Algebra 2 Module 4 Quadratic Equations

Section 4.1 The Parts of the Pendulum

Looking Back 4.1

We are going to begin and end this module with projects. The first project will help answer the essential question, “Is Edgar Allan Poe a mathematician?” and the driving question, “Does the prisoner in *The Pit and the Pendulum* really have time to escape?”

Let me summarize the story while we watch this short excerpt highlighting the dilemma of the prisoner. In the story, the prisoner escapes the pendulum just after his skin is scathed. The question is, given the information Edgar Allan Poe provides in the story, is it really possible the prisoner could escape at all?

In order to answer these questions, we will need to understand the parts of the pendulum and which parts effect the period of the pendulum.

Let’s look at a power point demonstration to analyze the parts of the pendulum.

The total mechanical energy in a pendulum, assuming there is no friction, is a combination of kinetic energy and potential energy. The potential energy of an object is the stored energy of its position. It is dependent on the mass (m) of an object and the height (h) of the object. Gravity is constant at 9.8 m/sec^2 or 32 ft/sec^2. The equation for gravitational potential energy is $PE = m \cdot g \cdot h$.

When does the pendulum have maximum potential energy?

The answer is just before it is dropped and then again when it stops at the opposite side of release before returning to the start position.

The kinetic energy of an object is due to mass (m) and speed (v) of the object. The equation for kinetic energy is $KE = \frac{1}{2} \cdot m \cdot v^2$.

When does the pendulum have maximum kinetic energy?

The answer is at the lowest point. At the bottom of the swing the pendulum is at its greatest speed. At this point the pendulum has zero potential energy with respect to its position.

There is a constant exchange of energy between these two forces as the pendulum swings back and forth. One swing out and back is called the period of the pendulum.

Looking Ahead 4.1

Gravity is a force acting on the pendulum. There is also a tension force and function, however, we will not include this in our experiment. We are going to investigate the three variables that affect the period of the pendulum. You will build a simple pendulum using a pencil, string, and washers. What factors do you think could possibly affect the period of the pendulum?

- The weight of the bob (mass/washers)
- The length of the string (chord)
- The angle of amplitude (drop from the pencil)

These three factors will be investigated one at a time. If you are testing the weight of the bob, keep the length of the string and the angle of the drop the same for one washer, two washers, and three washers.

The directions for the experiment will be found in the practice problems for this section. We will discuss the results tomorrow. Your other assignment is to read *The Pit and the Pendulum* and extract the important mathematical information that you think might help you figure out whether the prisoner can indeed escape. There are also audio instructions online that you can listen to if that is easier for you.

We will watch a brief video of students using a pendulum to perform the experiment. You will build a pendulum using a pencil, string and washers. You can use a protractor to measure the angle or simply cut right triangle out of cardboard and use that for the angle of amplitude.

In the 1600's there was no way to accurately record time. Galileo and his friends did experiments with the pendulum. They recorded the number of periods of the pendulum for one full rotation of the earth. They did this by observing a star at a fixed point on a cathedral to start the count and then completed it when the star again reached the same location 24 hours later.

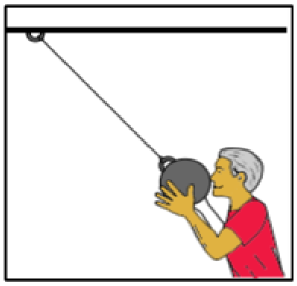
Galileo had ideas that conflicted with the known thoughts at the time. Though his ideas were correct, he was placed on house arrest by the church. Galileo thought that the way the pendulum makes all its vibrations, small or large, in equal time was such a marvelous property that allowed for the accurate recording of time. Galileo knew that the principles found in God's universe would aid in the telling of time. He continued his studies even under house arrest until he died.

Section 4.2 The Period of the Pendulum

Looking Back 4.2

Now that you have completed the experiments and tested the weight of the bob, the length of the string, and the amplitude (angle) of the drop, you should realize that the time for the period of the pendulum changes when the length of the string changes.

There are multiple ways to make a homemade pendulum. One way is by hanging a bocce ball from the ceiling. You can drop it from your nose and after one swing out and back it returns to your nose (it should not hit your nose). If you time it and have someone taller or shorter than you repeat the drop, the time will be about the same. The chord length and bob weight stay the same. It is the height and amplitude that change. The amplitude does not affect the period of the pendulum. However, if you change the chord length and release it from your nose again, you will see the time for the period of the pendulum is different.



Looking Ahead 4.2

Now that you know the string length affects the period of the pendulum you will change the length of the string. You will use just one washer and keep the angle of the drop the same for each trial. Then time the period.

The practice problems section has the instructions for the period of the pendulum and a table to record your data.

Next, you will graph the results to find the relationship between the period and length of the string. Once you determine the function that models this relationship you will understand the function for the period of the pendulum and its inverse as well.

Moreover, the important mathematical information from *The Pit and the Pendulum* given by the prisoner is laid out as follows:

(If you have not read the story please do now. Look online for a free audio version.)

