Module 3: Inverse Variation and Rational Expressions

Section 3.1 Inverse Variation Experiments

Looking Back 3.1

In the next module, we will learn about functions. We have already learned a great deal about linear functions. In this module, we will learn about another type of function, which is called an inverse (indirect) variation function.

In some linear functions, we learned that as the input increased at a constant rate, the output increased at a constant rate (positive slope). In other problems, as the independent variable increased at a constant rate, the dependent variable decreased at a constant rate (negative slope). Linear relationships that begin at $(0, 0)$ or cross through the origin are called direct variations. The equation for direct variations is $y = kx$ in which k is the constant of variation and x varies directly with y. That means they are directly proportional.

To find the constant of variation in a direct variation, solve $y = kx$ for k: $\frac{y}{x} = k$.

In the table below, k is the slope.

x	y	
1	2	$\frac{2}{1} = 2$
2	4	$\frac{4}{2} = 2$
3	6	$\frac{6}{3} = 2$
4	8	$\frac{8}{4} = 2$

The direct variation equation for the table above is $y = 2x$. When the quotient of y and x is constant, then x varies directly with y. When $y = \frac{k}{x}$, then x varies inversely with y. The variable k is still the constant of variation. To find the constant of variation (k), first isolate the variable k. Now, we have $xy = k$. From the table below you can see that k is 12.

x	y	$xy = k$
1	12	$1 \times 12 = 12$
2	6	$2 \times 6 = 12$
3	4	$3 \times 4 = 12$
4	3	$4 \times 3 = 12$

The inverse (indirect) variation equation for the table above is $y = \frac{12}{x}$.

When the product of x and y is constant, then x varies inversely with y.

Looking Ahead 3.1

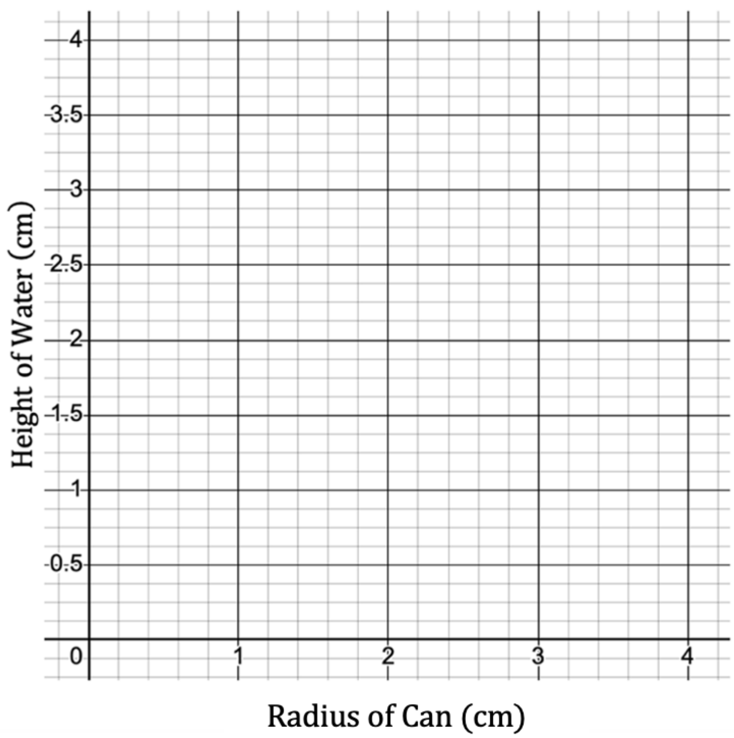
In this section, we will be doing another experiment that will provide a real-world example of an inverse variation to see what its graph looks like.

You will need the following materials:

- A tape measure, meter stick, or ruler
- Several cans of different sizes

1. Measure the radius of one can and record it in the table.
2. Fill a drinking glass half full of water and pour it into the can. Measure the height of the water and record it in the table (below).
3. Measure the radius of another can and record it in the table.
4. Pour the same amount of water from Step 2 in the can. Measure the height of the water and record it in the table.
5. Repeat Step 2 and Step 3 for several cans of different radii.
6. Graph the data with the measure of the radius of the can on the x-axis and the height of the water on the y-axis.

