY." The encoding matrix, only known by you and the receiver, is as follows:

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

What is the message you have been sent?

Decode the messages given the encoded messages and matrices.

a. Write the alphabetical letters of the encoded message as their numerical counterparts.
b. Write these as 1 x 2 row matrices.
c. Find the inverse of the encoded matrix to use as the decoded matrix.
d. Multiply each matrix of the encoded message by the decoded matrix.
e. Find the corresponding letters that go with each number of the decoded matrices.
f. The number 0 will be used for a space.

3. $[23 \quad 51]$ $[5 \quad 14]$ $[25 \quad 50]$ $[21 \quad 57]$ $[18 \quad 36]$ $[33 \quad 80]$ $[38 \quad 99]$ given the encoded matrix $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

4. $[17 \quad 26]$ $[49 \quad 76]$ $[2 \quad 3]$ $[37 \quad 60]$ $[11 \quad 19]$ $[9 \quad 14]$ $[50 \quad 75]$ given the encoded matrix $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

5. $[29 \quad 51]$ $[34 \quad 53]$ $[41 \quad 69]$ $[50 \quad 82]$ $[32 \quad 55]$ $[14 \quad 21]$ given the encoded matrix $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

6. $[29 \quad 46]$ $[40 \quad 60]$ $[38 \quad 67]$ $[52 \quad 85]$ $[53 \quad 91]$ given the encoded matrix $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

Encode the messages given the secret messages and the encode matrix.

a. Write the numbers of the encoded message as their corresponding alphabetical letter.
b. Write these as 1 x 2 row matrices.
c. Multiply each matrix of the encoded message by the 2 x 2 encode matrix.
d. Check these using the previous step to decode a message.
e. For a space use the number 0.

7. Secret message SHOW TIME using the encoded matrix $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$.

8. Secret message CELEBRATE using the encoded matrix $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

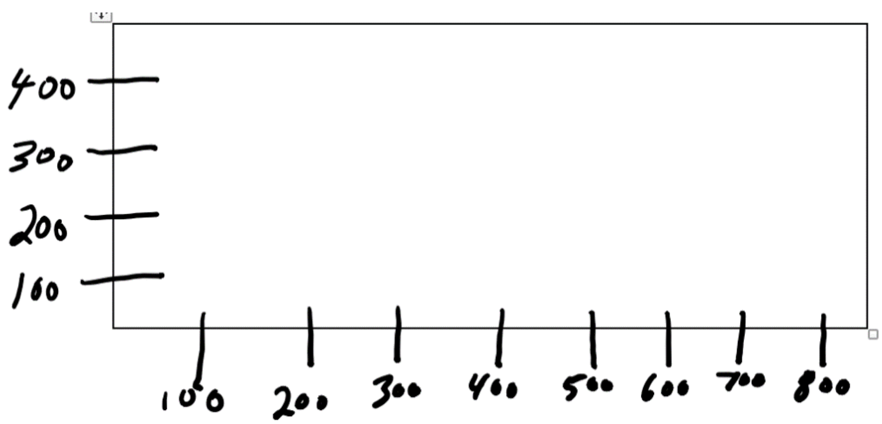
Section 2.13 Contour Maps

Practice Problems 2.13

The following steps will guide you through making a contour map. You will need the following supplies to complete it:

- Poster board
- Large cardboard box or corrugated cardboard sheets
- Exacto knife
- Glue