- There are 10-12 swings left (until it reaches his neck)
- The chord length is about 30-40 feet
- The rats will chew through the ropes in about 1 minute

Section 4.3 The Pit and the Pendulum

Looking Back 4.3

Now we have the formulas for the period of the pendulum. The prisoner tells us there is about 30-40 feet of chord. Since the length is given, the formula we will use to determine the period is given below:

$$p = 2\pi\sqrt{\frac{L}{g}}$$

Since the length is given in feet, $32.2\ ft/s^2$ will be used for gravity (g). The constant is 2π or $(2)(3.14\ldots) \approx 6.28$.

Edgar Allan Poe says the prisoner thinks the length is 30-40 feet. You will find the period for 5-40 feet. This will help determine if it only works for the 30-40 feet mentioned in the story or if it is possible for the prisoner to escape other lengths as well, namely 5-25 feet.

The story also says the rats can chew through the ropes in about 1 minute. You will test 1.5 minutes as well to see if any other times may work. If it works at 1 minute only, then Edgar Allan Poe may really know his math. However, if the prisoner can escape after 1.5 minutes then maybe he was just making a close guess.

Test all the above information for 10-12 swings (the numbers mentioned in the story). If it works between those and the prisoner can escape, then you can decide if you think the author knew his Math as well as his English!

Do the calculations in the practice problems section and then answer the questions and explain your reasoning. Student samples have been included as well. You will decide if you agree with their answers or not and explain why you do or do not.

Section 4.4 Factoring Quadratic Equations Without a Linear Term

Looking Back 4.4

In the previous section the square root function modeled the relationship between the length of the chord of a pendulum and its period. The quadratic function modeled the relationship between the period of the pendulum and the length of the chord. The graphs are reflections of each other over the line of reflection, $y = x$. The two functions are inverses of one another. You investigated this geometrically and algebraically in Algebra 1. The inverse of a quadratic function is a square root function. This principle will be used to solve quadratic equations that are missing the linear term.

Quadratic equations will be reviewed for the remainder of this module. The parent function of a quadratic equation is $y = x^2$.

The parent function of a quadratic equation is centered about the origin, so it is an even function. It is symmetric over the axis of symmetry, $x = 0$. The output falls in Quadrants I and II.

The standard form, or general form, for a quadratic equation is given below:

$$ax^2 + bx + c = y$$

In this equation, x and y are variables and a, b, and c are parameters.

The vertex form, or graphing form, for a quadratic equation is given below:

$$y = a(x - h)^2 + k$$

In this equation, x and y are variables and a, h, and k are parameters.

The equations above were investigated in Algebra 1, and it was found that a represents the stretch or shrink, h represents the horizontal shift, and k represents the vertical shift. In the transformation, (h, k) becomes the vertex.

In the standard form, ax^2 is the quadratic term, bx is the linear term, and c is the constant term. The quadratic equations being investigated in this section have no linear term, and therefore, are the easiest to solve. The solutions for x in these equations represent the x-intercepts. These help us graph the quadratic equations.

Looking Ahead 4.4

Example 1:	Solve for x in the equation $x^2 = 49$.

Example 2: Solve for x in the equation $3x^2 = 12$.

Example 3: Solve for x in the equation $x^2 = 5$.

Example 4: Solve for x in the equation $x^2 + 5 = 41$.

Example 5: Solve for x in the equation $(x - 1)^2 = 16$.

Example 6: Solve for x in the equation $x^2 = -1$.

Example 7: The Leaning Tower of Pisa has a height of 186 feet. If a bird drops a seed from its nest on top of the tower, how long will it take before it hits the ground? The falling object model is $-16t^2 + 186 = h(t)$.

Let h (height) be 0 when the seed hits the ground. Let t be the time in seconds.

Section 4.5 Factoring and the Zero-Product Property

Looking Back 4.5

Quadratic equations that have the linear term often require factoring in order to use the zero-product property to solve for x. The reason we factor equations and solve for x is many times so we can graph them. When we find the critical points, such as x and y intercepts or a maximum or minimum point we can analyze the equations.

There are many ways to factor equations which we will review here. We will also review using the zero-product property.

The first thing to do when factoring is to see if there is a greatest common monomial factor. After that you can use the zero-product property to solve for x and to the find the x-intercept(s).

The zero-product property states that "if $a \cdot b = 0$, then either $a = 0$ or $b = 0$."

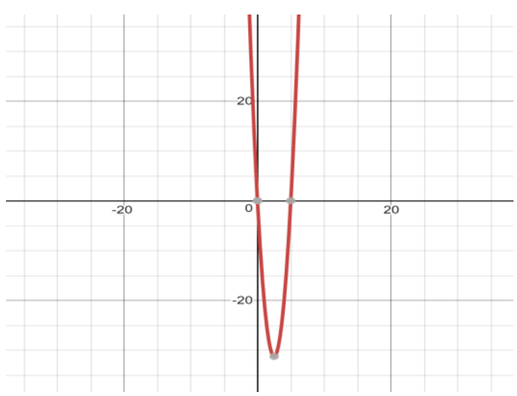
"If $a = 0$, then $0 \cdot b = 0$."

"If $b = 0$, then $a \cdot 0 = 0$."

When factoring, if $(x - a)(x - b) = 0$ then either $x - a = 0$ and $x = a$, or $x - b = 0$ and $x = b$.

