Algebra 2 Module 3 Introduction to Functions

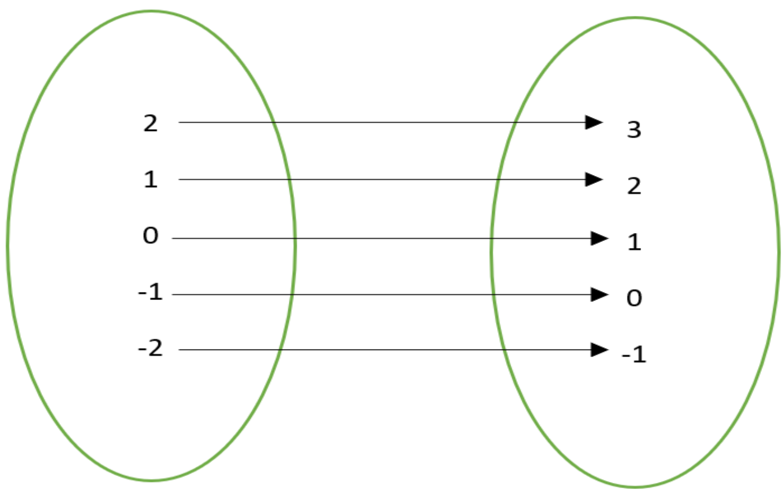
Section 3.1 Relations and Functions

Looking Back 3.1

A relation is a set of ordered pairs. The first coordinate is the input and the second coordinate is the output.

A function is a relation that has different values for the input, but the values for the output may remain the same. A function has each member of the domain mapped to exactly one member of the range. For each input, there is exactly one output.

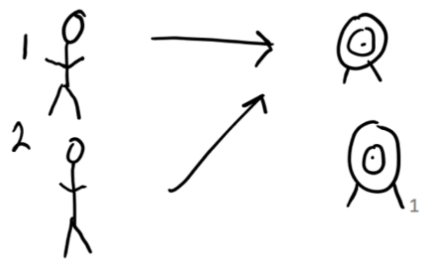
Example 1:	Is the mapping below a function or relation only?



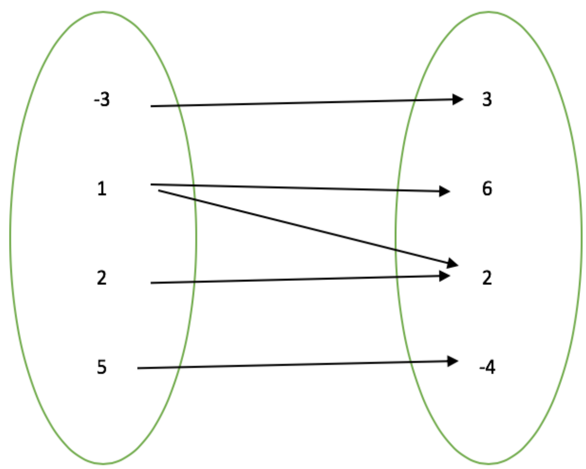
x	y
-2	-1
-1	0
0	1
1	2
2	3

This is called a one-to-one mapping. It is a function because no ____________________ values repeat.

An archer can only shoot at one target.

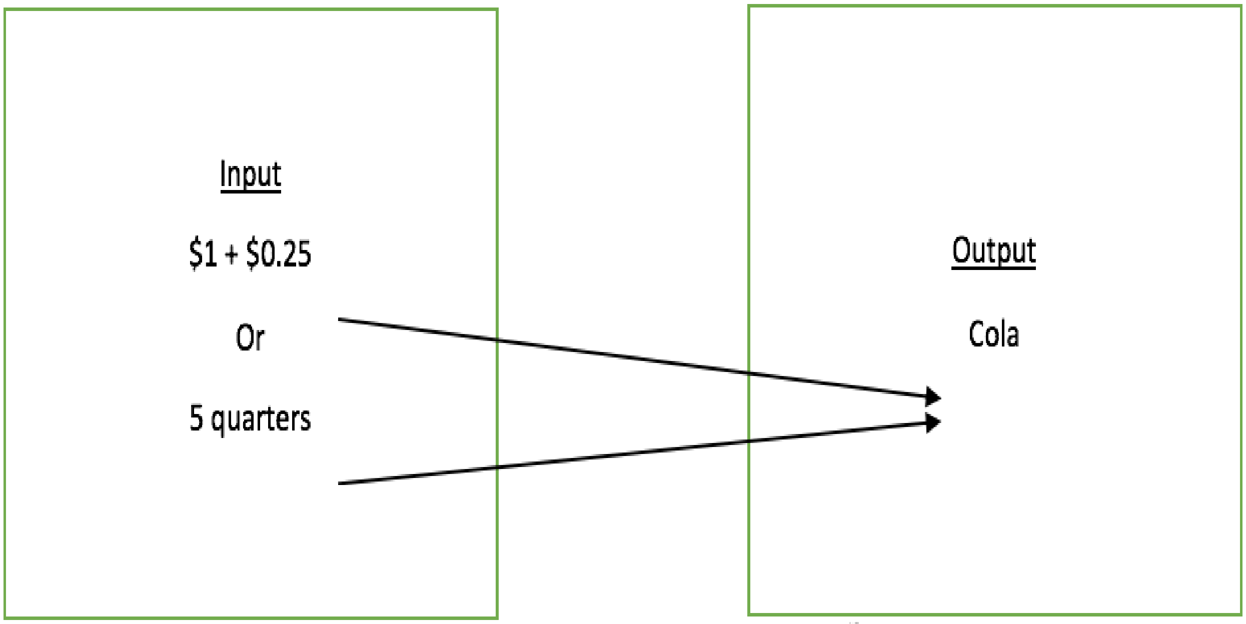


Example 2:	Is the mapping a function or relation only?

