

Algebra 2 Module 7 Rational Expressions, Functions, and EquationsSection 7.1 Defining Rational ExpressionsLooking Back 7.1

Rational functions are of the form $f(x) = \frac{p(x)}{q(x)}$, where both $p(x)$ and $q(x)$ are polynomial expressions. Therefore, $\frac{p(x)}{q(x)}$ is a rational expression and $f(x) = \frac{p(x)}{q(x)}$ is a rational equation. In the rational expression $\frac{p(x)}{q(x)}$, $q(x)$ cannot be equal to zero. Remember, a polynomial cannot have an exponent that is a negative, a radical, or a fraction. The exponent must be a non-negative integer (0, 1, 2, ... etc.).

Example 1: Which of the following expressions are rational expressions?

a) $\frac{5x}{x+2}$

b) $\frac{x}{4}$

c) $x^{\frac{1}{3}}$

d) $\frac{x}{\sqrt[4]{x}}$

Looking Ahead 7.1

Answer C from Example 1 is not a rational expression, but it is a rational power function. You learned about these in the previous module. It is not a polynomial because it has a degree that is a fraction. It can be written as a radical.

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

Sometimes the terminology gets confusing. An expression has no equal sign, whereas an equation does. A function uses function notation, such as $f(x)$, $g(x)$, and $h(x)$.

Example 2: Determine which of the following are expressions, functions, or equations.

a) $\frac{x}{y-4}$

b) $f(x) = \frac{4x-10}{2x+5}$

c) $\frac{2x}{6x+2} = \frac{x}{3x+1}$

d) $\frac{2x^3-1}{x^2-6x+4}$

A rational expression is simplified when the greatest common factor of the quotient of polynomials is 1.

Example 3: Simplify the following rational expressions.

a) $(x + 2 - x^2)^{-1}(5x - 6 - x^2)$

b) $\frac{x+2x}{x^2-4}$

c) $\frac{2x}{6x+2}$

d) $\frac{3^y \cdot 3^y \cdot 3^y}{3^y + 3^y + 3^y}$

This Practice Problems section includes an experiment. To perform this experiment, you will need pieces of spaghetti, tape, a ruler, a paper cup and Skittles® or M&M's®.

Section 7.2 Graphing Rational FunctionsLooking Back 7.2

A rational function is an equation that uses function notation equal to a quotient where the numerator and denominator are both polynomials.

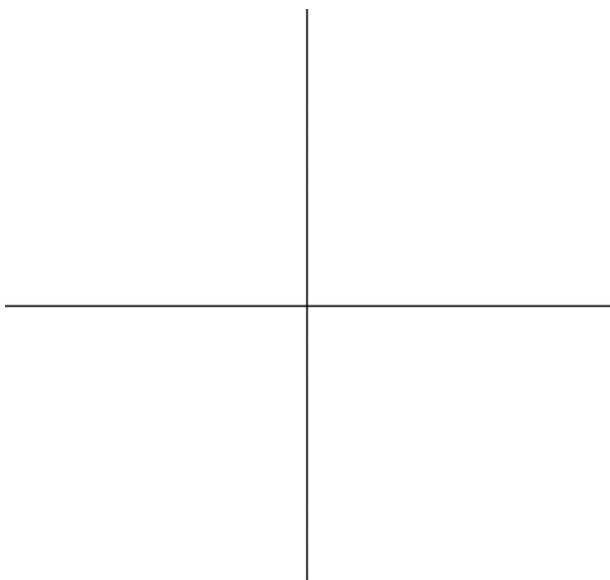
The simplest rational function is an inverse variation curve. The graph of the experiment in the previous Practice Problems section was a transformation of $f(x) = \frac{1}{x}$, which is the parent function for the simplest rational function, an inverse variation.

Looking Ahead 7.2

Example 1: Make the table and graph for the equation $f(x) = \frac{1}{x}$ and answer the questions.

x	$f(x)$
-1	
-0.1	
-0.01	
-0.001	
0	
0.001	
0.01	
0.1	
1	

x	$f(x)$
-10,000	
-1,000	
-100	
-10	
0	
10	
100	
1,000	
10,000	



- What does the curve appear to be?
- What value of x gives no value for $f(x)$?
- What are the asymptotes?