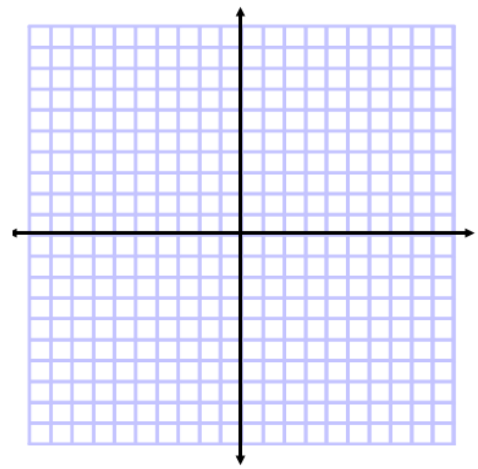
Example 1: Factor $y = 5x^2 - 25x$ and use the zero-product property to solve for x.

The x-intercepts are also called the roots of the equation, zeroes of the function, and solutions of the function.



The difference of squares is a special factoring pattern. It works if there is no linear term and the first and last terms are both perfect squares. A generic rectangle can be used to check the solution.

Example 2:	Factor $x^2 - 9 = y$ and use the zero-product property to solve for x. Then graph it.



Example 3:	Factor $4x^2 - 16 = y$ and solve for x using the zero-product property.

There are several ways to solve this quadratic equation from what we have previously learned. Difference of squares are represented by $a^2 - b^2$ and factor to $(a - b)(a + b)$.

Another way to factor, if the quadratic equation is a perfect trinomial square, is by finding the binomial square. A perfect trinomial square means there are three terms where the first and last terms are perfect squares and the middle term is double the product of the first and last term.

$$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$$

If both signs are negative or the first is positive and the second is negative, the binomial cannot be factored into a binomial square, even if the other conditions are met.

Example 4:	Factor $y = x^2 + 4x + 4$ and solve for x using the zero-product property.

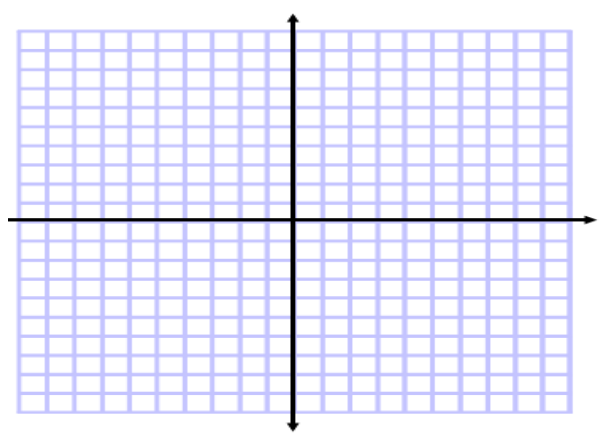
There is only one x-intercept for a binomial square. The parabola touches or bounces off the x-axis at that point. This means that $x = -2$ is also the axis of symmetry.

Many trinomials do not have special patterns for factoring but are still factorable. A guess and check method can be used for these.

Example 5:	Factor $x^2 + x - 12 = y$.

Example 6:	Factor $5x^2 + 22x - 15 = y$ and use the zero-product property to solve.

Example 7:	Factor the equation $x^2 - x - 6 = y$. Find the x-intercepts, the axis of symmetry and the vertex.



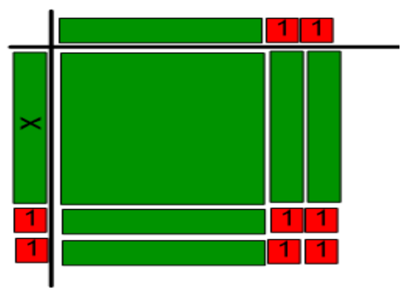
Section 4.6 Completing the Square

Looking Back 4.6

Some quadratic equations cannot be factored using any of the previously learned methods. However, there is a method that always works called "Completing the Square."

Completing the Square is also used to find the vertex of a quadratic equation. It is a reliable method and it is sometimes easier than any other method of factoring. Competing the Square gives the factored form which is the graphing form of an equation because it is of the form $a(x-h)^2+k=y$. It is also called the vertex form because (h,k) is the vertex of this form and easy to identify and graph.

A perfect square trinomial forms a square geometrically.



Sometimes the trinomial is not a perfect square trinomial so something must be added or subtracted in order to complete the square.

Example 1: Factor $y=x^2+4x+3$.

To factor the equation, use the following steps:

1. Take half of the coefficient of the middle term. It is 2. The binomial square is $(x+2)^2$.

2. Square half of the middle term. It is 4. That is one more than the last term (3) so 1 must be subtracted from 4 to get 3.

$$y=(x+2)^2-1$$

Let us check the work using the algebra tiles.

Example 2:	To complete the square, follow the steps below.