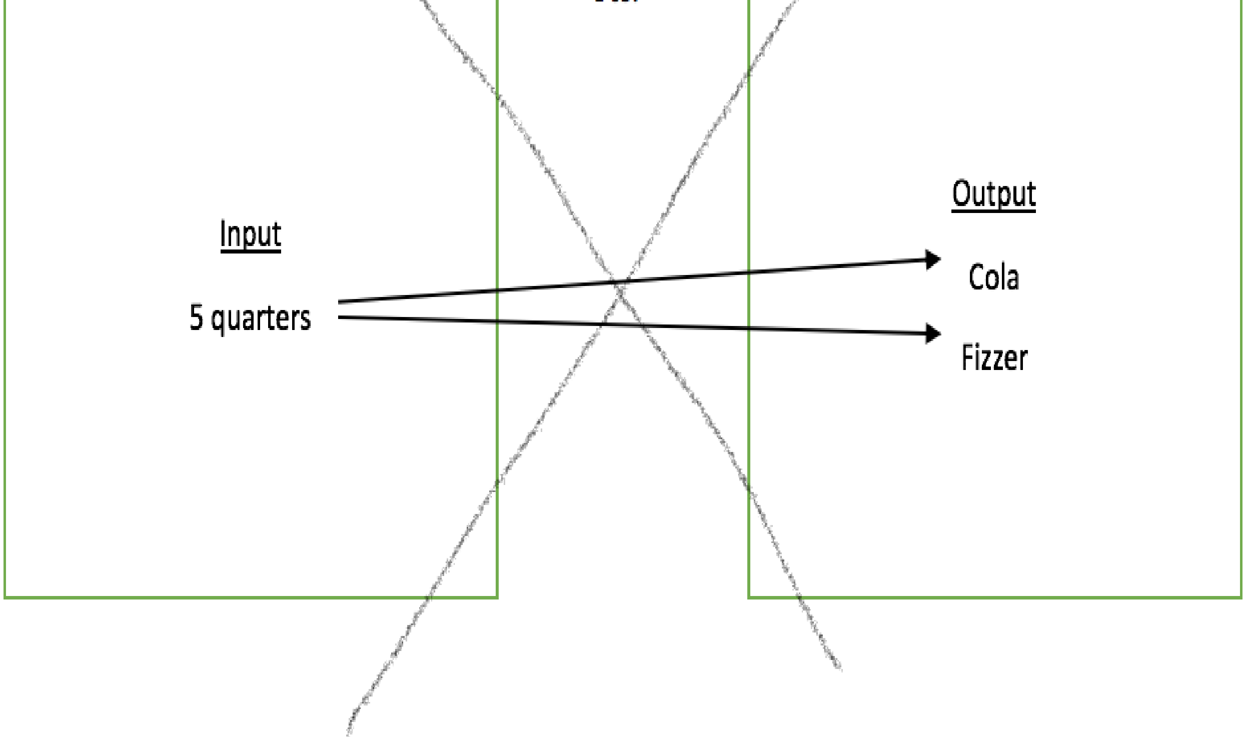


x	y
-3	3
1	6
1	2
2	2
5	-4

COLA MACHINE

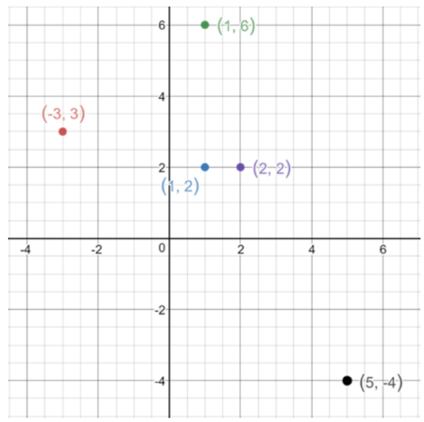


Yes!



No!

Making a graph from the table and mapping in Example 2 also demonstrates that it is not a function.

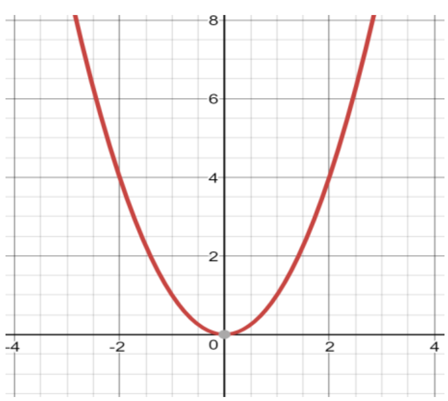


Looking Ahead 3.1

The input is also called the domain or independent variable. The output is also called the range or dependent variable.

If x represents the input, $f(x)$ represents the output. If t represents the input, $g(t)$ represents the output. For the output or range, $g(t)$, $h(m)$, and $k(s)$ represent function notation.

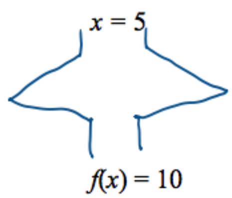
Example 3:	Is $y = x^2$ a function or relation only?



Example 4: Look at the completed table for the function and write the rule using function notation. (Write the equation.)

t	-6	-2	0	2	5	6	10
$f(t)$	-10	-6	-4	-2	1	2	6

Example 5: Write the possible rules for the function given the input and output.



Example 6: Find $h(t) = 3t - 5$ when t = -2, 0, and 4.

Section 3.2 Domain and Range

Looking Back 3.2

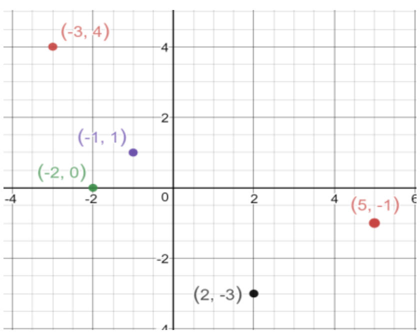
The terms input and independent variable represent the domain of a function. The terms output and dependent variable represent the range of the function.

The domain is all the possible input values of a function. The range is all the possible output values of a function.

Example 1: List the domain and range of the function.

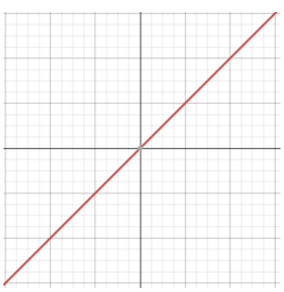
x	-3	-1	0	1	4	5
$f(x)$	-2	1	3	7	-2	1

Example 2: List the domain and range of the function.



Looking Ahead 3.2

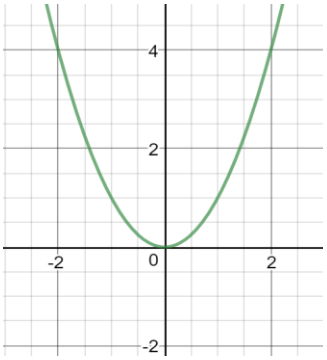
The parent function for a linear equation is $y = x$. The graph is as follows:



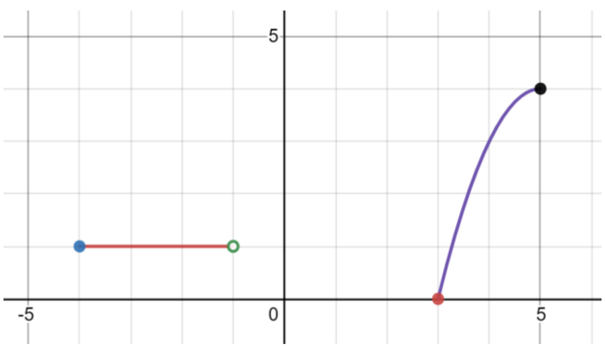
The values of x go from negative infinity $(-\infty)$ to positive infinity $(+\infty)$. It is possible to input negative and positive integers and zero to get a solution.

The values of y go from $-\infty$ to $+\infty$ as well. It is possible to get outputs that are positive and negative integers and zero.

Example 3: Find the domain and range of $y = x^2$ and write them using an inequality and interval notation.



Example 4: Find the domain and range of the piecewise function below. Use the union symbol for the conjunction "and."



Example 5: Find the domain and range of the function $g(x) = \frac{x+4}{x-1}$.

Section 3.3 Odd and Even Functions

Looking Back 3.3

A function is an odd function if $f(-x) = -f(x)$. A function is an even function if $f(-x) = f(x)$. A function can also be neither odd nor even.