Radius of Can in cm (x)	**Height of Water in cm (y)**



Consider the following questions when looking at the graph:

- Why do you think the graph looks this way?

- Why do you think it is called an inverse variation?

- Does it go through the origin? Will the radius ever be zero?

- Will the water level ever reach zero?

- Suppose the diameter of the can was measured instead of the radius. Would the data look different or the same?

- What is the constant of variation? How can we find it?

These questions will be further explored in the Practice Problems section.

Section 3.2 Direct and Inverse Variation Problems

Looking Back 3.2

We know that a direct variation is a relationship between two variables in which the quotient is constant. The equation is of the form $y = kx$. The variable k is the constant of variation and comes from the quotient of y and x because $\frac{y}{x} = k$. Therefore, an inverse variation is a relationship between two variables in which the product is constant. The equation is of the form $y = \frac{k}{x}$. The variable k is the constant of variation and comes from the product of x and y because $xy = k$. In the equation $y = mx$, the constant of variation is also the slope of the linear function. So, we see the constant of variation (k) is the same as the slope (m) when $y = mx$ (given $b = 0$).

Looking Ahead 3.2

Example 1:

In a direct variation, the variables vary ____________________, meaning that as the value of x increases or decreases, y or $f(x)$ increases or decreases proportionally. If you remember, x represents the ____________________ and y or $f(x)$ represents the ____________________. What is the constant of proportionality for the direct variation tables below?

x	y
1	2
2	4
3	6
4	8

x	y
1	-3
2	-6
3	-9
4	-12

Example 2: In direct variations, we scale up or scale down by the same factor since x is directly proportional to y. Demonstrate this using the table below.

Example 3: In an inverse variation, the variables vary inversely, which means there exists a constant of proportionality. For the inverse variation $y = \frac{k}{x}$, when $k = 3$ then $y = \frac{3}{x}$. The constant of variation, or constant of proportionality, is 3. As x scales up by a given factor, then y scales down by the same factor. Or if x scales down by a given factor then y scales up by the same factor since $x$is inversely proportional to y. Demonstrate this using the tables below.

$$y = \frac{3}{x}$$

x	y
1	3
2	1.5
3	1

$$y = \frac{3}{x}$$

x	y
-1	-3
-2	-1.5
-3	-1

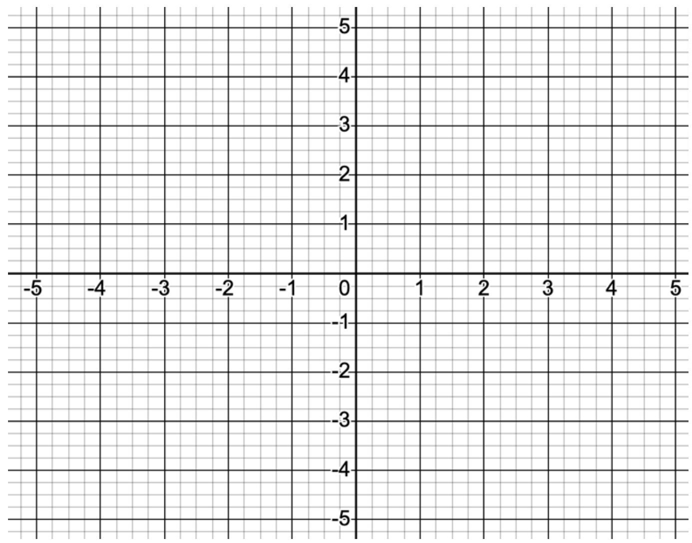
Example 4: Parent functions are where equations come from. For a linear equation, the equation is _____________________ in which $m = 1$ and $b = 0$. For a direct variation, the equation is _____________________ in which $k = 1$ and for an inverse variation it is _____________________ in which $k = 1$. As we add constants to these equations in locations, we develop families of functions around these parent functions. Other functions in the family are called the children.