1. Move the constant term of the standard form equation to the other side.

$$x^2 + 16x + 36 = 0$$

$$x^2 + 16x = -36$$

2. If the leading coefficient is 1, take half the coefficient of x (the linear term), square it and add it to both sides of the equation.

$$b = 16 \text{ and } \frac{1}{2}(16) = 8 \text{ and } 8^2 = 64$$

$$x^2 + 16x + 64 = -36 + 64$$

3. Rewrite the perfect square trinomial as a binomial square.

$$(x + 8)^2 = 28$$

4. Move the constant term back to the other side of the equation.

$$(x + 8)^2 - 28 = 0$$

The vertex of the parabola is (h, k) and $x = h$ is the axis of symmetry. The h represents how far left or right the graph has shifted from $x = 0$ (horizontal shift). The k represents how far up or down the graph has shifted from $y = 0$ (vertical shift). In the vertex form, the h value is subtracted and the k value is added, as shown below.

$$a(x - h)^2 + k = y$$

The vertex form of the equation $(x + 8)^2 - 28 = y$ is $(x - (-8))^2 - 28 = y$. In that equation, $(h, k) = (-8, -28)$.

In the equation $y = 2(x - 1)^2 + 6$, the value of h is 1 and the value of k is 6. The vertex is $(1, 6)$.

In the equation $y = 3(x + 4)^2 - 5$, the value of h is -4 and the value of k is -5. The vertex is $(-4, -5)$.

Looking Ahead 4.6

When the lead coefficient in a quadratic equation is not 1, it is a little more difficult to complete the square. The value of "a" is factored out on both sides of the equation before anything else to give the quadratic term a coefficient of 1.

Example 3: Complete the square for the quadratic equation $2x^2 - 4x + 5 = y$. Let $2x^2 - 4x + 5 = 0$.

Example 4: Complete the square to find the x-intercepts of $x^2 - 12x + 27 = y$.

Example 5: Complete the square to find the vertex of $2x^2 - 16x + 24 = 0$. Find the square root of both sides of the equation to solve for x. Sketch a graph of the quadratic equation using the vertex and x-intercepts.

Section 4.7 The Vertex Form

Looking Back 4.7

There is a formula to find the vertex form, which can be derived from the graphing form. Let us use a general standard form equation and convert it to vertex form so we can see the formula in terms of a, b, and c.

Given a standard form quadratic equation…

$$y = ax^2 + bx + c$$

1. Move the constant term to the other side.

$$y - c = ax^2 + bx$$

2. Factor out the value of "a."

$$y - c = a(x^2 + \frac{b}{a}x)$$

3. Find $\frac{1}{2}$ the coefficient of x and square it and then add it to the right and multiply it by a on the left.

$$y - c + a(\qquad) = a(x^2 + \frac{b}{a}x \qquad)$$

4. Find the missing term.

$$(\frac{1}{2}\left(\frac{b}{a}\right))^2 = (\frac{b}{2a})^2 = \frac{b^2}{4a^2}$$

5. Substitute this missing term in both sides to complete the square.

$$y - c + a\left(\frac{b^2}{4a^2}\right) = a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2})$$

6. Simplify the left side.

$$y - c + \frac{b^2}{4a} = a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2})$$

7. Find common denominators on the left side and write the right side as a binomial square.

$$y - \frac{4ac}{4a} + \frac{b^2}{4a} = a(x + \frac{b}{2a})^2$$

8. Isolate the y on the left side.

$$y = a(x + \left(\frac{b}{2a}\right))^2 + \frac{4ac}{4a} - \frac{b^2}{4a}$$

9. Combine common denominators on the right side.

$$y = a(x + \left(\frac{b}{2a}\right))^2 + (\frac{4ac - b^2}{4a})$$

10. Convert to vertex form.

$$y = a(x - \left(-\frac{b}{2a}\right))^2 + (\frac{4ac - b^2}{4a})$$

$$y = a(x - h)^2 + k$$

Now you can see that $h = -\frac{b}{2a}$ and $k = \frac{4ac-b^2}{4a}$, and also $k = \frac{4ac}{4a} - \frac{b^2}{4a}$ or $k = c - \frac{b^2}{4a}$. The vertex is $(-\frac{b}{2a}, c - \frac{b^2}{4a})$. You can memorize this or just memorize $h = x_v$, which is $h = -\frac{b}{2a}$ and substitute that value in the quadratic equation to find y_v.

Looking Ahead 4.7

Example 1: Use $(h, k) = (-\frac{b}{2a}, c - \frac{b^2}{4a})$ to find the vertex of the quadratic equation $y = 2x^2 - 4x + 5$

Example 2: Find the vertex of the quadratic equation $y = 4x^2 - 8x - 3$. Use $h = \frac{-b}{2a}$ to find x_v. Substitute that value in the original equation to find y_v.

Example 3: Name the vertex in the quadratic equation $y = (x - 3)^2 + 2$ and convert it to standard form.

Example 4: Name the vertex in the quadratic equation $-3(x+4)^2 - 8 = y$ and convert it to standard form.

Example 5: Given the vertex $(-6, -8)$ and the y-intercept $(0, 64)$, find "a," the graphing form, and then convert it to standard form for the quadratic equation.

Example 6: Given the vertex $(1, -4)$ and a point $(3, 8)$ on the parabola, find the value of "a" and then convert the vertex form of the quadratic equation to standard form.