Example 1:	Determine whether the function is even, odd, or neither.

$$f(x) = \frac{x^2}{x^4 + 4}$$

Example 2:	Determine whether the function is even, odd, or neither.

$$g(x) = x^2 + 3x$$

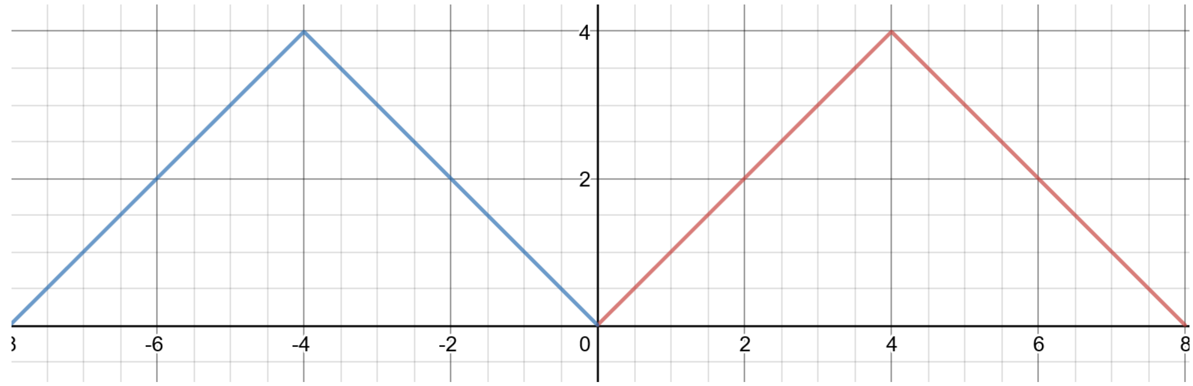
Example 3:	Determine whether the function is even, odd, or neither.

$$h(x) = x^3 - 2x$$

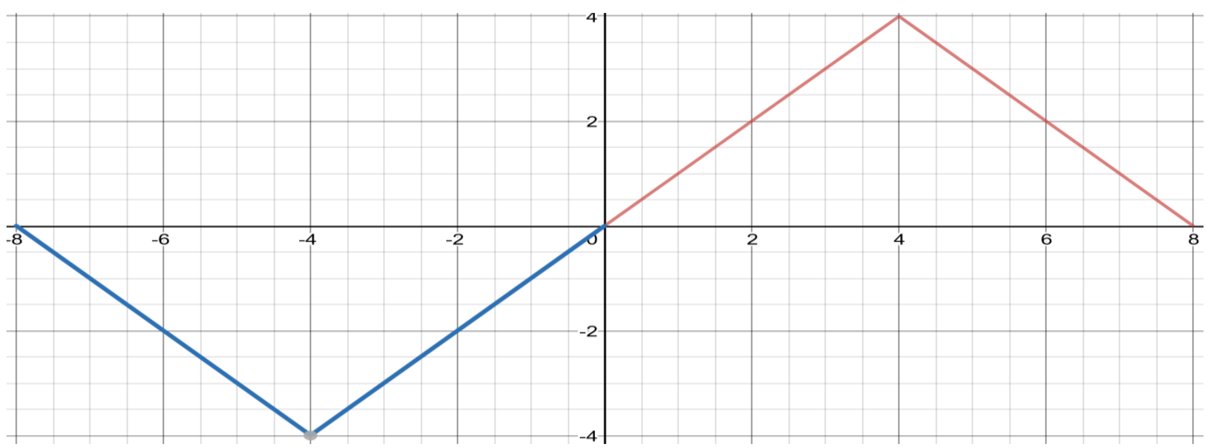
Looking Ahead 3.3

The graph of an even function is reflected over the y-axis. The graph of an odd function is reflected about the origin.

Example 4:	Is the following function an even or odd function?



Example 5:	Is the following function an even or odd function?



Section 3.4 Composition of Functions

Looking Back 3.4

The composition of f and g is the function $f(g(x))$. The composition of g and f is the function $g(f(x))$. The operation that combines f and g or g and f to produce their composite is called a composition.

Consider the two functions $f(x) = 2x$ and $g(x) = x^2$. To perform a composition of functions, perform the innermost first and work to the outermost next.

Example 1:	Find $f(g(x))$ when $x = 4$.

Example 2:	Find $g(f(-1))$.

Looking Ahead 3.4

Function composition gives the order of the functions working inside to outside.

Example 3: Given the following four functions, find the order of the functions if the initial input is 9 and the final output is 1:

$$f(x) = -(x-1)^2$$

$$g(x) = 2^x - 5$$

$$h(x) = \frac{x}{2} + 3$$

$$j(x) = \sqrt{x}$$

Example 4: Find $f\big(g(x)\big)$ if $f(x) = x^2 - 1$ and $g(x) = x + 1$.

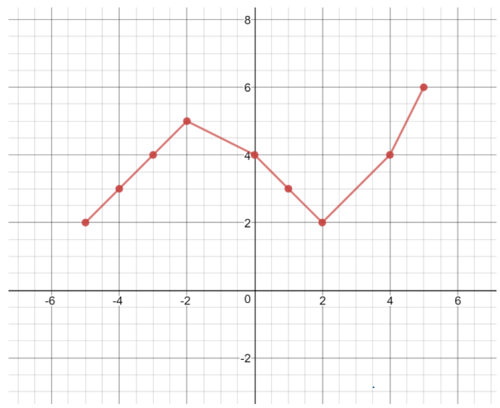
Example 5: Find $g(f(x))$.

Section 3.5 Inverse Functions

Looking Back 3.5

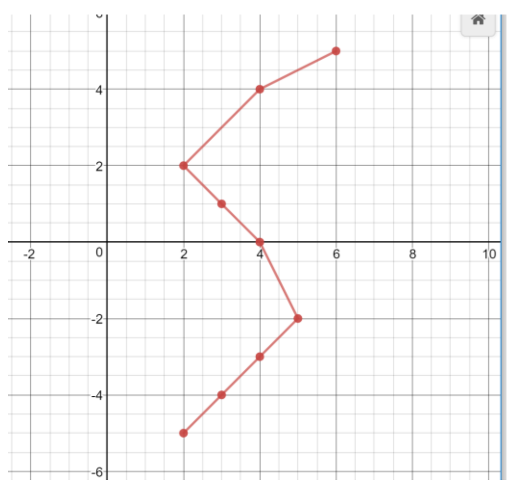
Inverse functions were first introduced in Algebra I. The table and graph for a piecewise function is given below.

x	y
-5	2
-4	3
-3	4
-2	5
0	4
1	3
2	2
4	4
5	6



The inverse function of a relation is a set of ordered pairs found by switching the coordinates of each ordered pair in the relation. The inverse graph of a function is the mirror image over the line of reflection, $y = x$. The function $y = x$ is its own inverse, $x = y$.

x	y
2	-5
3	-4
4	-3
5	-2
4	0
3	1
2	2
4	4
6	5



To find the inverse of an equation simply switch x and y. The inverse notation is y^{-1} or $f^{-1}(x)$. This does not mean $\frac{1}{y}$; y^{-1} is the function notation in this instance, not a negative exponent. There is no confusion with $f^{-1}(x)$, but we use y^{-1} because the x and y are switched for the inverse function.

If $y = \frac{4}{x-3}$, then the inverse is $x = \frac{4}{y-3}$.

However, the inverse is solved for y in terms of x so it is written as output in terms of input. When input values are substituted in the equation, the output is easily found.

$$x = \frac{4}{y-3}$$

$$(y-3)\,x = \frac{4}{\cancel{y-3}}\,\cancel{(y-3)}$$

$$\frac{(y-3)\cancel{x}}{\cancel{x}} = \frac{4}{x}$$

$$y - 3 = \frac{4}{x}$$

$$+3 \qquad +3$$

$$y = \frac{4}{x} + 3$$

$$y^{-1} = \frac{4}{x} + 3$$

The y is to the power of -1 because this is the inverse function. It can also be written as shown below:

$$f^{-1}(x) = \frac{4}{x} + 3$$

Looking Ahead 3.5

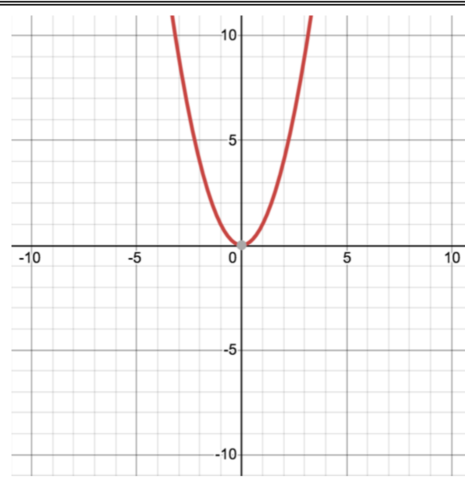
The inverse of a function may or may not be a function. If there is more than one output for every input, then the inverse is not a function.