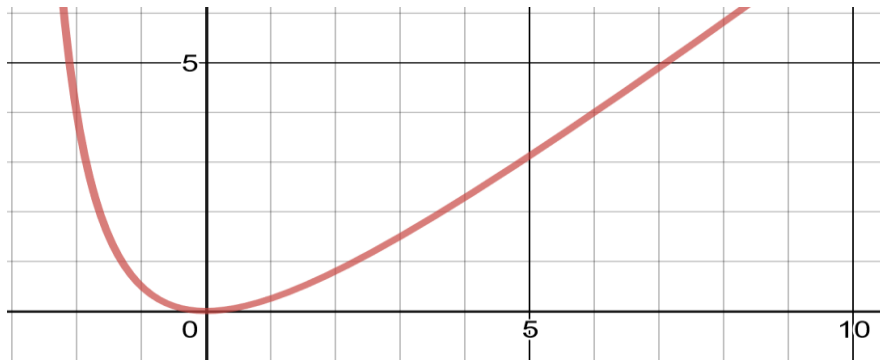
d) As x gets closer to zero from the left, what does $f(x)$ approach? As x gets closer to zero from the right, what does $f(x)$ approach?

e) As x takes on large absolute negative values, what happens to $f(x)$?

f) As x takes on large positive values, what happens to $f(x)$?

Some rational functions are continuous; you do not have to pick up your pencil to trace them on paper. Some are discontinuous; you do have to pick up your pencil to continue tracing them on paper.

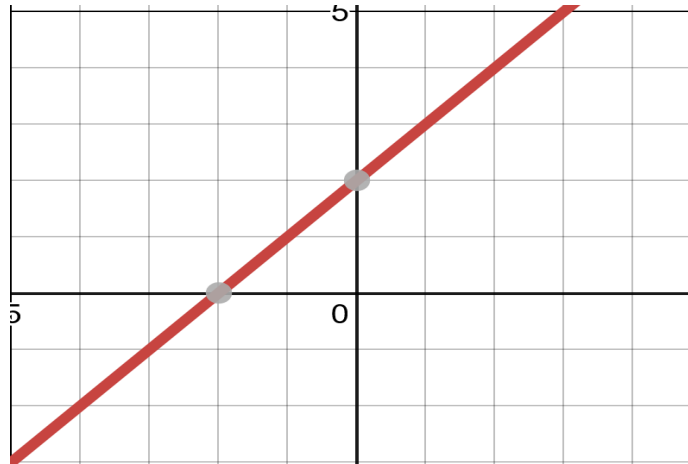
The graph of $f(x) = \frac{x^2}{x+3}$ is a continuous graph. The domain of this function is all real numbers.



Discontinuous graphs may have points of discontinuity that are removable. These points appear as holes in the graph, but do not always appear on the graphing calculator. Other discontinuous graphs may have a non-removable discontinuity. This jump in the graph is an asymptote that may not appear on a graphing calculator.

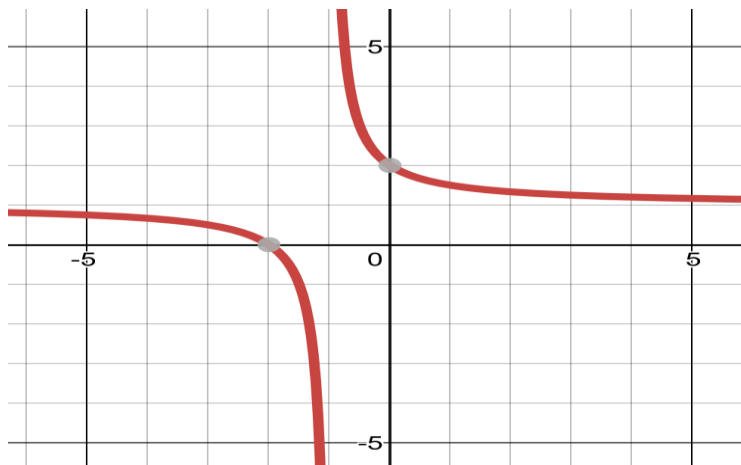
Example 3: Find the discontinuities for the following functions

a) $f(x) = \frac{x^2+3x+2}{x+1}$



The graph above has a removable discontinuity at $x = -1$. The function can be redefined at the point $x = -1$ so that $f(x) = 1$. The linear graph appears continuous on the graphing calculator. The hole in the graph that occurs at $(x - a)$ is in both $p(x)$, the numerator, and $q(x)$, the denominator. The domain of the function is all real numbers except $x \neq -1$.

b) $g(x) = \frac{x+2}{x+1}$



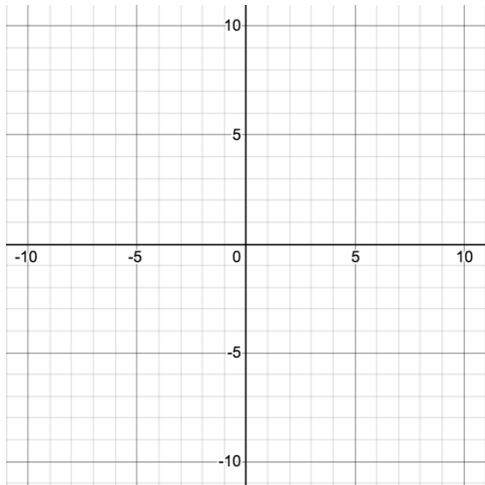
The graph above has a non-removable discontinuity at $x = -1$. The function cannot be redefined at the point $x = -1$ so that the graph is continuous. The hyperbolic graph is discontinuous over the two branches. The jump in the graph occurs at the asymptote when $x = a$. The domain of the function is all real numbers except $x \neq -1$.