Section 4.8 Imaginary Numbers

Looking Back 4.8

In previous mathematic courses you have learned quite a bit about the real number system. There is another number system that includes non-real numbers. These numbers are called imaginary numbers. An imaginary number is a real number multiplied by the imaginary unit i. The imaginary unit i is defined by the property $i^2 = -1$. Complex numbers have a real number part and an imaginary part. Therefore, imaginary numbers are a subset of complex numbers.

If $i^2 = -1$, then $\sqrt{i^2} = \sqrt{-1}$ and $i = \sqrt{-1}$.

Therefore, the square root of a negative number has a real part, which is numerical, and an imaginary part, which is i.

Example 1: Simplify $\sqrt{-49}$.

Example 2: Simplify $\sqrt{-32}$.

There are properties of imaginary numbers just as there are properties of real numbers. Any number to the zero power is 1, even an imaginary number.

$$i^0 = 1$$

$$i^1 = i$$

$$i^2 = -1$$

$$i^3 = i^2 \cdot i^1 = -1 \cdot i = -i$$

$$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$$

$$i^5 = i^3 \cdot i^2 = -i \cdot -1 = i$$

$$i^6 = i^3 \cdot i^3 = -i \cdot -i = i^2 = -1$$

$$i^7 = i^4 \cdot i^3 = 1 \cdot -i = -i$$

$$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$$

Let us see if there is a pattern from i^0 to i^7.

i^0	i^1	i^2	i^3	i^4	i^5	i^6	i^7
1	i	-1	$-i$	1	i	-1	$-i$

The pattern is and continues as…

1, i, -1, and $-i$

Therefore, for i^8 , i^9, i^{10}, and i^{11}, it is $i^8 = 1$, $i^9 = i$, $i^{10} = -1$, and $i^{11} = -i$.

Hopefully you can see that even numbered powers of i will be -1 or 1 and odd numbered powers of i will be $-i$ or i. The cycle repeats every four numbers. Let n be the exponent of i.

$$\text{If } \frac{n}{4} \text{ has a remainder of } \begin{cases} 0,\ then\ i^n = 1 \\ 1,\ then\ i^n = i \\ 2,\ then\ i^n = -1 \\ 3,\ then\ i^n = -i \end{cases}$$

Example 3:	Simplify the imaginary numbers.

a) i^{53}

b) i^{20}

c) i^{14}

d) i^{40}

Looking Ahead 4.8

Imaginary numbers, like real numbers, can be added, subtracted, multiplied and divided.

Example 4:	Add: $i + 4i$.

Example 5:	Subtract: $i - 4i$.

Example 6:	Multiply: $i \cdot 4i$.

Example 7:	Divide: $\frac{4i}{i}$.

Section 4.9 Complex Numbers

Looking Back 4.9

Complex numbers have already been defined as real numbers and imaginary numbers. They are of the form $a + bi$, where a is the real part and bi is the imaginary part. They follow the same rules as real numbers. To add and subtract, the real parts are added or subtracted, and the imaginary parts are added or subtracted as well.

Example 1: Simplify the complex number $2 + 3i + 4i - 8$.

Example 2: Simplify the complex number $(4 + 3i) + (8 - 2i)$.

Example 3: Simplify the complex number $(9 + i) - (22 - 5i)$.

Example 4: Multiply the complex number $(2 + 4i)(2 - 4i)$. This $(a + bi)(a - bi)$ form is the same as the form of the difference of two squares. Use the distributive property to solve.

Example 5: Multiply the complex number $(3 - i\sqrt{2})(3 + i\sqrt{2})$.

The standard form of a complex number is $a + bi$, where a is called the real component and bi is called the imaginary component. The complex conjugate of $a + bi$ is $a - bi$. When a complex conjugate pair is multiplied the product is a real number. Example 4 and 5 are examples of complex conjugates.

Example 6: Given the complex number $2 + 3i$ complete a) – d) below.

a) Name the real component.

b) Name the imaginary component.

c) Name the complex conjugate.

d) Multiply the conjugate pair.

Example 7: Divide the complex number $\frac{3-2i}{4+3i}$.

To divide you must first multiply by the conjugate of the denominator (simply change the sign). This results in a real number in the denominator. Simplify all powers of i and combine like terms in both the numerator and denominator. Write the answer in the form $a + bi$.

Section 4.10 The Quadratic Formula and the Discriminant

Looking Back 4.10

You have previously learned the quadratic formula in Algebra 1. It is shown below.

You began with the standard form of a quadratic equation, $y = ax^2 + bx + c$ and used completing the square to solve for x.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Let's review how to derive a generalized formula to determine the x-intercepts of a quadratic equation.

Standard Form: $y = ax^2 + bx + c$

Let $y = 0$ and solve for x to get the x-intercepts.

Subtract c on both sides:

Divide by a:

Complete the square:

Find common denominators:

Rewrite as a binomial square:

Take the square root both sides:

Isolate the x to solve for x:

Quadratic Formula:

The quadratic formula always works to find the x-values of any quadratic equation. In Algebra1 you learned there are three possible solutions, based on the discriminant. If $b^2 - 4ac$ is 0 there is one solution.