In the piecewise function from the Looking Back section, the table has different values for the input, but the inverse has repeat values for the input at the coordinates (2, -5) and (2, 2), (3, -4) and (3, 1), and (4, -3) and

(4, 4). The original piecewise function passes the Vertical Line Test, but the inverse does not. An easy check for this is the Horizontal Line Test. If a horizontal line passes through the original function in more than one point, then when the function is reflected over the axis of symmetry, $y = x$, the inverse function will not pass the Vertical Line Test.

Example 1: Graph the function $y = x^2$ and its inverse. Is the inverse a function?

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



Example 2: Find the inverse equation of $y = x^2$. Switch the x and y. Solve for y in terms of x and use function notation for the inverse equation.

Example 3: Verify that the functions $f(x) = 2x - 2$ and $g(x) = \frac{1}{2}x + 1$ are inverses.

Example 4: Verify that $g\big(f(x)\big)$ is equal to x as well for the inverse function in Example 3.

Section 3.6 Operations with Functions

Looking Back 3.6

Functions represent equations and these equations represent real numbers. Operations that can be performed on real numbers can also be performed on functions. Operations with functions follow the same rules of exponents and properties of real numbers.

You may add, subtract, multiply, or divide two or more functions.

Looking Ahead 3.6

Let $f(x) = 2x^2 + 3x$ and $g(x) = 5x - 3$.

Perform each operation on the given functions in the examples below.

Example 1: $f(x) + g(x)$

Example 2: $f(x) - g(x)$

Example 3: $f(x) \cdot g(x)$

Example 4:	$\frac{f(x)}{g(x)}$

Example 5:	Divide the function $f(x)$ by $g(x)$ if $f(x)$ equals $2x^2 + 4x$ and $g(x)$ equals $x^2 - 4$.

Why do we use $f(x)$ when we really mean y? The equation $y = x + 2$ is an equation when the two variables are taken separately. The equation $f(x) = x + 2$ is used to show that there is a relationship between the two values x and y and they "change together."

Johann Bernoulli first used "Xx" to describe these relationships in a letter to Gottfried Wilhelm Leibniz in 1697. In 1734, Leonhard Euler created the modern notation $f(\frac{x}{a} + c)$ to denote the function of $\frac{x}{a} + c$. Later, Euler used $f:(x)$ for a function of one variable and $f:(x, y)$ to denote the value of a function of two variables.

- The term Christian denotes a relationship between an individual and Jesus Christ where the individual is a disciple and follower of the Son of God. Symbols have been used throughout history to represent this relationship also. Two such symbols that are the first two letters of the word for Christ (called "Chi(x)Rho(P)") are PX in the following symbol:

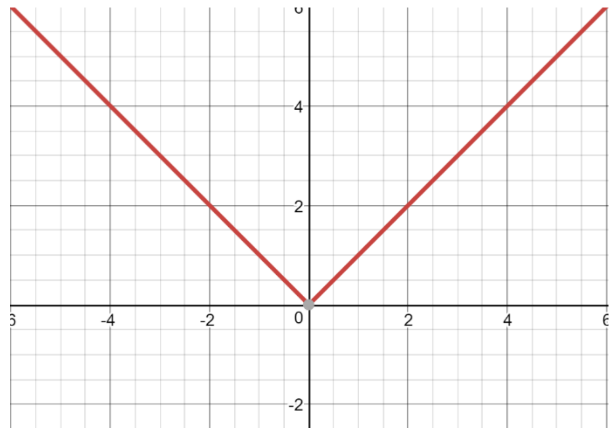
The monogram above (Chi(x)Rho(P)) was used by the early Christians and is attributed to the Roman Emperor Constantine, who claimed Christ as the King, and used it as a military symbol.

Section 3.7 Function Transformations

Looking Back 3.7

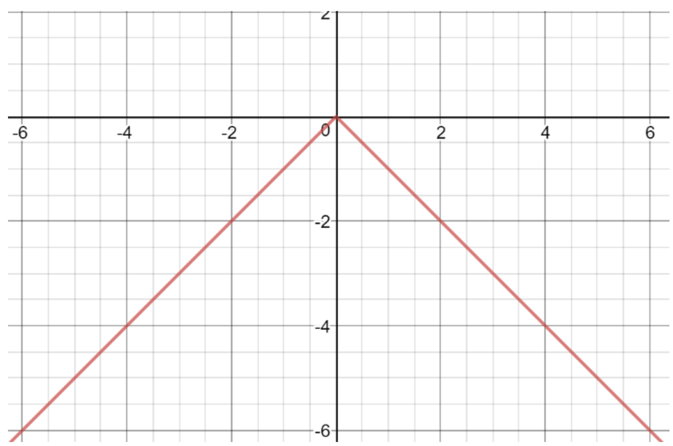
The parent function of an absolute value equation is $y = |x|$ or $f(x) = |x|$. It is symmetric over the y-axis and the vertex is at the origin.

x	$f(x)$
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3



How would $g(x) = -|x|$ transform the graph of $f(x)$. How does the table change?

x	$g(x)$
-3	-3
-2	-2
-1	-1
1	-1
2	-2
3	-3



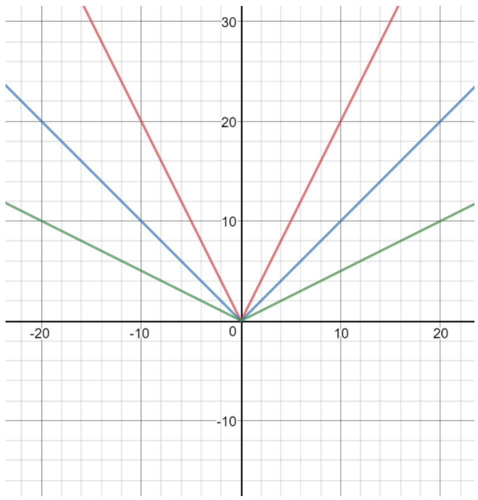
The negative sign in front of the function changes the value of the output to its opposite and flips the graph over the x-axis. When $a < 0$, the graph is reflected in the x-axis.

The variable a is used for the parameter in front of the function:

$$y = a|x|.$$

Example 1: Complete the table and graph for $j(x) = 2|x|$ and $k(x) = \frac{1}{2}|x|$. How does $|a| > 1$ transform the parent absolute value function? How does $0 < |a| < 1$ transform the parent absolute value function?

x	$j(x)$
-3	6
-2	4
-1	2
0	0
1	2
2	4
3	6



x	$k(x)$
-3	$\frac{3}{2}$
-2	1
-1	$\frac{1}{2}$
0	0
1	$\frac{1}{2}$
2	1
3	$\frac{3}{2}$

Multiplying by 2 doubles the output and appears to stretch the graph. Multiplying by $\frac{1}{2}$ cuts the output in half and appears to compress the graph.

