## Module 6: Quadratic Functions

## Section 6.1 Forms of the Quadratic Equations

Practice Problems 6.1
For Problem 1-10, follow the instructions/answer the question(s) to solve the problem.

1. Complete the table for $y=x^{2}$. If you use a calculator, put the $x$ in parenthesis (along with the sign when the number is negative).

| $y=(x)^{2}$ |  |
| :---: | :---: |
| If $x=1$, then $y=(1)^{2}=1$ |  |
| $x$ | $\boldsymbol{y}$ |
| If $x=-1$, then $y=(-1)^{2}=1$ |  |
| -3 |  |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |

2. Graph the equation $y=x^{2}$ using the ordered pairs from the table above.

3. On the graph in Problem 2, draw a dashed vertical line in the middle as a line of symmetry. What is the equation of this line?
4. The vertex of a parabola lies on the line of symmetry, which cuts the parabola into two equal parts. One part is a reflection of the other part. Is this a minimum point or a maximum point? Does the graph open upward or downward?
5. Complete the table for $y=-x^{2}$. If you use a calculator, put the $x$ in parenthesis but the negative sign outside the parenthesis.

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| -3 |  |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |

$$
\begin{aligned}
& \text { If } x=1 \text {, then } y=-(1)^{2}=-1 \\
& \text { If } x=-1, \text { then } y=-(-1)^{2}=-1
\end{aligned}
$$

6. Graph the equation $y=-x^{2}$ using the ordered pairs from the table.

7. Draw a dashed line vertically through the middle of the graph. This is the line of symmetry. What is the equation of this line?
8. The vertex is the highest or lowest point on the graph. How does the graph open if the vertex is a maximum? What are the coordinates of the vertex of $y=-x^{2}$ ? Is this a minimum or maximum point?
9. How does the graph of $y=-x^{2}$ relate to the graph of $y=x^{2}$ ?
10. What is a rule that could help determine if the parabola of a quadratic equation opens upward or downward?

For Problem 11-15, use the given information to solve the problem.
The quadratic equations below are in vertex form. Use multiplication and the Distributive Property to expand each quadratic equation below and combine any like terms. Write your solution in standard form. (Hint: Expand $(x+3)^{2}$ to $(x+3)(x+3)$ and multiply, then combine like terms.)
11. $y=(x+3)^{2}-4$
12. $y=(x-5)^{2}+2$
13. $y=-2(x+1)^{2}-1$
14. $y=\frac{1}{2}(x-2)^{2}$
15. $y=-(x+6)^{2}+22$

For Problem 16-20, solve the quadratic equation given and find the value(s) of $x$.
16. $x^{2}=100$
17. $x^{2}=49$
18. $x^{2}=10$
19. $x^{2}-3=78$
20. $x^{2}+5=30$

## Section 6.2 Quadratics and Area Problems

Practice Problems 6.2
For Problem 1-5, use the given information to solve the problem.
La'Daisha can only buy 14 linear-feet of fence. Using the perimeter formula $P=2(l+w)$, then $14=2(l+w)$.

1. Solve for the width in terms of the length.
2. Using the formula from Problem 1 and the values given for length in the table below, complete the table.

| Length $(l)$ | Width $(w)=(7-l)$ | Area $=l \cdot w$ |
| :---: | :---: | :---: |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |

3. Graph the length on $x$ and the area on $y$. What is the maximum point on the graph and what does it represent? What is the line of symmetry?

4. What are the $x$-intercepts? Where do you see them on the graph? Where do you see them on the table?
5. Find the quadratic equation that represents the area of the puppy pen in terms of the length.

For Problem 6-10, solve the word problem given.
6. If rate of change represents the change of $y$ with respect to $x$, what does it mean in terms of the puppy pen problem from the Lesson Notes (Example 1)? In other words, as the length increases, what happens to the area before the maximum point? As the length continues to increase, what happens to the area after the maximum point?
7. Does the line of symmetry always go through the vertex? Why or why not?



8. If the area of a pen is $A=2 l^{2}-4 l$, it can be written using function notation, $f(x)=2 x^{2}-4 x$ (we substituted $f(x)$ or $f$ of $x$ for $y$ ) when $x$ is the length. (Using this notation, $x$ represents length and $f(x)$ represents area.) Factor the quadratic to find the length and width of the rectangle in terms of $x$.