Example 1:	Solve for x in the equation $2x^2 + 4x + 2 = y$ using the quadratic formula.

If the discriminant, $b^2 - 4ac$ is a positive real number, then there are two solutions.

Example 2:	Solve for x in the equation $2x^2 - x - 3 = y$ using the quadratic formula.

Factoring gives the same result. However, some quadratic equations are not able to be factored. The quadratic formula always works when quadratic equations cannot be factored using other methods.

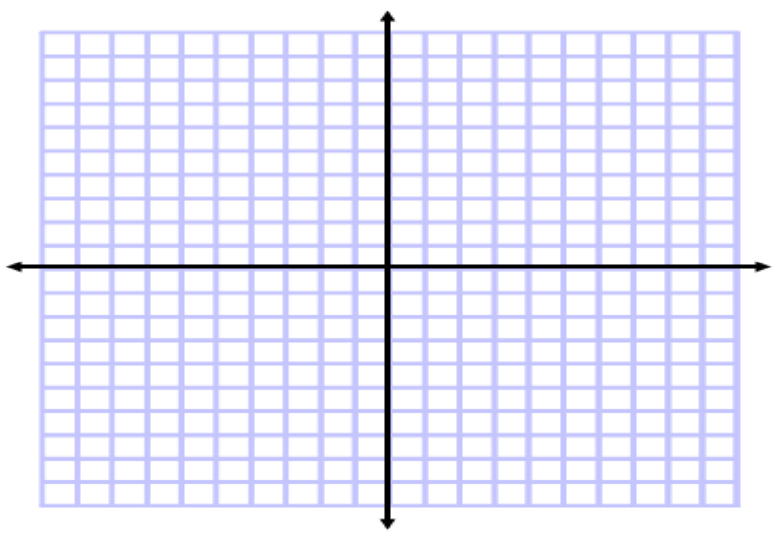
Sometimes the discriminant is a negative number. If there is a negative number under the radical sign, then there is a non-real solution and now you know the non-real solution is a complex number.

Therefore, if the discriminant $(b^2 - 4ac)$ is a negative number, the solution is a complex number.

Example 3:	Solve for x in the equation $2x^2 - x + 3 = y$ using the quadratic formula.

The two solutions are $\frac{1}{4} + \frac{\sqrt{23}}{4}i$ and $\frac{1}{4} - \frac{\sqrt{23}}{4}i$. There is no radical in the denominator. This parabola does not cross the x-axis at all.

The complex plane is an infinite 2-D plane that represents the complex numbers. The real component is on an x-axis and the imaginary component is on a y-axis. It is also called the z-plane.



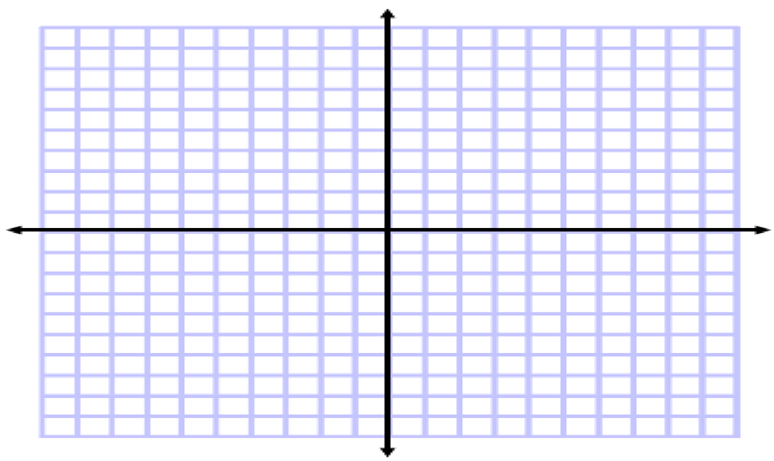
Example 4: Plot the given points on the complex plane.

a) $4 + 3i$

b) $-3 - 2i$

c) $-5 + i$

d) $-1 - 6i$



Completing the square can also be used to find complex numbers for x. Like the quadratic formula, it works when the factoring methods fail to work.

Example 5: Complete the square to solve for x in the equation $x^2 + 6x + 10 = 0$.

Looking Ahead 4.10

Let us further investigate the quadratic formula. We know the x-intercepts are $(\frac{-b-\sqrt{b^2-4ac}}{2a}, 0)$ and $(\frac{-b+\sqrt{b^2-4ac}}{2a}, 0)$. These x-values can be written as $\frac{-b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}$ and $\frac{-b}{2a} + \frac{\sqrt{b^2-4ac}}{2a}$. We know that $\frac{-b}{2a}$ is the x-value of the vertex and $x = \frac{-b}{2a}$ is the axis of symmetry.

Let us look at Example 2 again. The quadratic equation is $2x^2 - x - 3 = y$ and the x-intercepts are $x = \frac{3}{2}$ and $x = -1$. The vertex is $x_v = \frac{-(-1)}{2(2)}$ or $x_v = \frac{1}{4}$.

Example 6: Using the equation $2x^2 - x - 3 = 0$, find the values of $-\frac{\sqrt{b^2-4ac}}{2a}$ and $\frac{\sqrt{b^2-4ac}}{2a}$ and tell what they mean in terms of the graph of the equation.