The graphing form of the absolute value equation is $f(x) = a|x - h| + k$. The absolute value of "a" stretches or compresses the graph. The value of "h" is inside the function with the input. Let us investigate the transformation of the graph when h is positive and when it is negative. We will let $a = 1$ (the identity element for multiplication) and $k = 0$ (the identity element for addition).

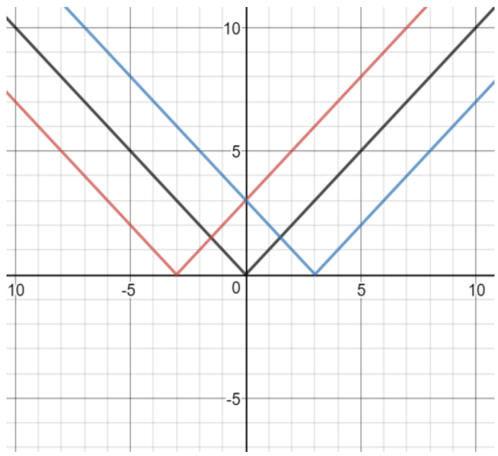
This will eliminate the parameters a and k and allow us to exclusively explore the effects of one variable, h.

Example 2:	How does $h = 3$ and $h = -3$ transform the equation $f(x) = |x|$?

When $h = 3$, $m(x) = |x - 3|$.

When $h = -3$, $n(x) = |x - (-3)|$; and $n(x) = |x + 3|$.

x	$m(x)$
-3	6
-2	5
-1	4
0	3
1	2
2	1
3	0
4	1
5	2
6	3
7	4
8	5
9	6



x	$n(x)$
-9	6
-8	5
-7	4
-6	3
-5	2
-4	1
-3	0
-2	1
-1	2
0	3
1	4
2	5
3	6

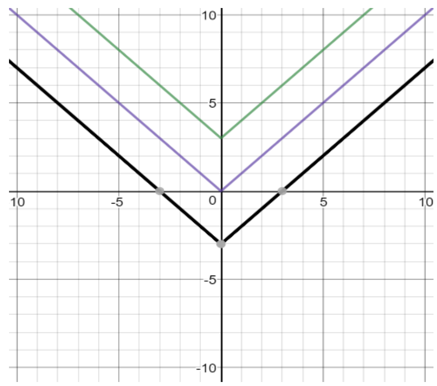
When h is positive the graph shifts to the right that number of spaces. When h is negative the graph shifts to the left that number of spaces. This can be tricky so be careful. The graphing form has a subtraction sign, and when you subtract a negative number it is the same as adding a positive number. For $n(x) = |x + 3|$, h is negative and the graph shifts left, even though there is a plus sign. For $m(x) = |x - 3|$, h is positive and the graph shifts right, even though there is a minus sign. This stays consistent for the transformation of all functions; "h" affects the input and the shifts are on the x-axis.

Finally, let us investigate the effect of the parameter k on the parent function. Again, let $a = 1$, but now h equals 0 to eliminate those variables leaving only k to be explored; k is outside the parent function and would seem to affect the output.

Example 3: How does $k = 3$ and $k = -3$ transform the equation $f(x) = |x|$?

$s(x) = |x| + 3$

x	$s(x)$
-3	6
-2	5
-1	4
0	3
1	4
2	5
3	6



$t(x) = |x| - 3$

x	$t(x)$
-3	0
-2	-1
-1	-2
0	-3
1	-2
2	-1
3	0

A positive k shifts the graph up. A negative k moves the graph down. The effect is on the output, and the shift occurs on the y-axis. This is the y-intercept since there is no horizontal shift. The sign is the same as the operation so there is no confusion here!

Looking Ahead 3.7

There are several forms of equations as shown below.

Parent Function	*Graphing Form*
$y = x$	$y = m(x - x_1) + y_1$
$y = x^2$	$y = a(x - h)^2 + k$
$y = x^3$	$y = a(x - h)^3 + k$

Example 4: The graph of $g(x) = 2|x - 4| + 3$ has the variables $a, h,$ and k where $a = 2$, $h = 4$, and $k = 3$. These parameters transform the parent function by shifting it horizontally right 4, stretching it by a factor of 2, and moving it up 3. What is the vertex of $g(x)$?

Section 3.8 Direct and Inverse Variation

Looking Back 3.8

A linear function of the form $y = kx$ is called a direct variation and y varies directly as x.

Several friends went out to the football field to calculate their walking rates. They started at the 0-yard line, or goal line, and walked 90 yards to the opposite 10-yard line. They each had a partner time them. Bryce walked 90 yards in 48.9 seconds. The students converted yards to feet as shown below.

$$90\ yds. \cdot \frac{3\ ft.}{1\ yd.} = 270\ ft.$$

Bryce walked 270 feet in 48.9 seconds. This means his walking rate is $\frac{270\ ft.}{48.9\ sec.} = 5.53\ ft./sec.$

Walking Rates

Name	**Walking Rates (ft./sec.)**
Bryce	5.53
Jordan	4.89
Tiffany	4.59
Tequia	4.76
Kylee	5.76
Kean	3.76
Trevor	5.2
Miranda	5.1
Malachi	7.01
Chris	5.3
Ethan	5.5
Trysten	5.5
Shane	5.61
David	5.32

1. Who is the fastest walker in the class? ________________________ How do you know?
____

2. Who is the slowest walker in the class? _______________________ How do you know?
____

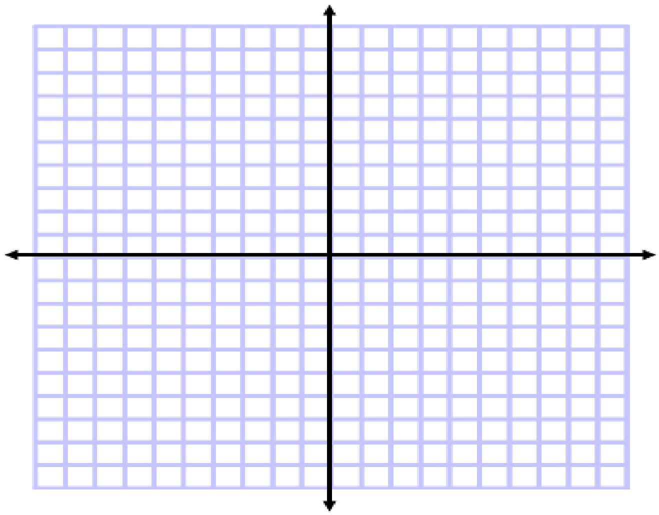
Below is the table using Jordan's walking rate in feet per second. Let x be the number of seconds and let y be the total feet walked.

$x\ (seconds)$	$y\ (total\ feet)$
0	0
1	4.89
2	9.78
3	14.67
4	19.56
5	24.45
x	$4.89x$

3. What is the equation that represents Jordan's total distance (y) when he has walked any number of seconds (x)? ____

4. What is the slope of the linear equation? _____ What does it represent? _________________

5. Draw the linear graph that represents Jordan's walking rate. Let the x-axis be seconds and the y-axis be total feet.



6. The relationship between a constant walking rate and distance is linear. Why?

____

____

The equation $y = kx$ is the same as $y = mx$. This means that $k = m$. The constant of variation is the slope. The walking rate represents the slope. This is a direct variation. The graph goes through the origin.

The graph of $y = mx$ is a line through the origin with a slope of m. The slope is constant. The coordinates (x_1, y_1) and (x_2, y_2) are two points on a line. The slopes are equal; $\frac{y_1}{x_1} = \frac{y_2}{x_2}$. These two ratios are proportional. In a direct variation, y is directly proportional to x and m is the constant of variation. The equation $y = mx$ can also be written as $m = \frac{y}{x}$ in terms of slope, or constant of variation.