9. A company sells vacation packages. The equation to model the total income for the vacation packages sold is $y=-45 x^{2}+430.5 x$.
a) If $x$ represents the number of vacation packages sold and $y$ represents the total income, what quadrant will the reasonable solutions for this equation fall into? What are the domain and range of the equation?
b) Complete the table and graph the equation.

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 9 |  |
| 9 |  |


10. How do you know the graph below is not a quadratic equation? Give at least three reasons.


For Problem 11-13, complete the table and graph the quadratic equation given. Answer the questions: Does the graph open upward or downward? Does it have a maximum or minimum?
11. $y=(x-1)^{2}+1$

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |


12. $y=x^{2}-2 x$

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |


13. $y=-x^{2}-4 x+5$

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| -5 |  |
| -4 |  |
| -3 |  |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |



For Problem 14 and 15, solve for $x$ in the quadratic equation given.
14. $(x+3)^{2}=36$
15. $(x-4)^{2}+1=17$

For Problem 16 and 17, convert the vertex form given to standard form.
16. $(x-2)^{2}=y$
17. $(x+5)^{2}-10=y$

For Problem 18-20, solve the word problem given.
18. The perimeter of a square is given by the function $p=4 \sqrt{a}$ in which $a$ is the area.
a) Find the perimeter of a square that has an area of $256 \mathrm{~m}^{2}$.
b) Find the area of a square that has a perimeter of 420 m .
19. The time, $t$ (in seconds), for an object to free-fall a distance, $d$ (in feet), is given by the function $t=\frac{1}{4} \sqrt{d}$ given no air resistance.
a) Find the time it takes for a 25 lb bowling ball to fall 16 m .
b) Find the time it takes for a 1 lb ball to fall 16 m .
20. The speed a tsunami wave can travel is modeled by the equation $s=256 \sqrt{d}$ in which $s$ is the speed in kilometers per hour and $d$ is the average depth of water in kilometers. What is the speed, $s$, when the average water depth of a tsunami is $d=0.512 \mathrm{~km}$ ? (Round to the nearest tenths place.)

## Section 6.3 The Zero-Product Property

Practice Problems 6.3
For Problem 1-6, use the given information to solve the problem.
Cassandra makes gift baskets. The equation $p=m(80-m)$ gives the profit she makes each month if she charges $m$ amount of money for each basket.
1-2. Make a table and a graph for the information and estimate the price for a basket that will produce a maximum profit.

| $m$ | $p$ |
| :---: | :---: |
| 0 |  |
| $\$ 10$ |  |
| $\$ 20$ |  |
| $\$ 30$ |  |
| $\$ 40$ |  |
| $\$ 50$ |  |
| $\$ 60$ |  |
| $\$ 70$ |  |
| $\$ 80$ |  |


3. What is the price that will give the most profit from the baskets?
4. What does the origin represent?
5. What possible reasons would explain why a more expensive basket gives less profit?
6. What is a possible explanation for $(\$ 80,0)$ ?

For Problem 7-12, use the Zero-Product Property to find the $x$-intercepts of the quadratic equation given.
7. $(2 x-3)(2 x+1)=0$
8. $(x-2)(x+8)=0$
9. $(4 x-4)(x+2)=0$
10. $(3 x-1)(x+4)=0$
11. $x(x+9)=0$
12. $x(4 x-4)=0$

For Problem 13-20, factor the quadratic given and use the Zero-Product Property to find the $x$-intercepts of the quadratic graph.
13. $y=3 x^{2}-3 x$
14. $y=25 x^{2}-25$
15. $y=x^{2}-5 x-14$
16. $f(x)=x^{2}+2 x-15$
17. $f(x)=5 x^{2}+10$
18. $f(x)=x^{2}+x-12$
19. $f(x)=x^{2}+6 x-16$
20. $f(x)=x^{2}-8 x+16$

## Section 6.4 Finding the Vertex of a Quadratic Equation

Practice Problems 6.4
For Problem 1-6, use the equation $x^{2}-x-6=f(x)$ to solve the problem.

1. Factor $x^{2}-x-6=f(x)$.
2. Use the Zero-Product to find the zeroes of the quadratic function in Problem 1.
3. What are the ordered pairs (coordinates) of the $x$-intercepts of the quadratic function?
4. What is the $x$-value of the vertex? How did you find it?
5. What is the $y$-value of the vertex? How did you find it?
6. Graph the $x$-intercepts and the vertex of the quadratic function to draw a sketch of the parabola.