We know that x_v is $\frac{1}{4}$. If we subtract $\frac{5}{4}$ from the vertex we get $x_v - \frac{5}{4}$, or $\frac{1}{4} - \frac{5}{4} = \frac{-4}{4} = -1$, which is the x-intercept to the left of the axis of symmetry.

If we add $\frac{5}{4}$ to the vertex, we get $x_v + \frac{5}{4}$, or $\frac{1}{4} + \frac{5}{4} = \frac{6}{4} = \frac{3}{2}$, which is the x-intercept to the right of the axis of symmetry.

Therefore, the first half of the quadratic formula is the axis of symmetry and the last half of the formula is the distance between the axis of symmetry and the x-intercepts. So, when the discriminant is zero the distance is zero and the vertex is the x-intercept. When there is a non-real solution these distances lie on the complex plane.

Now we know where the quadratic equation comes from and also what it represents.

Section 4.11 Projectile Motion

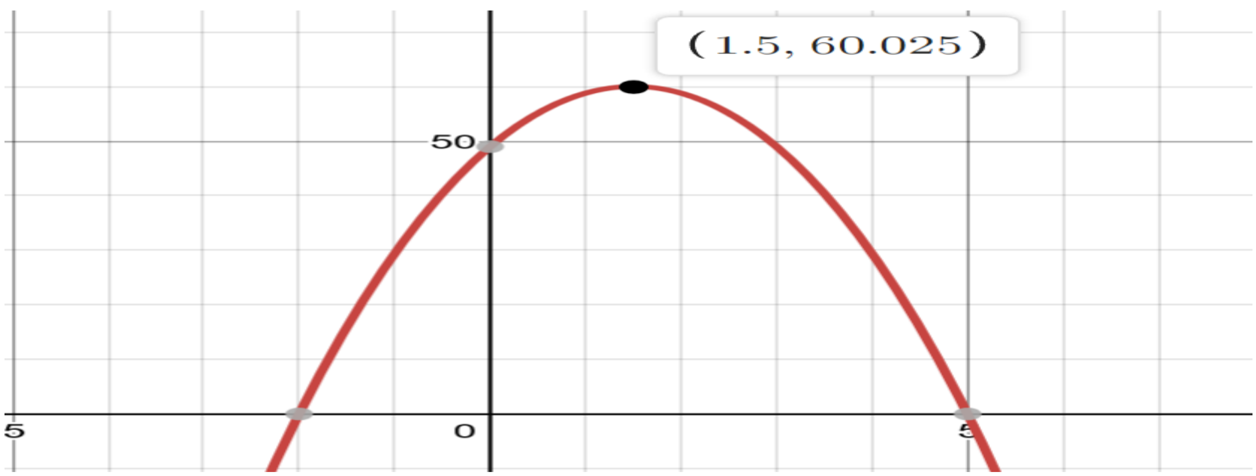
Looking Back 4.11

Projectile motion is the motion of an object that rises or falls under the influence of gravity. In a free fall, the main force acting on the object is the earth's downward pull of gravity. An object that is launched into the air, such as a shot-put or a baseball, is called a projectile, and the path it follows is called a trajectory.

In Physics, Pre-Calculus, and Calculus you will learn about the horizontal and vertical components of launch and use vectors to investigate these. You will also explore acceleration and velocity.

For now, we will analyze the different parts of the projectile motion equations and graphs. Projectile motion has the trajectory of an upside-down parabola. This is a quadratic equation where "a" is negative.

Example 1: The projectile motion formula for a rock that is thrown from a cliff is $r(t) = -4.9t^2 + 14.7t + 49$ where $r(t)$ is the height of the rock, t is the time and -4.9 is one-half of gravity due to acceleration in meters per second squared.



Let's discuss some critical points on the graph.

a) What does $(-2, 0)$ and $(5, 0)$ represent?

b) What does the point $(0, 49)$ represent?

c) What does $(1.5, 60.025)$ represent?

d) What does a, b, and c represent?

Looking Ahead 4.11

Given just the equation (and no graph) you can still find all the critical points from what you have learned about quadratics.

Example 2: Use factoring and the quadratic equation to find the x-intercepts given the equation $r(t) = -4.9t^2 + 14.7t + 49$.

Example 3: Use $-\frac{b}{2a}$ to find x_v and substitute it in the equation $r(t) = -4.9t^2 + 14.7t + 49$ to find y_v.

Example 4: Use completing the square to find the vertex of $r(t) = -4.9t^2 + 14.7t + 49$.

In 1961 NASA sent the robotic probes Pioneer, Voyager, and Galileo out into space and throughout the solar system. For the first time in history space was brought to earth. One of the men responsible in part for making space understandable to those on earth was Robert Jastrow.

Born in New York in 1927, Jastrow would go on to receive a degree in Physics from Columbia University. He joined the National Aeronautic Space Administration (NASA) in 1958 and was head of NASA's Goddard Institute for Space Studies for 20 years. Jastrow pioneered the field of robotics probes and lunar exploration and authored numerous books. In recognition of all his work, Robert Jastrow was awarded NASA's medal for scientific achievement.