Example 1: If y varies directly with x and $y = 15$ when $x = 36$, find x when $y = 25$.

Looking Ahead 3.8

A function of the form $y = \frac{k}{x}$ is called an inverse variation, and y varies inversely with x. This can also be written as $xy = k$. The constant of variation, also called the constant of proportionality, is the product of x and y.

Example 2: The electrical resistance of a wire varies directly with its length and inversely with the square of its diameter. An 80 mm. length of wire has a diameter of 5 mm. and a resistance of 15 ohms (**Ω**). What is the diameter of 100 mm. of the same type of wire that has a resistance of 12 ohms (**Ω**)?

Let $R =$ the resistance in ohms, $d =$ diameter (mm.) and $L =$ length (mm.). Let $k =$ the constant of proportionality.

Example 3: Write the equation of the variation described.

a) The volume of a cube is directly proportional to the product of the length, width, and height.

b) Newton's Law of Gravity states that the force between two spherical bodies varies directly as their masses, m_1 and m_2, and inversely as the square of the distance between their centers, r. Let K be the constant of proportionality.

Section 3.9 Linear Functions

Looking Back 3.9

Linear functions are foundational to the study of all higher-level functions. These were studied extensively in Algebra I and will be reviewed through the practice problems for the remainder of the module. In this section, we will investigate a real-world problem that can be solved using linear functions.

A farmer needs to irrigate his fields. He needs at least 200 L of water per hour to irrigate all his acres of crops. He wants to figure out the diameter size of the pipe needed for this volume of water.

Looking Ahead 3.9

In this investigation, rice will be used to simulate the flow of water. Funnels of different sizes will represent the pipe openings for water flow. If you decide to try the experiment on your own, you can make funnels using cardboard patterns or you can buy plastic funnels of different sizes. If you make them, roll cardboard corner to corner and then measure the diameter of the opening. Staple some tight and some loose for different size diameters. Make sure the rice flows steadily and does not get caught on the staples. You can fill empty 1-liter bottles with rice to be poured through the funnel. We will watch the video of the experiment. After that, we will discuss the results of a sample experiment. We will complete the tables with experimental data.

Example 1: Simulation of Water Flow Rate Using Rice

As previously described, in this investigation you will be using rice to simulate the flow of water.

The cardboard funnels will represent the different size openings of the pipes:

#1 has a diameter of approximately 1.5 cm.

#2 has a diameter of approximately 1.9 cm.

#3 has a diameter of approximately 2.3 cm.

1. The cardboard box in the video has a volume of 1,000 cubic centimeters, which is a capacity of 1 liter. Firstly, the rice will be poured by a team member into the liter box to fill.

2. The rice will then be poured through Funnel #1 by another team member at a constant rate from the liter box into a larger box to prevent spillage.

3. Another team member will time how long it takes for the entire liter to flow through the funnel into the box. Be consistent with each start time and with how you pour the liter box of rice into the funnel in order to control all the variables.

4. There will be three trials each for Funnel #1-3 and the trials will be added together for each and divided by 3 to get an average rate of flow in liters per second. They will be recorded in the table below.

Funnel Opening	Time for Trial 1	Time for Trial 2	Time for Trial 3	Average Time
1				
2				
3				

A rate compares the change in one quantity to the change in another. For example, if 2 liters of water flow through the funnel in 9.3 seconds then the flow rate is 2 L/9.3 sec. = 0.215 L/sec.

Use the fact that there are 60 seconds in one minute to convert the L/sec. flow rate above to L/min. flow rate:

0.215 L/sec. · 60 sec./min. = 12.9 L/min.

So, in five minutes the volume of flowing water would be 12.9 L/min. · 5min. = 64.5 L.

Convert your L/sec. to L/min. below and compare the table for volume of water for 1, 2, 3, 6, 9, and 10 minutes.

Time (minutes)	**Funnel #1**	**Funnel #2**	**Funnel #3**
1			
2			
3			
6			
9			
10			

Which funnel size(s) will give the farmer at least 200L of water per hour to irrigate his fields?

A manufacturing plant needs 3,000 Liters of water for parts to build components in an 8-hour day of work. Which size pipe diameter will yield the needed amount of water?

Section 3.10 Point-Slope Form

Looking Back 3.10

Understanding the form of linear equations is foundational to understanding all other functions.

Let us investigate further, understand more and grow wiser in our use of functions to problem-solve in our daily lives.

We know slope is m. We also know that a point on a line is (x, y) so it makes logical sense that if we know one point on a line and the slope, we can find the equation of a line.

$$m = \frac{y - y_1}{x - x_1}$$

The denominator can be cleared, and the following equation can be written:

$$(x - x_1)m = y - y_1$$

This is called point-slope form.

With a little rearranging and the commutative property, we can solve for y.

$$m(x - x_1) + y_1 = y$$

Or

$$y = m(x - x_1) + y_1$$

Graphing form is shown below.

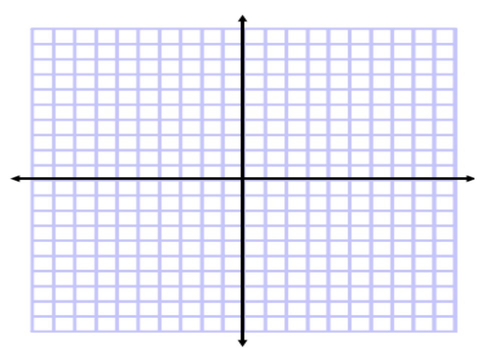
$$y = a(x - h) + k$$

This means $m = a$ (the slope is the stretch or shrink). The horizontal shift is x_1 and the vertical shift is y_1.

Looking Ahead 3.10

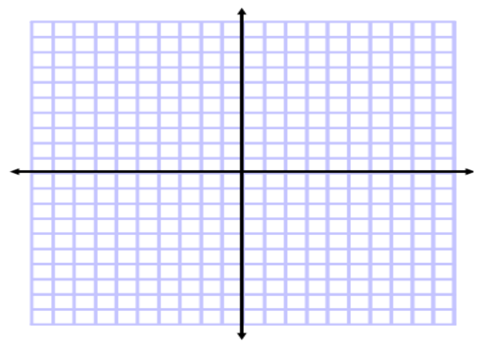
Example 1:	Given the point $(5, -2)$ on a line and the slope of the line, $m = 4$, find the equation of the line.

Example 2:	Graph the line given the point $(5, -2)$ and the slope of 4 or $\frac{4}{1}$.



Example 3: Given the point (2, 8) and the slope $m = 2$, find the equation of the line.

Example 4: Using a graph, show that $g(x) = \frac{1}{2}x - 4$ is not an inverse of the function found in Example 3.



Example 5: Using your knowledge of the composition of functions, show that the functions f and g from Example 3 and 4 are not inverses.

Section 3.11 Absolute Value Functions

Looking Back 3.11

The absolute value of a number is the distance of a number from the origin. In Algebra I, we solved absolute value equations and inequalities on a number line. Here, we will investigate absolute value functions and their transformations.