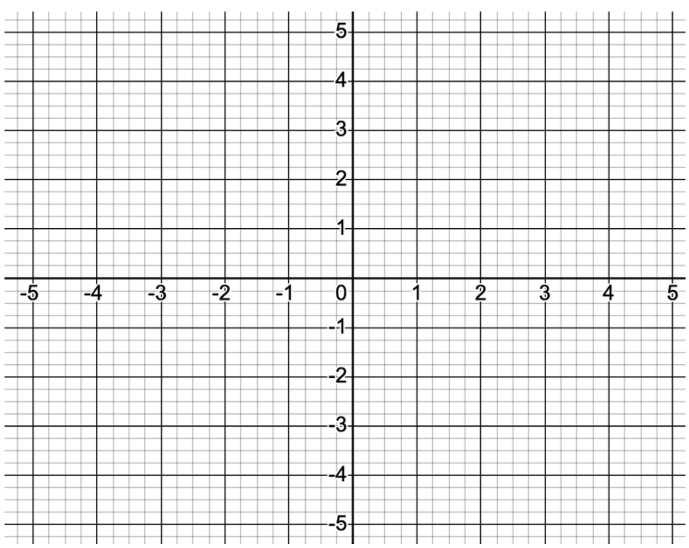
Make a table for the parent function of a direct variation and the parent function of an inverse variation and draw the graphs. Use the values $-3, -2, -1, 0, 1, 2, 3$ for x and let $k = 1$.

$$y = kx$$

$$y = x$$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



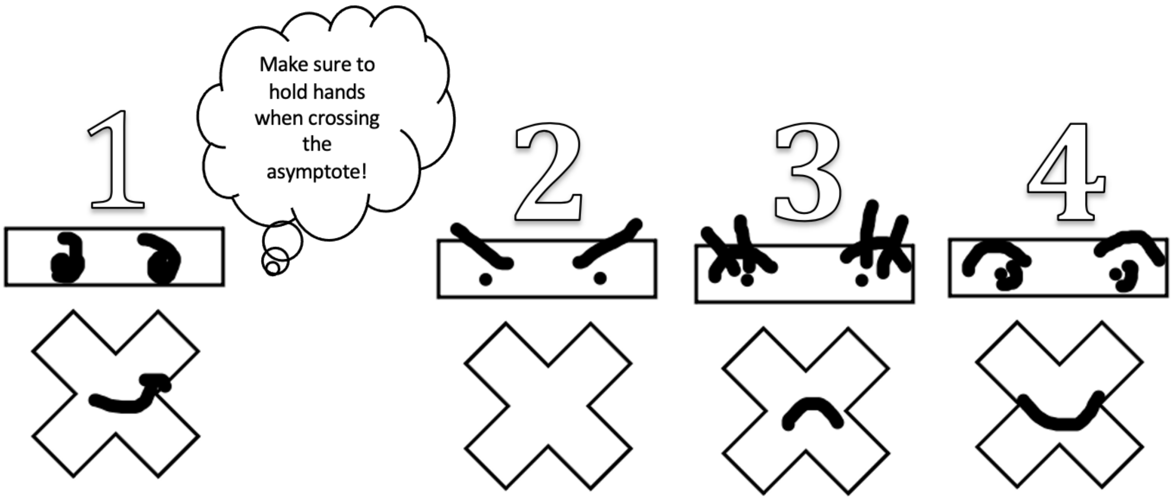


$$y = \frac{k}{x}$$

$$y = \frac{1}{x}$$

x	y
-3	
-2	
-1	
0	
1	
2	
3	

What does the parent function say to the children on the first day of school?



Example 5: The variables x and y are directly proportional when $x = 5$ and $y = 10$. Find the missing values for $(x, 14)$ and $(10.5, y)$. (Hint: Solve for k first.)