Section 7.3 Finding Equations of Rational FunctionsLooking Back 7.3

In the previous section, you investigated changing parameters in rational functions and explored its effects. The parent function of a rational function is $f(x) = \frac{1}{x}$. The asymptotes are $x = 0$ and $y = 0$. The equation for the graphing form of a rational function is $y = \frac{a}{x-h} + k$.

Example 1: Below is the new equation that is a transformation of the parent function that has been translated down 3 units on the graph (vertically). Draw the graph and check your work or use a graphing calculator to graph the equation.

$$g(x) = \frac{1}{x} - 3$$

Looking Ahead 7.3

When the transformation of a function is a translation up or down (vertically), the horizontal asymptote changes. The asymptotes for Example 1 are $x = 0$ and $y = -3$. An up or down vertical translation of the graph changes the y -asymptote.

When the transformation of a function is a translation right or left (horizontally), the vertical asymptote changes.

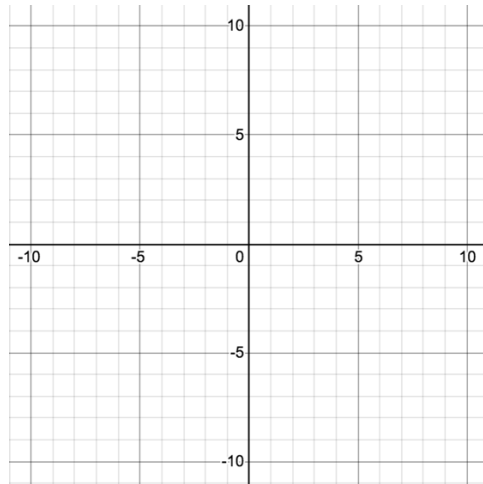
Example 2: What are the equations of the asymptotes for the hyperbolas?

a) $h(x) = \frac{1}{x-2.5}$

b) $j(x) = \frac{4}{x+5}$

Example 3: Below is the new equation that is a transformation of the parent function. What are the x and y asymptotes? Draw the graph of the equation and check it using a graphing calculator.

$$k(x) = \frac{1}{x-2} - 4$$



Example 4: Name the steps needed to complete the graphs below that are transformations of the function $f(x) = \frac{1}{x}$.

a) $j(x) = \frac{4}{x+5}$

b) $n(x) = \frac{-3}{x+1} + 4$

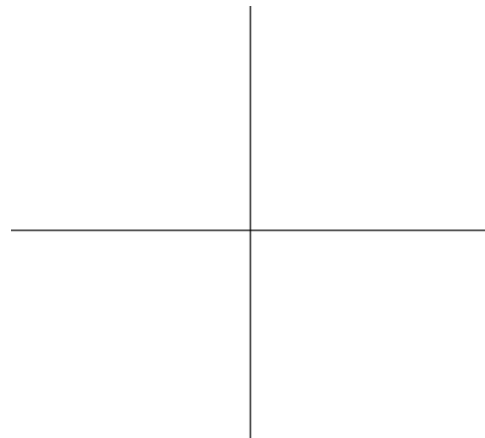
Section 7.4 Transformations of Rational FunctionsLooking Back 7.4

The parent function of a rational function is $f(x) = \frac{1}{x}$. The general or graphing form of a rational function is $f(x) = \frac{a}{x-h} + k$, where the parameter a causes a vertical stretch or shrink by a scale factor of a . The parameter h causes a horizontal translation right or left and the parameter k causes a vertical translation up or down.

Example 1: When $a = 3$, the graph stretches (which we have learned in a previous section). When $a = 0.5$, what does this do to the parent function?

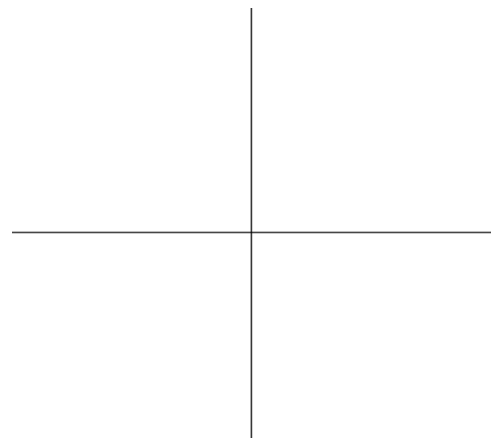
$$\text{Graph } g(x) = \frac{0.5}{x}$$

x	$g(x)$
-10	
-5	
0	
5	
10	



Example 2: When $h = 4$, how does this transform the graph of the parent function?