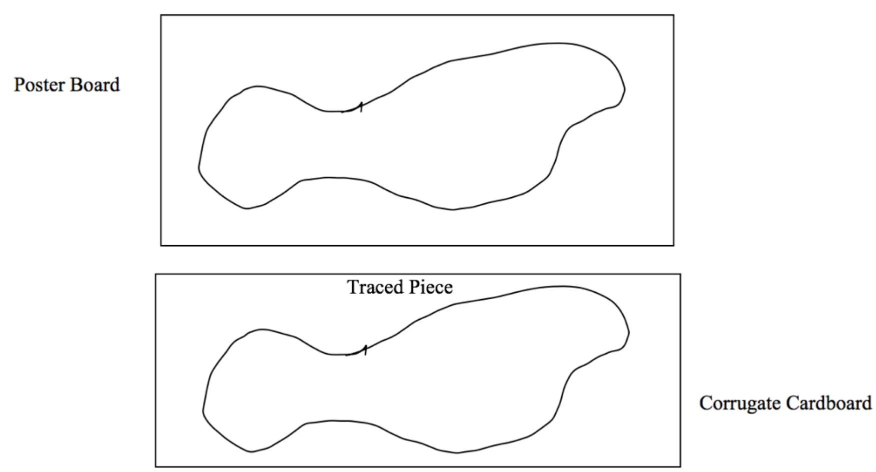
1. Cut the poster board in half. Along the length, or *x*-axis, draw tic marks in 1 cm. increments. Let 1 cm. = 100 ft. and mark the "feet" along the bottom. Draw 1 cm. increments along the left side, or *y*-axis. Beginning at 0 inches in the left-hand corner, label each tic mark in 100 ft. increments.



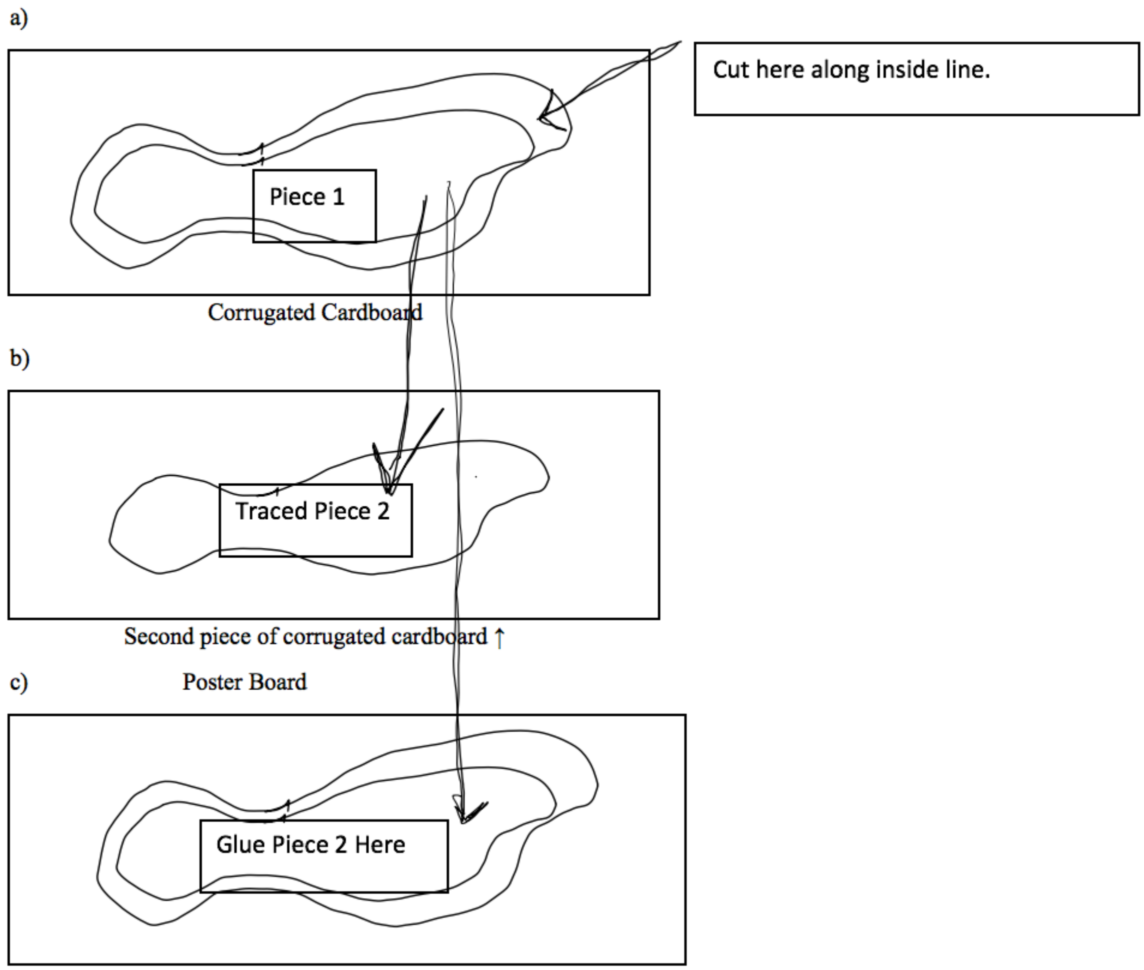
This will be the base of the model.