Jastrow said, "Now we see how the astronomical evidence supports the Biblical view of the origin of the world. The details differ, but the essential elements in the astronomical and Biblical accounts of Genesis are the same: the chain of events heading to man commenced suddenly and sharply at a definite moment in time, in a flash of light and energy."

All this merging of mathematical and scientific studies leads us back to God, the creator of both.

Quote from *The Enchanted Loom: Mind in the Universe* by Robert Jastrow.

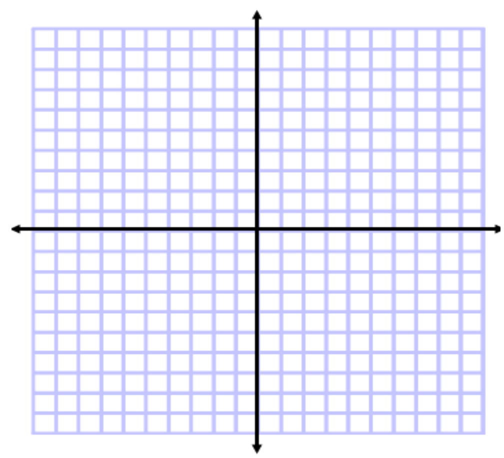
Section 4.12 Quadratic Inequalities

Looking Back 4.12

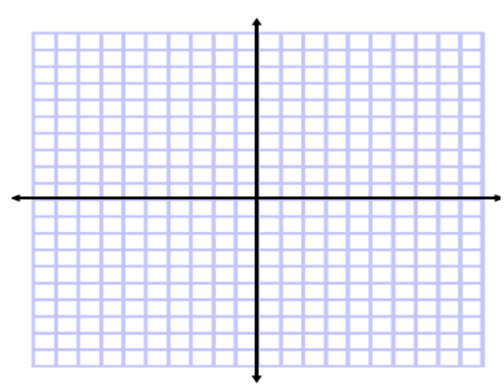
Graphing quadratic inequalities looks a bit different than graphing linear inequalities. With linear inequalities the infinite solutions lie above or below the boundary line and may or may not include the line. The boundary line for a quadratic equation is the parabola. The infinite solutions lie inside or outside the parabola and may or may not include the parabola.

Let us first solve the inequalities algebraically.

Example 1: Graph the quadratic equation $x^2 - 6x + 5 < y$ using any method. Use a dashed or solid line for the parabola. Pick a point inside or outside the parabola to test for validity. If it is valid, shade that portion of the inequality. If it is not valid, shade the other portion of the inequality.



Example 2: Graph the inequality $y \leq x^2 + 6x + 7$.



Section 4.13 Quadratic Math Hands

Looking Back 4.13

Now that you have completed the study of quadratic functions you will be able to derive the equation(s) that model(s) the parabola(s) in your hand(s).

There are some straight lines and some curved lines in your hands. There are some people who practice palm reading by looking at those lines. They make their profit from "reading" the lines of the hands to foretell a person's future. This craft had its origins in India many years ago. King Saul, the first king of Israel, attempted to know his future by visiting a medium. This displeased God (I Samuel 28). Only God knows our future and He tells us so in Jeremiah 29:11: "For I know the plans I have for you," declares the Lord, "Plans to prosper you and not to harm you, plans to give you a hope and a future." Not only does He know the number of lines in your hand, He knows the number of hairs on your head (Matthew 10:30).

To find the equations that model the lines of your hand, you will first need to take a picture of your hand face up and send it to your computer.

Open the application DESMOS.com, which is a free download on your computer. Press the red icon to start graphing. There is a graph on the right. You can use the "$+$" or "$-$" sign to change the scale or you can change it manually by pressing the wrench above the operation signs to set the parameters for the axis.

Press the "$+$" sign on the upper left and scroll down to "Image" to upload the picture of your hand. Click on the image until the blue border appears and move it so the left corner of the picture is on the origin.

Locate at least three points on the hand. The vertex is either at the top of a parabola opening downward, or at the bottom of a parabola opening upward. That is one point. Find two others where the x and y axes meet to avoid calculating with decimals or fractions. Try to locate the integer points if possible.

Click on the line under the hand information (height, width, center of hand). Put the vertex in parenthesis here and click "Enter" and it will appear on the graph. Check the label box if you want the numerical values of (x, y) to appear on the graph. Add another point below this on the left. Check the label box and click "Enter." Pick a point somewhere near the opposite end of the parabola.

Use the vertex to substitute for (h, k) in the equation $y = a(x - h)^2 + k$. Then use the other point to substitute for (x, y) and solve for a. Once you know the value of a, substitute it in for a in the formula along with the vertex. Then you will get the vertex form of the equation of your hand.

Type this equation in the next line on the left. Click on the keyboard icon on the bottom left of the screen to bring up the key. To get the square for the quadratic, click a^2. Once you type the equation a parabola will appear. You must put in the domain restrictions so the parabola does not appear outside the picture. At the end of the equation, click the "ABC" on the keyboard to get the braces symbol on the bottom. Use an inequality to input the domain. Click "123" at the bottom left to get the inequality symbols. Hopefully, you found a match!