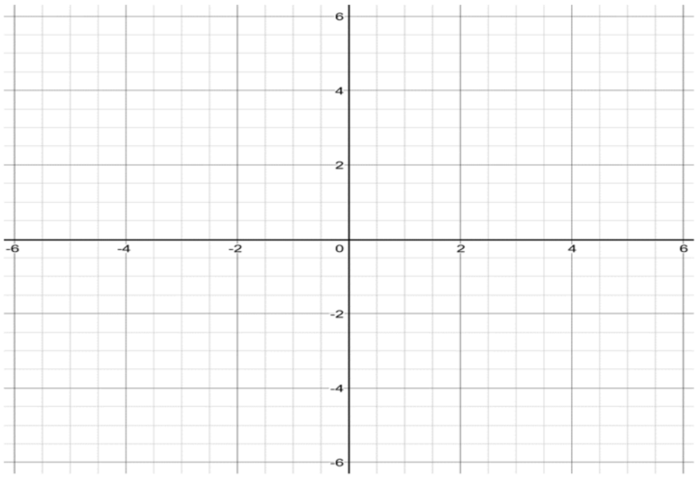
An absolute value function has several similarities to a quadratic function. There are a few differences as well. The inverse of a quadratic function is not a function unless domain restrictions are in place. This is true also for an absolute value function.

Example 1: Find the inverse of $f(x) = |x+3|$ for $x \le -3$.

Example 2: Find the inverse of $f(x) = |x-1| + 2$ for $x \ge 1$.

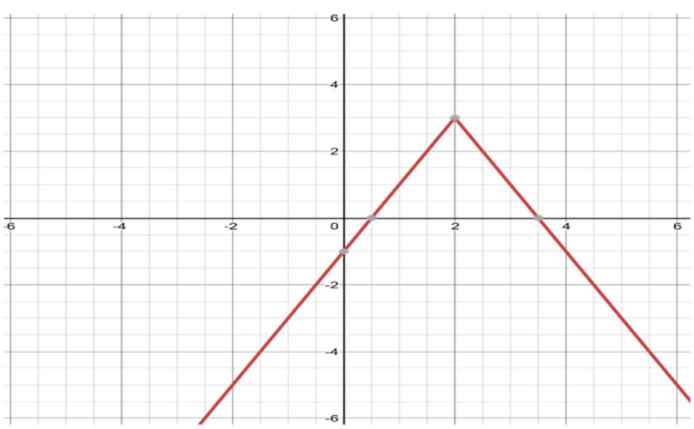
Looking Ahead 3.11

Example 3: Graph the points of the absolute value function $y = |x-2|+3$. What is the point of the vertex?



x	y
-1	6
0	5
1	4
2	3
3	4
4	5
5	6

Example 4: Change the equation in Example 2 to the equation $y = -2|x-2|+3$. What effects will $a = -2$ have on the function? Does it change the vertex?

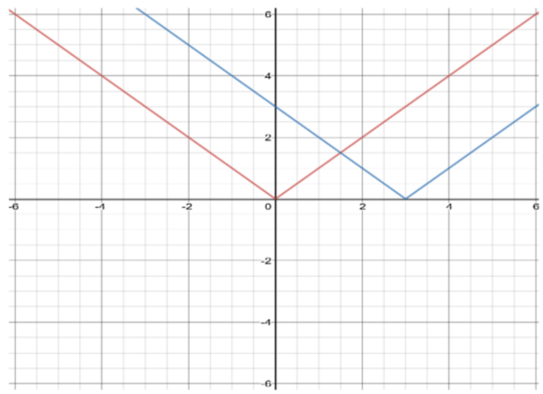


x	y
-2	-5
-1	-3
0	-1
1	1
2	3
3	1
4	-1
5	-3
6	-5

Example 5: Consider the graphs of $y = |x|$ and $y = |x - 3|$.

$y = |x|$

x	y
-6	-6
-5	-5
-4	-4
-3	-3
-2	-2
-1	-1
0	0
1	1
2	2
3	3
4	4
5	5
6	6



$y = |x - 3|$

x	y
-3	6
-2	5
-1	4
0	3
1	2
2	1
3	0
4	1
5	2
6	3

Example 6: If $y = |x|$ and $T(x, y) \to (x + 3, y)$, find the image of $(-5, 5)$.

Section 3.12 Step Functions

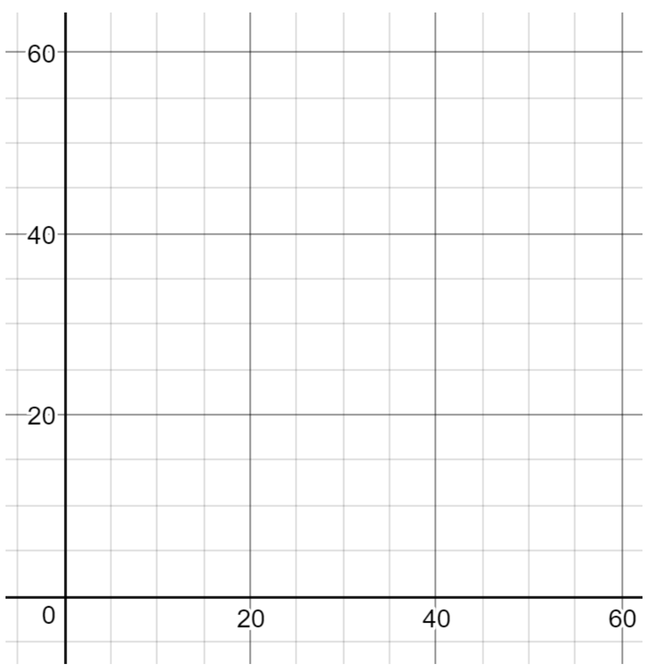
Looking Back 3.12

A step function is exactly as it sounds: a function whose graph looks like a set of steps.

Example 1: Let us suppose there is a group rate to a water park that changes based on the amount of people going to the park. The price is as follows:

Price	Number in Group
$64	1-9
$53	10-19
$42	20-24
$31	25-30

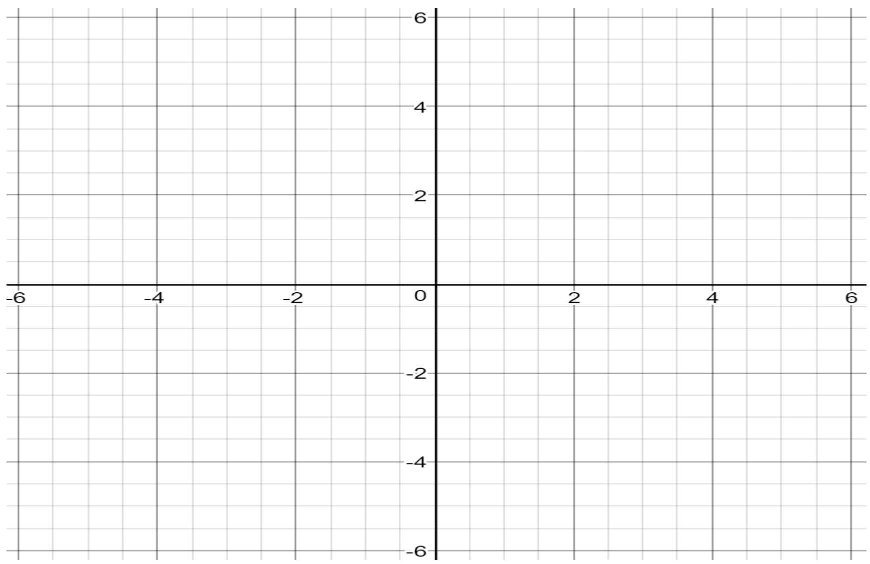
The graph would have intervals to represent the prices, open circles for what is not included and closed circles to show inclusion.