2. Cut a large shape out of the corrugated cardboard to fit in the poster board.

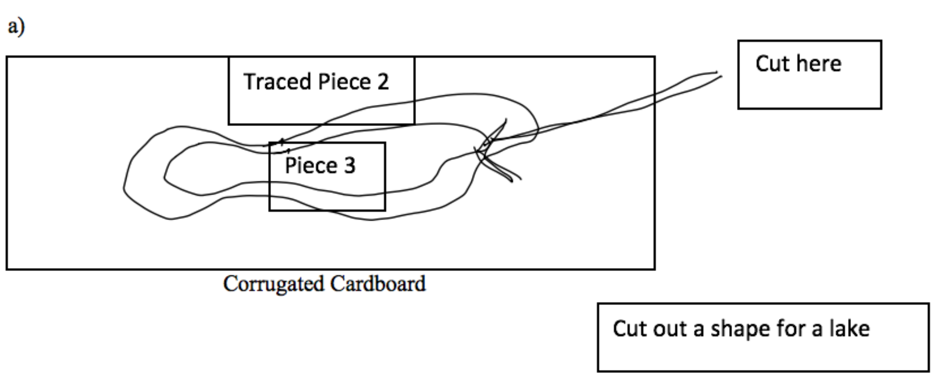
This will be the base of your land mass. Trace it onto another piece of corrugated cardboard before gluing it onto the poster board.



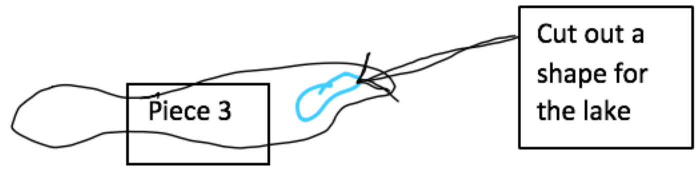
3. On the traced piece, move about 1”- 2” (inches) inside and draw the same contour line and cut it out using the inside line. Trace what you just cut out onto another piece of corrugated cardboard before gluing it onto the base of the poster board.



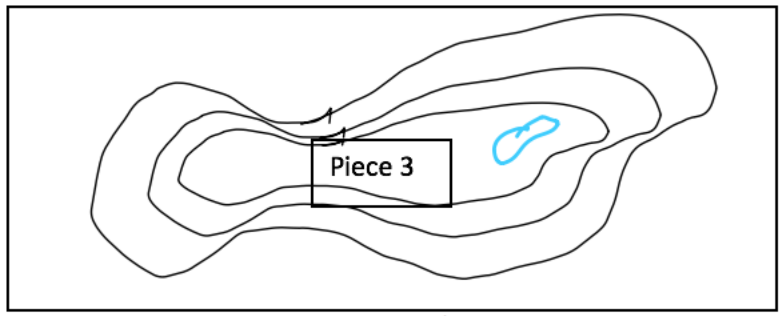
4. Draw a different shape smaller than Traced Piece 2 and cut it out. Cut a small shape out of it to represent a lake. Glue it on the top of Piece 2 on the poster board.



b)