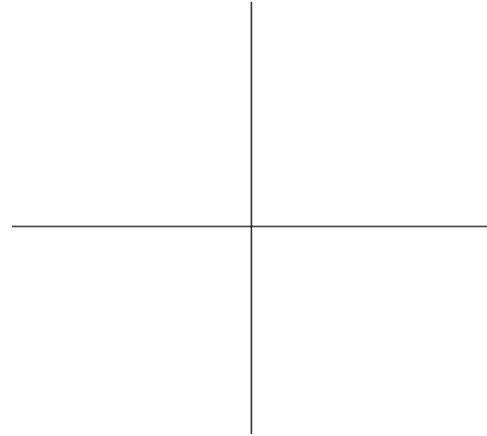
x	$j(x)$
-12	
-8	
-4	
0	
4	
8	
12	



Example 3: When $k = -6$, how does this transform the graph of the parent function?

$$m(x) = \frac{1}{x} - 6$$

x	$m(x)$
-10	
-5	
0	
5	
10	

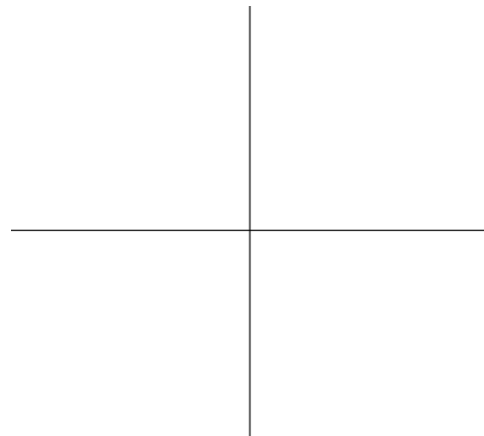


Looking Ahead 7.4

The domain of a rational function has values for which the denominator, $q(x)$, is not equal to zero. For example, in the rational function $f(x) = \frac{x+1}{x-2}$, all values of x work except $x = 2$. Therefore, the domain is all real numbers, but x does not equal 2.

Example 4: Graph the function $f(x) = \frac{x+1}{x-2}$ after completing the table. Find the vertical and horizontal asymptotes.

x	$f(x)$
-3	
-2	
-1	
0	
0.5	
1	
1.5	
2	
3	
4	
5	



If there is a non-removable discontinuity at $x = a$, then there is a vertical asymptote at $x = a$.

This is a little more complex than the previous equations because the form does not have the value of a as a constant.

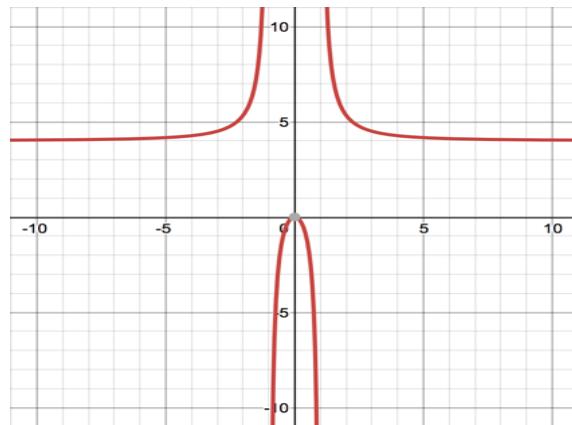
Example 4 had a vertical asymptote and a horizontal asymptote.

There are vertical asymptotes at the real zeroes of the denominator, $q(x)$. There can be any number of vertical asymptotes. However, there is at most one horizontal asymptote. We will discuss further how to find the vertical asymptotes in Example 5.

Example 5: Find the x and y intercepts of the function and the vertical and horizontal asymptotes.

$$f(x) = \frac{4x^2}{x^2 - 1}$$

The graph is shown below.



a) The x -intercept is when $f(x) = 0$. This occurs when $p(x) = 0$ and $q(x) \neq 0$.

b) The y -intercept is when $x = 0$.

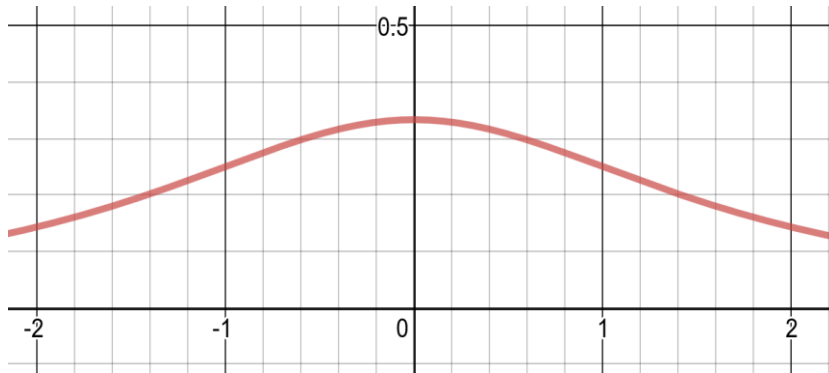
c) The vertical asymptotes are the real zeroes of the denominator that do not have removable discontinuities. That means that $p(x)$ and $q(x)$ do not have the same factor.

d) Let a be the lead coefficient of $p(x)$. Let b be the lead coefficient of $q(x)$. If the degree of the of $p(x)$ (call it m) and the degree of the of $q(x)$ (call it n) are equal, then the horizontal asymptote is the line $y = \frac{a}{b}$.