For Problem 7-14, use the Zero-Product Property to find the solutions for $x$.
7. $(x-2)(8 x-5)=0$
8. $\quad x(x+4)=0$
9. $3(6 x+1)(2 x+5)=0$
10. $(x-7)(x+2)=0$
11. $(x-4)(x+1)=0$
12. $6(x-4)(x+2)=0$
13. $(5 x+6)(x-1)=0$
14. $x(x-1)=0$

For Problem 15-20, factor the quadratic and then use the Zero-Product Property to find the $x$-values. Find the vertex of the quadratic.
15. $x^{2}-3 x-28=0$
16. $9 x^{2}-16=0$
17. $x^{2}+2 x+1=0$
19. $x^{2}-25=0$
20. $3 x^{2}+3 x-6=0$

## Section 6.5 Horizontal Shifts in Quadratic Equations <br> Practice Problems 6.5

For Problem 1-5, tell whether the graph shifts right or left on the $x$-axis from the parent function $y=x^{2}$ and tell how many units it shifts.

1. $y=(x-3)^{2}$
2. $y=(x+1.4)^{2}$
3. $y=\left(x-\frac{3}{4}\right)^{2}$
4. $y=(x+5)^{2}$
5. $y=(x-4)^{2}$

For Problem 6-10, use the equations from Problem 1-5 to solve the problem.
Sketch the graphs of the equations above on the same coordinate grid below using a different colored pencil for each parabola.

11. What quadrants do all the graphs lie in and why?

For Problem 12-19, factor the quadratic equation given into a binomial square and then find the horizontal shift from the origin of the parent function.
12. $x^{2}+14 x+49=y$
14. $x^{2}+22 x+121=y$
15. $x^{2}+16 x+64=y$
16. $x^{2}-18 x=-81$ (Hint: Write it in standard form first.)
17. $x^{2}-10 x=-25$
18. $x^{2}-2 x+1=0$
19. $(x-5.5)^{2}=y$
20. How many x-intercepts do the parabolas in Problem 12-19 have?

Section 6.6 Vertical Shifts in Quadratic Equations
Practice Problems 6.6
For Problem 1-7, solve the word problem given.

1. Does the equation $y=(x-5)^{2}$ affect the $x$-value or the $y$-value of the vertex? How does it change the graph of the parent function?
2. Does the equation $y=x^{2}-4$ affect the $x$-value or $y$-value of the vertex? How does it change the graph of the parent function?
3. In the equation $y=(x-h)^{2}$, what does $h$ represent: the $x$-value or the $y$-value of the shifted vertex?
4. In the equation $y=x^{2}+k$, what does $k$ represent: the $x$-value or $y$-value of the shifted vertex?
5. In the equation $y=(x-h)^{2}+k$, the values of the vertex are $(h, k)$. How do you know what the signs are?
6. Which way does the graph of $y=x^{2}-6.2$ shift and how far from the parent function does it shift?
7. Which way does the graph of $y=x^{2}-4.5$ shift and how far from the parent function does it shift?

For Problem 8-15, name the vertical shift for the parent function in the quadratic equation given.
8. $y=(x-3)^{2}+2$
9. $y=x^{2}+4$
10. $y=(x+4)^{2}-7$
11. $f(x)=x^{2}-9$
12. $f(x)=(x+1.2)^{2}-6.8$
13. $f(x)=\left(x-\frac{1}{4}\right)^{2}-\frac{3}{8}$
14. $f(x)=(x+2)^{2}+11$
15. $f(x)=x^{2}-3$

For Problem 16-20, use the equation $f(x)=(x+4)^{2}-6$ to solve the problem given.
16. What is the value of $h$ ? What is the horizontal shift?
17. What is the value of $k$ ? What is the vertical shift?
18. What is the vertex, $(h, k)$ ?
19. Is the parabola opening upward or downward?
20. Is the vertex a minimum or a maximum?

## Section 6.7 Vertex (or Graphing) Form

## Practice Problems 6.7

For Problem 1-4, name the vertex in the equation given.