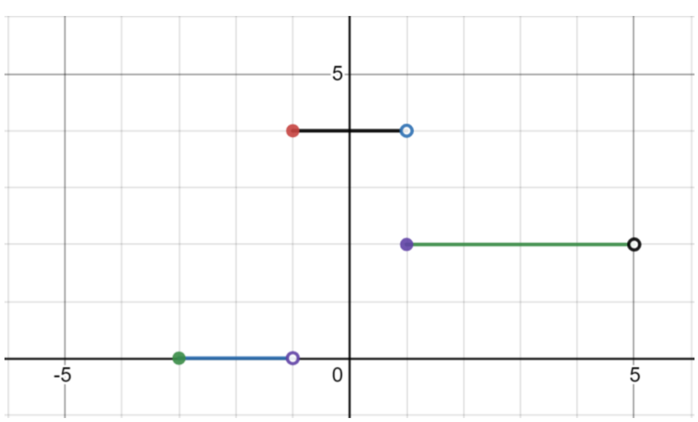
Looking Ahead 3.12

The Greatest Integer Function is the most familiar step function and is a function f defined for all real numbers such that $f(x)$ is the greatest integer less than or equal to x. It is written $f(x) = [x]$. It is also called the rounding down function or floor function because for any integer x, the greatest integer less than or equal to x is x. If the real number is not an integer, round down to find $[x]$.

Example 2: Graph the greatest integer function also called the constant piecewise function for domain and range from -6 to 6.

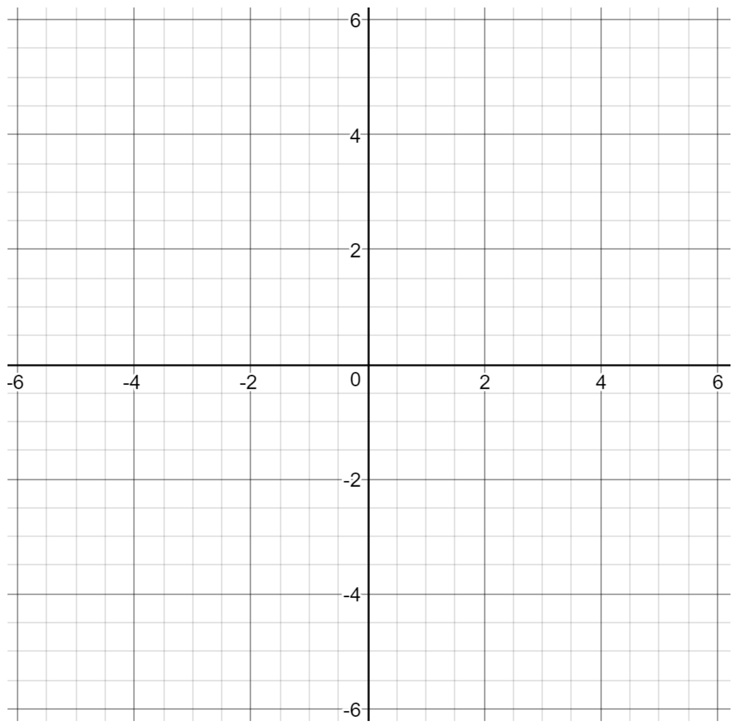


Example 3: Write the interval for the step function using inequality notation.



Another step function is the Smallest Integer Function. It is also called the Least Integer Function. This is a function f defined for all real numbers such that $f(x)$ is the smallest integer greater than or equal to x. It is written $f(x) =]x[$. It is also called the rounding up function or ceiling function because for any integer x, the greatest integer greater than or equal to x is x. If the real number is not an integer, round up to find $]x[$.

Example 4:	Graph the smallest integer function (or least integer function). Evaluate $]-3.1[$ and $]4.7[$.

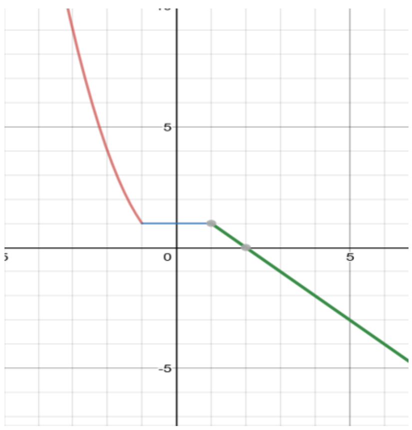


Section 3.13 Piecewise Functions

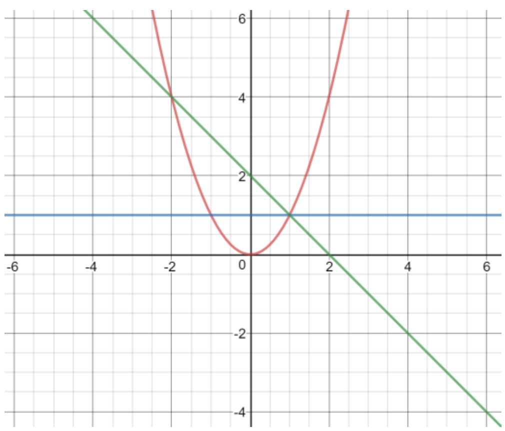
Looking Back 3.13

A piecewise function is exactly what it sounds like. It is a function made up of pieces. A piecewise function is defined as a function f made up of at least two equations (pieces), each of which applies to a different part of a domain. These pieces may all be linear or a combination of linear, quadratic, cubic, exponential, etc. Because it is various functions, a parent function does not exist. It can be continuous or discontinuous.

Example 1: Below is the graph of a piecewise function. List the equations of each piece along with the interval of each using inequality notation.



Below is a graph showing how all three above would look together on one graph:



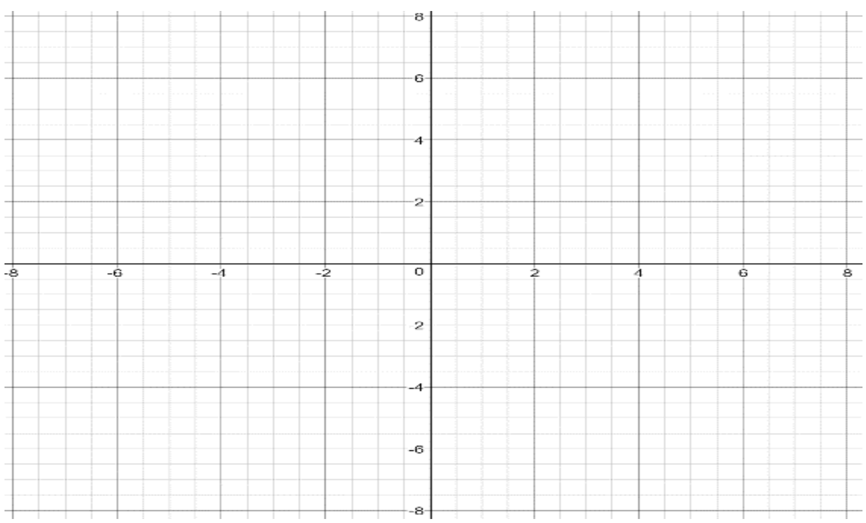
We are finding the intersection or continuous function as we move from one to another.

Looking Ahead 3.13

Some graphs are discontinuous because they have breaks, which are holes or gaps.

Example 2: Draw the graph of the piecewise function. List the domain using interval notation.

$$f(x) = \begin{cases} (x-2)^2\,;\, x \ \geq 2 \\ x+2\,;\, x < 0 \end{cases}$$



Example 3: Draw the graph of the piecewise function. What is the domain and range?

$$f(x) = \begin{cases} x^2\,;\, x \geq 0 \\ x+2\,;\, x < 0 \end{cases}$$