Example 6: Find the x and y intercepts of the function and the vertical and horizontal asymptotes.

$$g(x) = \frac{1}{x^2 + 3}$$

The graph is shown below.



a)

b)

c)

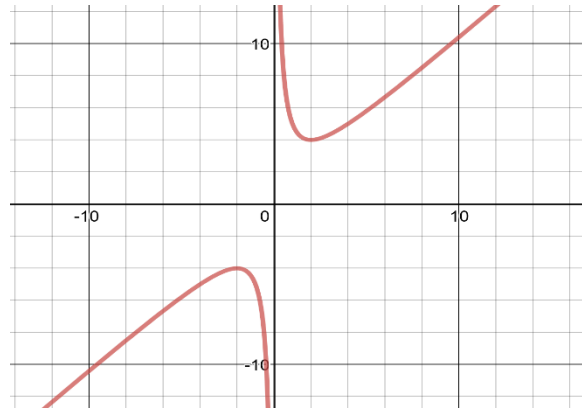
d)

If the degree of $p(x)$ is less than the degree of $q(x)$, then $m < n$ and the horizontal asymptote is 0.

Example 7: Find the x and y intercepts of the function and the vertical and horizontal asymptotes.

$$h(x) = \frac{x^2 + 4}{x}$$

The graph is shown below.



a)

b)

c)

d)

If the degree of $p(x)$ is greater than the degree of $q(x)$, then $m > n$ and there is no horizontal asymptote.

These rational functions are of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$, the numerator, and $q(x)$, the denominator, have no common factors. This means the equation is in simplest terms.

To summarize:

Let the degree of $p(x)$ be m and let the degree of $q(x)$ be n .

1. If $m = n$, then the horizontal asymptote is the line $y = \frac{a}{b}$.
2. If $m < n$, then the horizontal asymptote is the line $x = 0$.
3. If $m > n$, then there is no horizontal asymptote.

Section 7.5 Direct VariationLooking Back 7.5

You have learned about direct variation in previous courses. This is a relation in which the quotient of the independent variable and dependent variable is constant. It can be written in the form $y = kx$ or $\frac{y}{k} = x$, where k is the constant of variation. You have also seen this in the form $y = mx$, where m is the slope, which is constant, and the graph is linear.

In this section, we will investigate the relationship between the density and sugar content of a beverage. The density of a solution depends on concentration. Concentration is the amount of solute (solid) that is dissolved in a solvent (liquid). Volume will be plotted on the x -axis and mass will be plotted on the y -axis. This establishes the relationship by recognizing a pattern that exists. Density is proportional to concentration. The concentration here is the sugar content. Density is the physical property that identifies a substance or the ratio of mass per unit of volume. Therefore, the density of sugar solutions must be determined firstly followed by the percentage of sugar in the beverage.

In this first experiment, students followed the steps to calculate the density (slope) of 5% sucrose solution. The same procedure is followed for 10%, 15%, and 20% sucrose solution.

1. Place a graduated cylinder on a balance and record the mass.
2. Fill the cylinder to the 10.0 mL mark with a given percentage of sucrose solution.
3. Measure the mass of the solution with the graduated cylinder and record it in the table.
4. Subtract the mass of the graduated cylinder from the mass of the solution with the graduated cylinder to get just the mass of the solution, and then record it in the table.
5. Add sucrose solution to the 20 mL mark of the graduated cylinder and repeat Steps 3 and 4.
6. Repeat Step 5 for 30, 40, 50, 60, 70 and 80 mL. of solution.
7. Graph the volume of the standard solution on the x -axis and the mass of the solution on the y -axis

Example 1: Below is the data from a group of students that used a standard solution of 5% sucrose. Complete the table. Find the slope and line of best fit. What does the slope represent?

Volume of 5% Sucrose Solution (mL)	Mass of Graduated Cylinder and Solution (g.)	Mass of 5% Sucrose Solution (g.)
10.0	104.16	10.06
20.0	113.95	
30.0	123.43	
40.0	134.64	
50.0	145.06	
60.0	155.06	
70.0	165.12	
80.0	175.49	

Looking Ahead 7.5

The same procedure used above is used by the students to find the density for a beverage sample of diet cola.