1. $y=(x+2)^{2}-4$
2. $y=(x+1)^{2}+3$
3. $y=(x-4)^{2}-1$
4. $y=(x-5)^{2}+2.5$

For Problem 5-8, plot the points of the vertex and sketch the graph of the equation which is given in vertex form.
5. $y=(x+2)^{2}-4$

6. $y=(x+1)^{2}+3$

7. $y=(x-4)^{2}-1$

8. $y=(x-5)^{2}+2.5$

9. How does the equation $y=-(x-5)^{2}+2.5$ change the graph of Problem 4 ?

For Problem 10-13, name the horizontal and vertical shift from the origin of the parent function $y=x^{2}$ in each of the quadratic equations that are written in vertex form.
10. $y=(x+6)^{2}-8$
11. $y=(x+5)^{2}+7$
12. $y=(x-1)^{2}-4$
13. $y=(x-2.5)^{2}+5$

For Problem 14-17, substitute $y=0$ and solve for $x$ to find the $x$-intercepts.
14. $y=(x+1)^{2}-4$
15. $y=(x-5)^{2}$
16. $y=x^{2}-16$
17. $y=(x-6)^{2}-9$

For Problem 18-20, solve the word problem given.
18. What is the vertex in $y=(x+10)^{2}-2$
19. Does the parabola open upward or downward in $y=-2(x+4)^{2}-1$ ?
20. Does $y=-4(x-3)^{2}$ have a vertex that is a minimum point or a maximum point?

## Section 6.8 Factoring to the Vertex Form

Practice Problems 6.8
For Problem 1-7, solve word problem given.

1. Can $x^{2}+6 x+2=y$ be factored into two binomial factors? Why or why not?
2. Factor $x^{2}+6 x+2=y$ by completing the square. Write the equation in vertex form.
3. What is the vertex of $x^{2}+6 x+2=y$ ?
4. Expand the factored form of $x^{2}+6 x+2=y$ to see if it was factored correctly.
5. Sketch the graph of $x^{2}+6 x+2=y$.

6. Factor $x^{2}-x-2=y$ into two linear factors. Find the $x$-intercepts and the vertex of the equation and sketch the graph.

7. Factor $x^{2}-x-2=y$ by completing the square. Do you get the same vertex as in Problem 6 ?

For Problem 8-14, complete the square to write the quadratic equation in vertex form and find the vertical shifts of the equation.
8. $x^{2}+12 x-11=0$
9. $x^{2}-8 x-15=0$
10. $x^{2}-6 x-51=4$
11. $x^{2}+4 x=21$
12. $x^{2}-18 x-24=4$
13. $x^{2}-18 x-17=2$
14. $x^{2}-65=8 x$

For Problem 15-20, set $y=0$ and solve the equation for $x$ to find the $x$-intercepts of the equation.
15. $y=(x-2)^{2}$
16. $y=(x+4)^{2}-3$
17. $y=(x-7)^{2}-1$
18. $y=x^{2}-8$
19. $y=(x-1)^{2}-4$
20. $y=(x+4)^{2}-25$

## Section 6.9 Irrational Numbers and Quadratics <br> Practice Problems 6.9

For Problem 1-4, use the Venn Diagram from the Lesson Notes to name the set or sets of numbers to which the number given belongs.

1. $-\frac{13}{3}$
2. -2
3. $\sqrt{\frac{1}{2}}$
4. 413

For Problem 5-8, give two exact solutions for $x$ in the quadratic equation given.
5. $x^{2}=22$
6. $x^{2}+2=11$
7. $(x-5)^{2}=36$
8. $2(x+1)^{2}-3=20$

For Problem 9-11, give the decimal approximation for the irrational values of $x$. Round to the hundredths place.
9. $(x-1)^{2}+7=52$
10. $3(x+2)^{2}+8=45$
11. $5(x+3)^{2}-9=47$

For Problem 12 and 13, use the information given to answer the question given.
12. The function $h(t)=-4.9(t-0.3)^{2}+2$ models the height of a softball thrown in fast-pitch softball. The function, $h(t)$, represents the height in meters at any time, $t$, in seconds.
a) When the pitcher released the ball, the time was $t=0$; what height was the ball released from?
13. The relationship between a car's stopping distance when braking and its speed is modeled by the equation $y=0.006 x^{2}+0.12 x$ in which $x$ represents the speed in kilometers per hour and $y$ represents the distance in meters.
a) What is the stopping distance when the car travels at $90 \mathrm{~km} / \mathrm{h}$ ?

For Problem 14 and 15, fill in the blanks to describe the transformation of the parent graph $y=x^{2}$.

Use the term "projectile motion," which means the movement of something through the air such as a football or rocket. In the equations of the projectile motion problems, let $x$ represent time and $y$ represent height.
14. $y=-4.9(x-3.1)^{2}+10$

The number -4.9 is the same as $\frac{1}{2}\left(-9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)$ and represents the effects of gravity. In the transformation, it is a reflection in the $x$-axis. There is a horizontal translation of $\qquad$ units, and a vertical translation of
$\qquad$ units.
15. $y=-16(x-2)^{2}+18$

The number -16 is the same as $\frac{1}{2}\left(-32 \frac{\mathrm{ft}}{\sec ^{2}}\right)$ and represents the effects of gravity. It also represents a
$\qquad$ stretch of 16 units and a reflection in the $x$-axis. There is a $\qquad$ translation of

18 units and a $\qquad$ translation of 2 units.