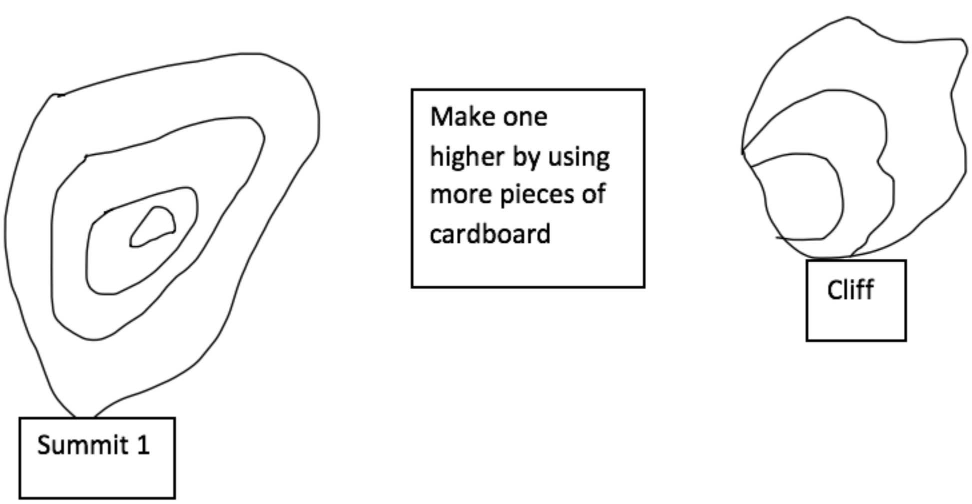
c)

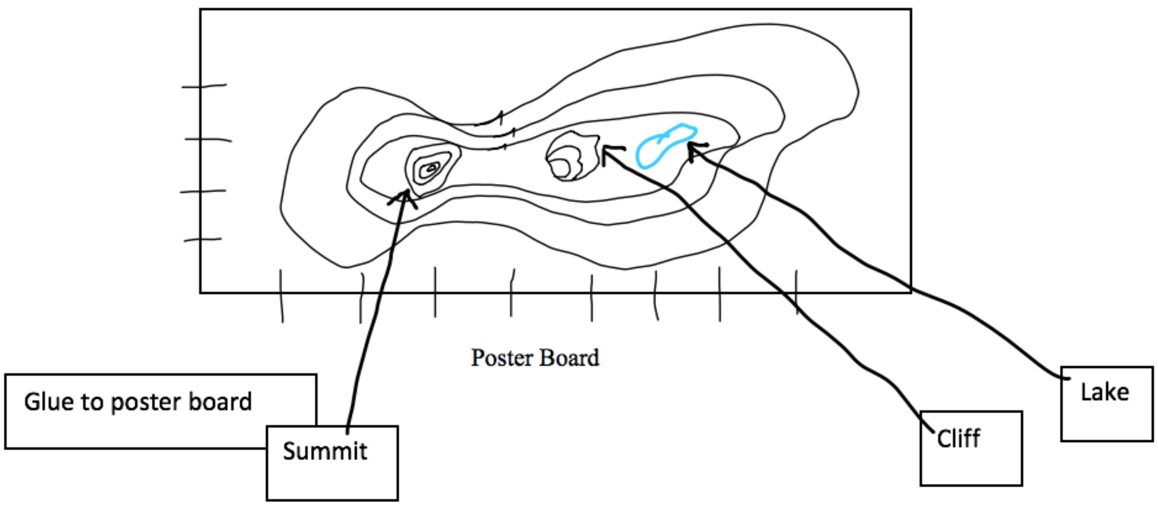


Poster Board

5. Cut out a few layers of smaller pieces of cardboard to make two summits. A summit is a point on a surface that is higher than all the points around it. Mathematically speaking, a summit is a local maximum in elevation. Make one summit a cliff.

A cliff usually goes straight down so glue the pieces of cardboard aligned to one side. Place both summits on the poster board and glue them down.





6. The x and y coordinates for each location are labeled on the two-dimensional plane. Measure the width of the cardboard to find the z coordinate. Locate the ordered triples for the following representations:

Lake: Summit: Cliff:

Section 2.14 Module Assessment

Use the matrix to answer Problem 1-5.

$$A=\begin{bmatrix}1 & 2 & -3\\ 0 & 4 & -\frac{3}{7}\\ 8 & 9 & 5\end{bmatrix}$$

1. What is the value of the element in cell $a_{1,2}$?

2. What is the value of the element in cell $a_{3,3}$?

3. Write the product matrix if A is multiplied by the scalar $\frac{1}{4}$.

4. Write the product matrix if A is multiplied by the scalar - 3.

5. Write the product matrix if A is multiplied by the identity matrix $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$.