The mass of the graduated cylinder is 94.31 g. The mass of the graduated cylinder and 10 mL of the diet Cola beverage is calculated and the two are subtracted and repeated for 20, 30, 40, 50, 60, 70, and 80 mL of beverage.

Example 2: Below is the data from the diet Cola experiment. Complete the table. Find the slope and line of best fit of the beverage graph. What does the slope represent?

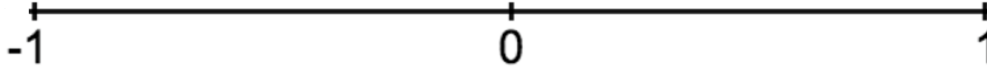
Volume of diet Cola (mL)	Mass of Graduated Cylinder with diet Cola (g.)	Mass of diet Cola (g.)
10.0	104.52	
20.0	114.65	
30.0	125.26	
40.0	135.94	
50.0	145.40	
60.0	155.70	
70.0	166.42	
80.0	176.42	

Once the density of a beverage is calculated, the percentage of sugar of the beverage can be calculated. The beverage label provides the grams of sugar and the amount of fluid ounces in the drink. If this is divided by density, the labels cancel out, leaving the amount of sugar per gram of beverage. Multiply this by 100% to get the percentage of sugar in the entire drink.

Example 3: Twenty fluid ounces of flavored water has a measured density of 1.668 g/mL. The nutritional label says there are 32 g. of sugar in 591 mL of the beverage. Calculate the percentage of sugar in the beverage.

Section 7.6 Correlation CoefficientLooking Back 7.6

In Algebra 1, you learned about the correlation coefficient.



High Correlation

No Correlation

High Correlation

A -1 or 1 on the scale means there is a high correlation (there is a strong relationship between the variables). It would be reasonable to conclude that there is a cause and effect relationship between the independent and dependent variable. A perfect fit for experimental data and calculated data is $+1$ or -1 . If the correlation coefficient is 0 , no relationship exists between input and output.

The distance the experimental data is from the line of best fit is called a residual. If this number is high, the actual value of the data may be far from the predicted value of the data. The smaller the sum of the residuals, the better fit the data is to the line of best fit.

Most calculators will find residual values for you, but we will do it by hand firstly. We will be using the data from a Cola experiment to determine if the density of the beverage is related to its sugar content.

Correlations can be positive (as input increases, output increases) or negative (as input increases, output decreases). A 0 on the scale means there is no correlation between the input and the output. You will soon find the type of correlation that exists in the beverage sample experiment.

Looking Ahead 7.6

Example 1: Follow the steps and complete the table to use a linear regression model and statistical data to see if the experiment shows a relationship between density and sugar content of a Cola. Remember, x is the volume of the beverage and y is the mass of the beverage. The actual volume and mass of the solution have been put in the table below for you.

- Put the volume of the standard solution in the table below in Lists and Spreadsheets and label the column x . Put the actual value of the mass in the table below in Lists and Spreadsheets and label the column y .
- Find the linear regression equation ($y = mx + b$) and let that be \hat{y} (pronounced, “y hat”). This is the predicted value of y based on the equation. Complete the table below for \hat{y} .
- Find the difference between the predicted value, \hat{y} , and the actual value of y from the experiment. This is called the residual. Complete the table below.
- Find the squares of the residuals and complete the table for $(y - \hat{y})^2$.
- Add up all the squares of the residuals. This is called the sum of the squares of the residuals or $SS_{\text{res}} =$ _____.
- Add up all the values of the y -coordinates and divide by the total number of y values. This is called the mean or \bar{y} (pronounced, “y bar”). This mean is _____.
- Find the difference between the y values and the mean. This is called the deviation. Complete the table below for $y - \bar{y}$.
- Find the squares of the deviation and complete the table below for $(y - \bar{y})^2$.
- Add up all the squares of the deviations. This is called the sum of the squares of the deviations or $SS_{\text{dev}} =$ _____.
- The SS_{res} is a minimum of all possible values. The deviation is from the horizontal or constant function. The residual is from the line of regression and is smaller and if you remove the fractional difference, or SS_{dev} , of one from the other you get an even better fit. This is called the coefficient of determination.

$$r^2 = \frac{SS_{\text{dev}} - SS_{\text{res}}}{SS_{\text{dev}}}$$

$$r^2 = \underline{\hspace{2cm}}$$

11. The r is called the correlation coefficient and it measures the fit of the data to the function. If it is 1 or -1 , it is a perfect fit. The correlation between the function and data can be strong or weak, or not at all if $r = 0$.

$$\sqrt{r^2} = \underline{\hspace{2cm}}$$

12. The function that identifies a pattern in Chemistry that is a theoretical law would give an exact fit, but when you collect data from an experiment, it is not an exact fit. Is your correlation a strong correlation or a weak correlation? Is it a positive correlation or a negative correlation?