For Problem 16-19, sketch the graph of the quadratic function given.
16. A function with zero $x$-intercepts 17. A function with two $x$-intercepts
18. A function with one $x$-intercept that has the vertex in Quadrant II.
19. A function with one $x$-intercept in which the vertex lies on the $x$ and $y$ axes and the parabola lies in Quadrant III and Quadrant IV.
20. A function with one $x$-intercept in which the vertex lies on the $x$ and $y$ axes and the parabola lies in Quadrant I and Quadrant II.

Section 6.10 Projectile Motion
Practice Problems 6.10
For Problem 1-3, use Example 2 from the Lesson Notes to solve the word problem given.

1. The equation for the height of the basketball is $h(t)=-16 t^{2}+64 t+7.5$. The constant is 7.5 , which can be found at the beginning (top) of the table. What does 7.5 represent?
2. In order to find the exact maximum height, what form could the expanded or standard form be converted to?
3. What could the equation possibly be if the son was laying on the ground throwing the ball over his head from 3 feet?

For Problem 4-7, use the given information to solve the problem.
The equation for the jump height of a mosquito is $h(t)=-16 t^{2}+6 t$.
4. Why is there no constant value in the equation?
5. Factor the equation by taking out the Greatest Common Factor and using the Zero-Product Property to find the $x$-intercepts of $h(t)=-16 t^{2}+6 t$.
6. Find the vertex of the equation $h(t)=-16 t^{2}+6 t$.
7. The table and graph of the equation in Problem 6 are below:

| $\boldsymbol{t}$ <br> (time in seconds) | $\boldsymbol{h}(\boldsymbol{t})$ <br> (height in feet) |
| :---: | :---: |
| -0.625 | -10 |
| -0.5 | -7 |
| -0.375 | -4.5 |
| -0.25 | -2.5 |
| -0.125 | -1 |
| 0 | 0 |
| 0.125 | 0.5 |
| 0.25 | 0.5 |
| 0.375 | 0 |
| 0.5 | -1 |
| 0.625 | -2.5 |
| 0.75 | -4.5 |


a) The increments (intervals) are 0.125 seconds ( $\frac{1}{8}$ th of a second). Why are they so small?
b) Why do negative values for the height not make sense?

For Problem 8-10, solve the word problem given.
8. A gerbil named Godspeed can jump a height of $h(t)=-16 t^{2}+16 t+0.3$ in feet. How high will Godspeed be after 1 second?
9. A golf ball is dropped from the top of a hotel next to a golf course. The number of seconds after it has been dropped is $t$. The equation that represents the height of the drop (in meters) is $h(t)=-4.9 t^{2}+147$.
a) What does $h(0)$ mean? What is the value of $h(0)$ ?
b) Solve for $h(t)=10$. What does your solution mean? There are two solutions: which solution represents when the golf ball is 10 meters above the ground?
c) When does the ball hit the ground?
10. A rocket is shot into the air from the ground.
a) What are the coordinates of the starting point?
b) If the rocket stays in the air for 4.5 seconds until it reaches the maximum height of 110 meters, what is the vertex of its parabola?
c) After the maximum height, the rocket falls back to the ground and lands at 9 seconds. What are the landing coordinates of the rocket?
d) Using -4.9 for the effects of gravity, write an equation for the rocket launch using the vertex form of the quadratic equation $y=a(x-h)^{2}+k$ or $h(t)=a(t-h)^{2}+k$.
e) Find $h(2)$.
f) Find $t$ when $h(t)=44$. What does this represent in terms of a real-world problem? Use the vertex form of the equation from part d) to solve this.
11. If you build a water-bottle rocket and launcher, follow the steps in the Lesson Notes, Examples 3-5 to find the velocity and maximum height of your water-bottle rocket.

It might be difficult to build a water-bottle rocket launcher, so we will do something a little bit easier here. We will launch a stick of gum to model projectile motion. Take a clothespin and tape one arm of it to a tabletop so the clip is on the edge of the table. Put a piece of gum on the other side of the clothespin. Pull the gum side down to meet the other side of the clothespin on the table. Then release the arm of the clothespin to launch the gum.