Fill in the blanks.

6. The Identity matrix is a _______________ ____________________.

7. The dimensions of a product matrix when a 2 x 3 matrix is multiplied by a 3 x 4 matrix are _________________.

8. The determinant for $\begin{bmatrix}4 & 5\\ -2 & 1\end{bmatrix}$ is _______________.

9. If $e = \begin{bmatrix}d & b\\ c & a\end{bmatrix}$ then e^{-1} is ___________.

10. Using matrices to solve the system of equations $\begin{matrix}3x + y = 5\\ 2x - y = 0\end{matrix}$.

x = ___________ y = ______________

Use the matrices to answer Problem 11- 20.

$$A = \begin{bmatrix} 4 & 5 \\ -3 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

11. Find det. A

12. Find det. B

13. Find the product of $A \cdot B$.

14. Find the product of $B \cdot A$.

15. Find A^{-1}

16. Find B^{-1}

17. Find A + B

18. Find B – A

19. Find $2 \cdot A$

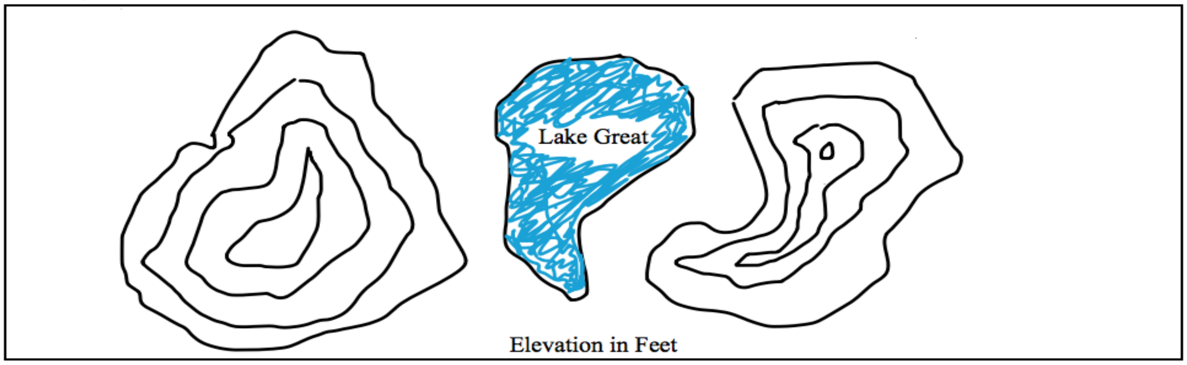
20. Find $\frac{1}{2} \cdot B$

21. Heather is buying flowers for her wedding. Carnations are $1.25 each, lilies are $2.25 each and roses are $2.75 each. For the ceremony, she needs to buy 12 carnations, 20 lilies and 24 roses. For the reception, she needs to buy 24 carnations, 48 lilies and 12 roses. Set up a matrix and calculate how much she will spend on flowers for her wedding.

Section 2.15 Module Project

The module assessment is a project to draw a profile map given a topographic or contour map and to draw a contour map given a three-dimensional model.