Volume (ml.)	Actual Value Mass (g.)	Predicted Value Mass	Residual	Residual Squared	Deviation	Deviation Squared
x	y	\hat{y}	$y - \hat{y}$	$(y - \hat{y})^2$	$y - \bar{y}$	$(y - \bar{y})^2$
10.0	10.22					
20.0	21.13					
30.0	31.04					
40.0	41.12					
50.0	51.36					
60.0	61.61					
70.0	72.20					
	288.64					

This chemistry experiment only examined the relationship between the density and the sugar content of a beverage. Assumptions are made about the other ingredients in the drink and their effect on its density. What are these assumptions and are they valid?

Section 7.7 Inverse VariationLooking Back 7.7

In the experiment to find the amount of sucrose in a beverage, density was the average mass of solution divided by the average volume of the standard solution ($d = \frac{m}{v}$). Therefore, density is the slope since $v = dm$.

We will now review inverse variation, which you have explored in previous math courses and investigated again in previous modules in this course.

Inverse variation is a relation in which the product of the independent and dependent variable is constant and can be written in the form $xy = k$, where k is the constant of variation. Therefore, it can also be written as the quotient, $y = \frac{x}{k}$.

Looking Ahead 7.7

Ohm's Law, which states that $I = \frac{V}{R}$, is an example of an inverse variation. The amount of current, I (amperes/amps), is equal to the quotient of the voltage, V (voltage/volts), and resistance, R (Ohms/ Ω). The current flows in a circuit when the voltage is applied to the resistance. Voltage is the electromotive force (emf = voltage). Current varies directly as to voltage and inversely as to resistance. These are the three building blocks of electricity.

Example 1: A blow-dryer is set on a maximum setting of 8.3 amps on a 120-volt line. What is the resistance of the heating coils?

Example 2: In Europe, the power lines are 240 volts. If someone on vacation uses the blow-dryer from Example 1 with a resistance of 14.46 ohms, what would be the flow of the current in amps?

Example 3: The 120 volt blow-dryer can be used in Europe with the proper amount of resistance. What resistance would work for an 8.3 amp setting at 240 volts?

Section 7.8 Simplifying Rational ExpressionsLooking Back 7.8

You have seen that a rational expression is $\frac{p(x)}{q(x)}$, where $q(x)$ is not zero. To find the real zero, the rational expression must be simplified. As you have previously seen, the denominator may cancel out leaving only the numerator, which could be a power function: linear, quadratic, cubic, etc.

Looking Ahead 7.8

In this section, you will simplify rational expressions. Simplifying is the first step before performing operations on expressions. In the following sections, you will add, subtract, multiply, and divide rational expressions. Once equivalent forms of rational expressions are found, a rational equation is formed and can be solved for the unknown variable.

Example 1: Which forms of expressions are equivalent to $\frac{(x+2)(x-1)}{2x+1}$?

a) $\frac{x(x+2)(x-1)}{x(2x+1)}$

b) $\frac{(x+2)(x-1)(x+1)}{2x+1(x+1)}$

c) $\frac{(x+2)(x-1)}{2x+3}$

Example 2: Simplify the expression $\frac{(2m-3)(m+3)}{(m+3)(5m-1)}$.

The expression in Example 2 was already in factored form and could be easily simplified. Sometimes, expressions have to be factored firstly and then simplified.

Example 3: Simplify the expression $\frac{x^2+x-2}{3x^2+7x+2}$ by factoring it firstly.

Section 7.9 Adding Rational ExpressionsLooking Back 7.9

A rational expression is a fraction and to add or subtract fractions you must have a common denominator.

$$\begin{array}{ccc} \frac{1}{2} + \frac{3}{7} & \text{and} & \frac{x}{2} + \frac{y}{7} \\ \frac{7}{14} + \frac{6}{14} & \text{and} & \frac{7x}{14} + \frac{2y}{14} \\ \frac{13}{14} & & \frac{7x+2y}{14} \end{array}$$

Looking Ahead 7.9

Example 1: Add the rational expressions:

$$\frac{1}{x} + \frac{3+x}{x+1}$$

Though $x^2 + x$ factors to $x(x + 1)$, it does not cancel with anything in the numerator and can be left as is.

Example 2: Add the rational expression:

$$\frac{2x}{x-2} + \frac{1}{x+5}$$

Example 3: Add the rational expressions:

$$\frac{3-x}{(x+3)(x-3)} + \frac{2x}{x-3}$$

Example 4: Add the rational expressions:

$$\frac{2}{x+1} + \frac{x-x^2}{x^2-1}$$

Section 7.10 Subtracting Rational ExpressionsLooking Back 7.10

Subtracting rational expressions is similar to adding rational expressions. You still have to find a common denominator before subtracting the numerator. The difference is that when you subtract you are taking the opposite of the expression after the equal sign so all the signs must change to their opposite.