Answer Problem 12-20 for the Gum Launch.
12. Have two timers time the Gum Launch starting at the release from the table and ending when it hits the ground.

Timer 1 $\qquad$
Timer 2 $\qquad$

Find the average time for the Gum Flight. $\qquad$
13. We can use the vertical motion formula to find the initial velocity of the Gum Launch:

$$
h(t)=-16 t^{2}+v_{0} t+s
$$

The height of the gum from the ground is $h(t)$ for some time, $t$.
What does $v_{0}$ represent?

What does $s$ represent? $\qquad$
14. Since the gum is being launched from a tabletop, measure the height of the table from the ground to the top. What is the initial height, $s$, in inches. $\qquad$
15. Convert the initial height in inches to feet. Remember that 1 foot $=12$ inches.
16. What is the height of the gum from the ground when the launch is finished, or when the stick of gum hits the ground? $\qquad$
17. Set $h(t)=0$ and substitute the average launch time, $t$, in the vertical motion formula:

$$
h(t)=-16 t^{2}+v_{0} t+s
$$

Find the initial velocity, $v_{0}$.
18. What is the time halfway through the flight? This is the $x$ - value of the vertex. Remember the vertex $\left(x_{v}, y_{v}\right)$ is $(t, h(t))$ in our Gum Launch problem.
19. The maximum height of the gum is the $y_{v}$, or maximum vertical displacement. Substitute $x_{v}$ or time, $t$, which is the time halfway through flight, into the equation $h(t)=-16 t^{2}+v_{0} t+s$ along with the initial velocity, $v_{0}$, and the initial height, $s$, and solve for $h(t)$.
20. Sketch the parabola of the Gum Launch. Put time on the $x-$ axis and height on the $y-$ axis. Label the maximum point and the $x$ - intercepts and the axis of symmetry.

## Section 6.11 Converting to Standard Form

Practice Problems 6.11
For Problem 1-4, use the given information to solve the problem. The equation for a ball thrown in the air is $h(t)=-4.25(t-2)^{2}+25$.

1. Is the vertex a minimum or a maximum height? Does the parabola open upward or downward? What is the vertex? (Height is in meters and time is in seconds.)
2. What is the $y$-intercept (starting point) of the ball?
3. When does the ball hit the ground?
4. Write the equation in standard form. What does the constant represent?

For Problem 5-8, convert the vertex form of the quadratic equation to the standard form.
5. $y=(x-3)^{2}+8$
6. $y=(x+6)^{2}+5$
7. $y=2(x+1)^{2}+3$
8. $y=-(x-2)^{2}$

For Problem 9-14, write the quadratic equation in standard form given the zeroes.
9. $x=9$ or $x-7$
10. $x=-9$ or $x=-5$
11. $x=5$ or $x=-2$
12. $x=4$ or $x=-3$
13. $x=-1$ or $x=4$
14. $x=0$ or $x=-6$

For Problem 15-22, find the vertex of the factored quadratic equation given:
Step 1: Factor the quadratic equation

Step 2: Find the zeroes using the Zero-Product Property

Step 3: Find the $x$-value of the vertex which is halfway between the zeroes

Step 4: Substitute the $x$-value in the quadratic equation and solve for the $y$-value of the vertex $\left(y_{v}\right)$
15. $y=5 x^{2}+5 x$
17. $f(x)=x^{2}+3 x+2$
19. $f(x)=x^{2}-2 x-15$
21. $f(x)=x^{2}+2 x-8$
16. $y=2 x^{2}-10 x$
18. $f(x)=x^{2}-x$
20. $f(x)=x^{2}+11 x+30$
22. $f(x)=x^{2}+4 x-5$

## Section 6.12 The Quadratic Formula

## Practice Problems 6.12

For Problem 1-5, identify the values of the terms $a, b$, and $c$ in the quadratic equation given.

1. $3 x^{2}-\frac{1}{3} x+2=0$
2. $-4 x^{2}-17=0$
3. $-4.8 x^{2}+22 x-3=0$
4. $-3 x^{2}+17 x=0$
5. $x^{2}-4 x+3=0$

For Problem 6 and 7, follow the instructions given to solve the problem.
6. Use the Quadratic Formula to find the zeroes of the equation $x^{2}+2 x-8=0$. Graph the $x$-intercepts and find and graph the vertex to connect the parabola.