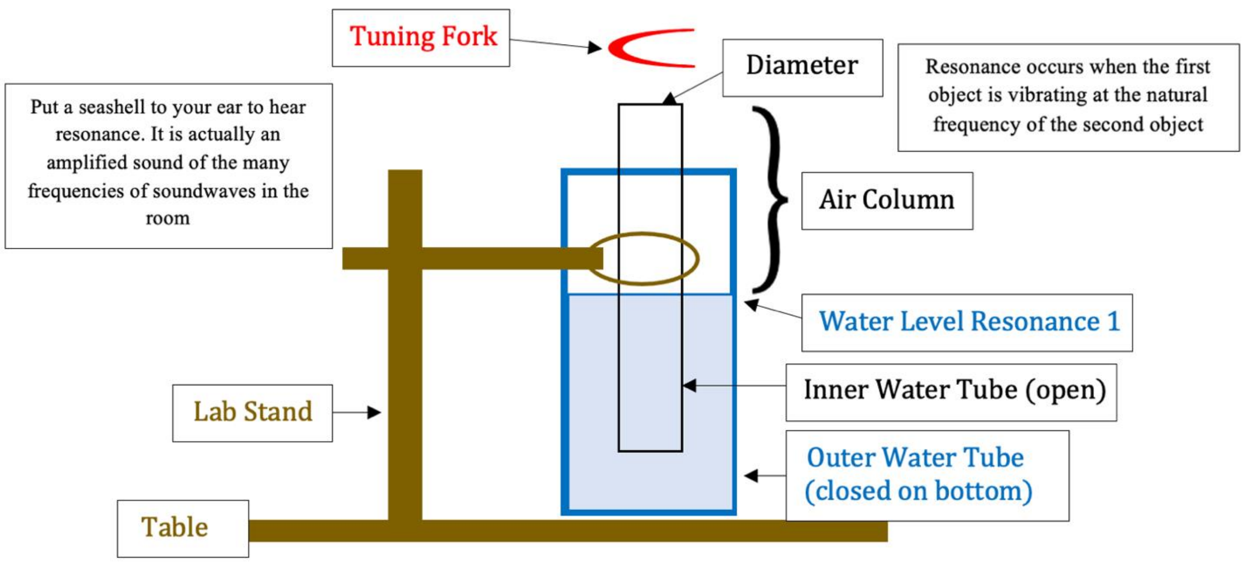
Section 3.3 Inverse Variation in the Real World

Looking Back 3.3

In the previous module, a tuning fork experiment was discussed in the Practice Problems. If you would like to buy a tuning fork and try this experiment, a tuning fork of 440Hz frequency is recommended. That would be the frequency of the musical note A_4.

Sound travels too fast to be measured by simple devices, but may be found indirectly by determining the wavelength of a known source. A tuning fork vibrates at a known and constant frequency and produces sound waves of the same frequency. There may be several resonant lengths in one tuning fork.

The wavelength of sound waves can be determined by holding the tuning fork over an air column of a glass tube that is open and slipped into another glass tube that is closed. To measure the wavelength of sound waves, move the tuning fork up and down until a resonance is found. It will be a very loud volume or increase in sound. The air is in resonance with the tuning fork. At that point, measure the length of the top of the innermost tube (L_1). You will be able to follow the instructions below to calculate the speed of sound in the air.



Looking Ahead 3.3

1. Set up the experiment according to the diagram above. Make sure the Inner Water Tube will easily move up and down. The Inner Water Tube may be glass or plastic as long as it is open at both ends. One end is above the water and the other end is in the water.

2. Strike the Tuning Fork with a rubber mallet. Move the Inner Water Tube up and down and the Tuning Fork up and down over the tube until you hear a loud sound. The sound heard is the resonance.

3. Measure in centimeters from the water level to the top of the tube and call this L_1, the length labeled "Air Column" in the diagram.

4. The length of the column previously described needs a small correction due to what is called the "end effect." Measure in centimeters the diameter (d) of the opening of the Inner Water Tube and multiply it by four-tenths. Then add this to the length measured, labeled "Air Column" (L_1), to get the corrected length (L).

$$L = 0.4d + L_1$$

Use the length of the resonance air column as the wavelength.