Looking Ahead 7.10

Example 1: Subtract the rational expressions: (Firstly, find the Lowest Common Denominator.)

$$\frac{x}{3x} - \frac{6}{x^3}$$

Example 2: Subtract the rational expressions:

$$\frac{x}{4x} - \frac{4}{x^2}$$

Example 3: Subtract the rational expressions:

$$\frac{5}{x+1} - \frac{2}{x-5}$$

Example 4: Subtract the rational expressions:

$$\frac{4+x}{x(x-2)} - \frac{x+3}{5x}$$

Section 7.11 Multiplying Rational ExpressionsLooking Back 7.11

Multiplying rational expressions is the same as multiplying fractions. Since $x \cdot y$ can be written xy and $x^2(y^3)$ can be written x^2y^3 , simply multiply the numerator and multiply the denominator. No common denominator is needed. However, both the numerator and denominator must be written in factored form to determine if anything divides out before multiplying.

Looking Ahead 7.11

Example 1: Simplify the rational expression:

$$\frac{x^2 + 4x + 3}{x^2 - x - 2}$$

Example 2: Multiply the rational expression:

$$\frac{x^2 + x - 12}{(x - 5)^2} \cdot \frac{x - 5}{x - 3}$$

Example 3: Multiply the rational expression:

$$\frac{(4x - 1)(x + 5)}{4(x^2 - 3)} \cdot \frac{10(x^2 - 3)}{(9x - 1)(x + 5)}$$

Section 7.12 Dividing Rational ExpressionsLooking Back 7.12

Dividing rational expressions is much like dividing fractions. To do so, you must first multiply by the reciprocal of the fraction following the division sign. Factor each numerator and each denominator and simplify as much as possible. Lastly, multiply the numerators and multiply the denominators of the rational expressions.

Example 1: Divide the rational expressions:

$$\frac{x^2 - 6x}{(x + 6)^5} \div \frac{x^2 - 1}{x^2 + 7x + 6}$$

Looking Ahead 7.12

If a rational expression has a fraction in the numerator and the denominator, then it is a complex fraction. To simplify a complex fraction, write the numerator and denominator as a single fraction, each with its own common denominator. Next, write the numerator and denominator as the division of two fractions. Lastly, follow the steps used in Example 1.

Example 2: Divide the complex fraction:

$$\frac{\frac{x + 3}{5}}{2 - \frac{2}{x}}$$

Example 3: Divide the complex fraction when $x \neq 0$ and $x \neq -3$:

$$\frac{\frac{3}{x+3}}{\frac{3}{x} + \frac{1}{x+3}}$$

There is another method that can be used to divide rational expressions. You can find the same Least Common Denominator of both the numerator and denominator and multiply the numerator and denominator by the LCD to clear the denominator of each. Then you should get one fraction that can be simplified.

Let us try Example 3 again using this method.

Example 4: Divide the rational expression when the LCD is $x(x+3)$:

$$\frac{\frac{3}{x+3}}{\frac{3}{x} + \frac{1}{x+3}}$$

As you can see, both methods give you the same answer, so going forward, use the method that works best for you!

Section 7.13 Solving Rational EquationsLooking Back 7.13

Rational equations are two rational expressions that are equivalent. One way to solve a rational equation is by finding the Least Common Denominator and cancelling it with the numerator on both sides of the equation. Since a rational expression is a fraction or ratio, a rational equation is two equivalent ratios, which is a proportion. Therefore, proportional thinking may be used to solve rational equations as well.

Looking Ahead 7.13

Example 1: Solve the rational equation:

$$\frac{x-3}{3} = \frac{2}{x-2}$$

Example 2: Solve the rational equation:

$$\frac{3x}{3-x} = \frac{x}{x-3}$$

Rational thinking is the art of arriving at true belief. When we think rationally, we can understand God and His universe more fully. Rational equations and rational thinking can be used to help us analyze and solve real world problems.

Example 3: If you have 100 mL of a solution that is 20% pure acid and 80% pure water, how many milliliters of pure acid do you need to add to make the solution 50% acid?

Rational equations can be used to solve mixture problems. They can also be used to solve amortization problems. These problems involve repaying a loan over time where payments are made in installments rather than at one time. Interest is added to the payments. The term is the time over which the loan is repaid. The initial amount of the loan is called the principal. The formula for calculating the payment amount per period for a loan is shown below:

$$A = P \frac{r(1+r)^n}{(1+r)^n - 1}$$

This is derived through an understanding of geometric series which you have studied previously and will study again. For now, understand the following terms:

A = amount per period

P = initial principal (loan amount)

r = interest rate per period

n = total number of payment periods

Example 4: What would the monthly payment be on a 5-year, \$10,000.00 car loan with a nominal 6.5% annual interest rate?

In the Practice Problems section, you will need to use all of the methods you learned for solving quadratic equations to solve the rational equations.