7. Sketch a graph for each of the following situations that demonstrates discriminants:
a) The discriminant is negative $(-)$ :
b) The discriminant is positive $(+)$ :
c) The discriminant is 0 :

For Problem 8-12, tell how many real solutions the quadratic equation given has. Find the discriminant first.
8.

$$
x^{2}-12 x+36=0
$$

9. $x^{2}-7 x+10=0$
10. $3 x^{2}-6 x+3=0$
11. $2 x^{2}+7 x-15=0$
12. $2 x^{2}-3 x+4=0$

For Problem 13-18, use the Quadratic Formula to solve the quadratic equation given.
13. $2 x^{2}-2 x-40=0$
14. $4 x^{2}+6 x-108=0$
15. $x^{2}-7 x+10=0$
16. $2 x^{2}+7 x-15=0$
17. $x^{2}-8 x+16=0$
18. $3 x^{2}-6 x+3=0$

For Problem 19 and 20, solve the word problem given.
19. A cheetah pounces on his prey with an initial velocity of 49 feet per second. How long is the cheetah in the air if he lands on the back of his prey 4 feet above the ground? Use the equation $h(t)=-16 t^{2}+v_{0} t$ in which $v_{0}$ is the initial velocity of the cheetah and $t$ is time.
20. Mitch throws the discus at school. The equation of the throw is $h(t)=-16 t^{2}+90 t+6$. What is the initial height of the throw (in meters)? What is the initial velocity of the throw? After how many seconds does the discus hit the ground?

## Section 6.13 Using Functions for Graphic Design <br> Practice Problems 6.13

The picture of Mr. Swiharts Ace Flyer to be drawn below is made up of twelve numbered lines. Draw the lines of the picture that go with each equation. Program these lines into your calculator to see if the calculator will draw the design (tweak as necessary.)

Everything we see in God's universe can be written in mathematical code. Game designers use these codes to program animation.

Scientists and engineers use quadratic equations to find the projectile motion of rockets and the orbital path they follow.

After you find the equations for the rocket's travel along with the domain restrictions that limit the size of the lines, try your own graphic design. Please note that a vertical line through $x=4$ would have a range restriction if you only want it to be six units long. For example, $x=4\{-1 \leq y \leq 5\}$.

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Name $\qquad$ Picture $\qquad$

| No. | Function | Equation | Domain |
| :---: | :---: | :---: | :---: |
| 1 | Quadratic | $y=-2 x^{2}+5$ | $-1 \leq x \leq 1$ |
| 2 | Linear | $y=-3 x+6$ | $1 \leq x \leq 2$ |
| 3 | Linear | $y=-1 x+2$ | $2 \leq x \leq 5$ |
| 4 | Linear | $y=-3$ | $-5 \leq x \leq 5$ |
| 5 | Linear | $y=1 x+2$ | $-5 \leq x \leq-2$ |
| 6 | Linear | $y=3 x+6$ | $-2 \leq x \leq-1$ |
| 7 | Linear | $y=x-3$ | $-3 \leq x \leq 0$ |
| 8 | Linear | $y=-x-3$ | $\mathbf{0} \leq x \leq 3$ |
| 9 | Linear | $y=-6$ | $-3 \leq x \leq 3$ |
| 10 | Quadratic | $y=-x^{2}+2$ | $-1 \leq x \leq 1$ |
| 11 | Circle Top | $y=\sqrt{1^{2}-x^{2}}-5$ |  |
| 12 | Circle Bottom | $y=-\sqrt{1^{2}-x^{2}}-5$ |  |
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Use the graph paper below to draw your own emoticon or design. Make sure you end lines at points where the $x$ and $y$ axes meet. Number the lines of the design to match the numbers on the coding instructions sheet.


Use the lined paper below to write the equations that match the lines of your numbered design. Write the function as linear, absolute value, cubic, quadratic or circle. A horizontal line is $y=a$ and a vertical line is $x=a$ where $a$ is any number. Use domain restictions for the horizontal lines $\{a \leq x \leq b\} a$ and use range restictionsfor the vertical lines $\{a \leq x \leq b\}$ where $a$ and $b$ are real numbers.

| Name | Picture |  |  |  |
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## Section 6.14 Module Review

For Problem 1-4, solve the word/matching problem given.

1. Does the parabola of $y=-3 x^{2}+4$ open upward or downward? Why?
2. Match the equation with its form.
a) $a x^{2}+b x+c=y$
i. Factored Form
b) $(a x+c)(b x+d)=y$
ii. Standard Form
c) $a(x-h)^{2}+k=y$
iii. Quadratic Equation
d) $\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}=x$
iv. Vertex Form
3. What is the $y$-intercept of the equation $y=x^{2}+6$ ? Let $x=0$.
4. What is another name for the $y$-intercept in $y=x^{2}+6$ ? Is it a minimum point or maximum point?