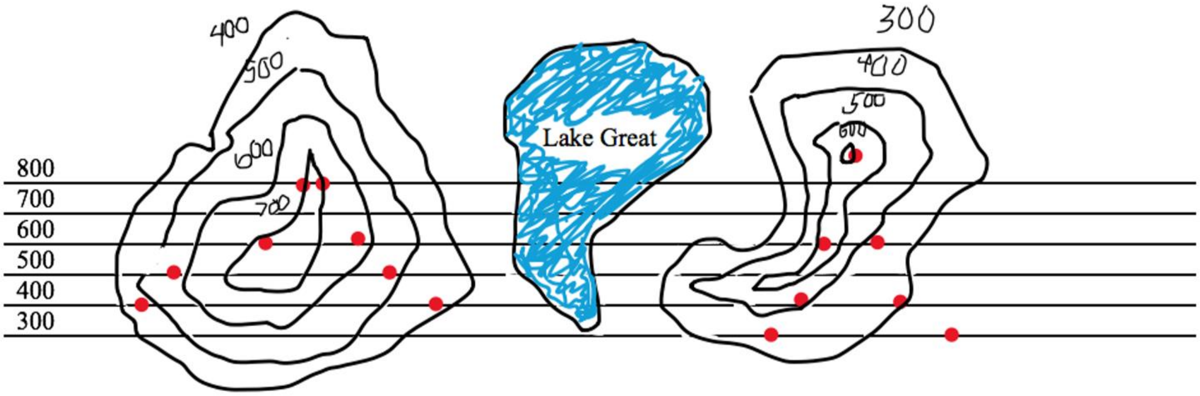
Topographic maps use contour lines to represent the relief or terrain of a region. Topographical maps show both natural and man-made features while contour maps show only natural features. For this activity, you will first investigate a contour map.



The rings in the contour map represent the elevation of the mountain in feet at that point. If a point is between two rings, then the elevation can be estimated to the nearest ring.

To draw a topographic profile:

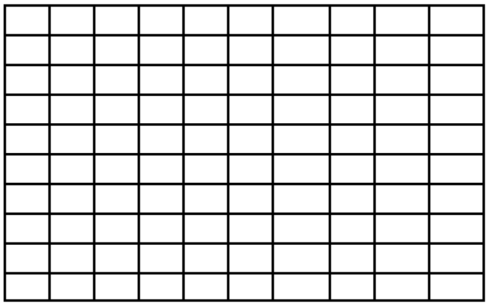
1. Sketch a line horizontally across the contour rings on the map. Name the line from left to right.
2. Put the edge of an index card on the line. Mark the left and right side as the map.
3. Move left to right and make a mark on the card anytime a contour line touches or crosses the paper or card.
4. Move from top to bottom on the index card and label every line with the elevation on the contour map from lowest to highest.
5. Draw a dotted line from each tick mark on the card down to the line of its elevation and put a dot there
6. Connect the dots to draw the profile of the contour map.

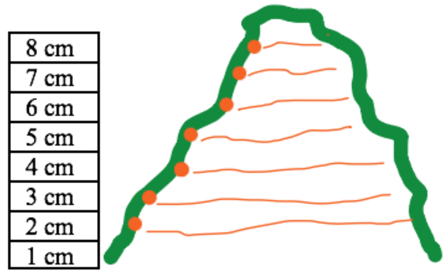


Use playdough to build a three-dimensional model of a mountain. Make it about the size of one can of playdough.

Use graph paper that has centimeter squares on it. Let a 1-centimeter square represent 100 square feet.

1. Mark the x-axis so that every tick mark represents 100 feet.
2. Mark the y-axis so that every tick mark represents 100 feet.
3. Place a centimeter ruler beside the playdough model, and poke a hole in the playdough at each centimeter height from bottom to top (1 cm., 2cm., etc.)
4. Place the playdough model on the graph paper and trace the base. Draw an arrow pointing to the holes in the side of the playdough as a marker to align after each slice.
4. Use dental floss or thread and slice through the playdough horizontally at the bottom hole.
5. Remove the bottom slice. Place the base of the playdough on the paper inside the original tracing. Make sure the holes are aligned with the arrow. Trace the base of the playdough again. Then slice through the playdough horizontally again at the next hole.
6. Continue this process all the way up the holes from top bottom to top and trace the base on the graph paper one inside the other after each slice.
7. Mark the contour lines so that each ring represents 100 feet.
8. To locate a point on the contour map, use the ordered triple moving on the x-axis and the y-axis. The z-axis is equivalent to the contour line.




8 cm
7 cm
6 cm
5 cm
4 cm
3 cm
2 cm
1 cm