For Problem 5-8, factor the quadratic equation given and use the Zero-Product Property to find the $x$-intercepts of the equation.
5. $y=x^{2}-1$
7. $y=x^{2}+x-20$
6. $y=x^{2}+6 x+9$
8. $y=2 x^{2}-2$
9. What is a special name for the binomial in Problem 5?
10. What is another name for the form of Problem 6 if the original equation is a square binomial?

For Problem 11 and 12, solve the word problem given which refers to a previous problem.
11. Use long multiplication to multiply the factors in Problem 7.
12. Use a geometric array to multiply the factors in Problem 8 which will demonstrate how to find the factors of the quadratic equation in Problem 8. Then use the Distributive Property to multiply the area by 2.

For Problem 13 and 14, use the Quadratic Formula to find the $x$-intercepts of the equation given.
13. $x^{2}+4 x-1$
14. $x^{2}-6 x+4$

For Problem 15-20, solve the word problem given.
15. The vertex of a quadratic equation is $(2,-6)$. Substitute the vertex in for the proper variables in the equation $y=a(x-h)^{2}+k$.
16. Now the equation from Problem 15 is $y=a(x-2)^{2}-6$. Another point on the parabola is $(0,5)$; substitute this point in for $x$ and $y$ in the equation and solve for $a$.
17. Now that you know $a, h$, and $k$, write out the equation for Problem 15 and 16 in vertex form.
18. Using the vertex and one other point on the parabola helps you find the vertex form of the quadratic equation. Convert the equation you found in Problem 17 to the standard form of the quadratic equation.
19. When a punter kicks a football, a quadratic equation is used to model the average height of the punt at $h(t)=-5.3(t-2.2)^{2}+26.5$ in which $t$ is time in seconds and $h(t)$ is height in yards.
a) How high is the ball from the ground when the punter kicks it? When $t$ is 0 , what is $h(t)$ or $h(0)$ ?
b) After how many seconds does the punt reach its highest point? How many seconds later does the highest point occur after the punt? What is the vertex of the punt?
c) How many seconds after it is kicked does the ball hit the ground? Find $h(t)=0$. That is how long the ball is in the air. It is called "hang time." Let $h(t)=0$ and solve for $t$. Use Square roots to answer the question.
20. Use the quadratic formula to find the $x$-intercepts of $y=2 x^{2}+7 x-3$.

## Section 6.15 Module Test

For Problem 1-3, solve the word/graphing problem given.

1. What is the vertex of the parabola in $y=(x-2)^{2}+5$ ?
2. What is the $y$-intercept of the parabola in $y=(x-2)^{2}+5$ ?
3. Using the vertex and the $y$-intercept, sketch the graph of the parabola: $y=(x-2)^{2}+5$


For Problem 4-8, describe the shifts of the quadratic equation from the parent function $y=x^{2}$.
4. $y=(x+3)^{2}$
6. $y=x^{2}+2.1$
7. $y=(x-4)^{2}$
8. $y=(x-5)^{2}+7.4$
9. What is the vertex of the parabola in Problem 8?
10. Does the parabola in Problem 8 open upward or downward? How do you know?

For Problem 11-14, factor the quadratic equation given. Use the Zero-Product Property to find the $x$-intercepts of the equation.
11. $y=16 x^{2}-9$
12. $y=x^{2}+6 x+5$
13. $y=x^{2}+4 x-45$
14. $y=x^{2}+x-6$

For Problem 15-20, solve the word problem given.
15. What is the vertex of the equation $y=x^{2}+4 x-45$ ?
16. The equation $y=x^{2}+4 x-45$ is in standard form; convert it to vertex form now that you know the vertex.
17. Use the Completing the Square method to factor $y=x^{2}+4 x-45$; is the equation the same as the vertex form in Problem 16?
18. The length of a rectangle is 1 meter more than its width. The area is 12 square meters.
a) Let $l=$ length and $w=$ width. Write an equation for the area of the rectangle in terms of its width.
b) Use the Quadratic Formula to solve for $w$.
c) What solution makes sense for the width of the rectangle? What is the length of the rectangle?

For Problem 19 and 20, use the Quadratic Formula to solve for $x$ when $y=0$.
19. $y=x^{2}-4 x$
20. $y=x^{2}+2 x-3$