5. Do three trials of the experiment. Add the lengths together and divide by three to find the average length.

6. Do the calculations below:

Frequency of Tuning Fork: 400 Hz

Measured Resonant Lengths (L_1):

Trial 1: _______________ cm

Trial 2: _______________ cm

Trial 3: _______________ cm

Total: _______________ $\div 3 =$ ___________ (Average L_1 in cm)

d (diameter of tube): _______________ cm
Substitute the value for d and the average of L_1 to calculate the corrected length (L) in the formula:

The corrected length is $L = 0.4d + L_1$

7. The wavelength is four times the length of the air resonance column:

$$\lambda = 4(0.4d + L_1) \text{ or } \lambda = 4L$$

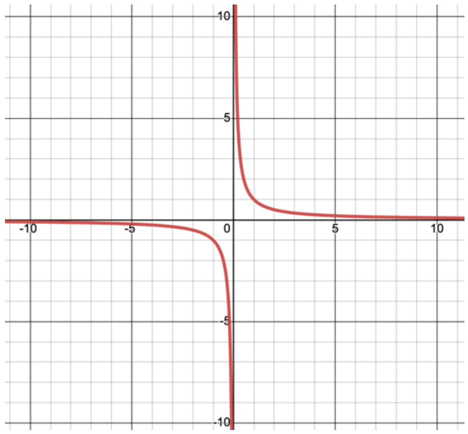
8. Now you can calculate the speed of sound with the formula $v = \lambda f$ using the frequency of the Tuning Fork and the calculated wavelength. The speed of sound or velocity is measured in meters per second. Use $100 \text{ cm} = 1 \text{ m}$ to convert centimeters to meters.

9. How close is your solution to 343 m/s, which is the estimated speed of sound in air at standard room temperature? To make a correction for this, look at your thermostat to see the temperature of your house. If your thermostat is in Fahrenheit, use the conversion formula that you derived earlier for converting Fahrenheit temperature to Celsius, $\frac{5}{9}(F - 32°) = C$. Now, use the equation for corrected velocity, or speed of sound in air, $v = 331.4 + 0.6T_c$ in which T_c is the room temperature in Celsius. How close is your solution to that?

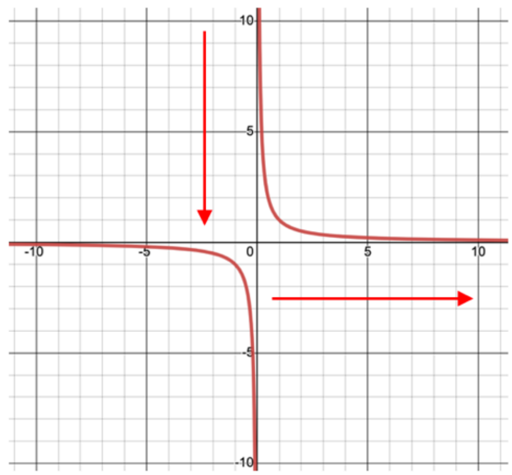
<u>Section 3.4 Graphs of Inverse Variation</u>

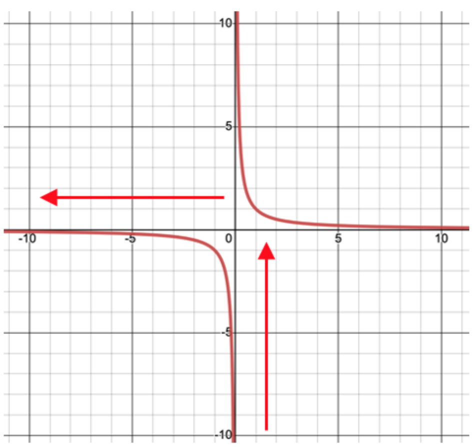
<u>Looking Back 3.4</u>

The graph of $y = \frac{1}{x}$ looks as follows:



As we substitute larger and larger values in for x, the fraction $\frac{1}{x}$ gets smaller: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$; the values of y decrease. On the graph, as x gets larger and larger, it moves to the right; as y gets smaller, it moves down. As x is increasing, y is decreasing; as x is moving right, y is moving down in the first quadrant (as the red arrows on the graph to the right indicate). The values of y will never be 0. This can be difficult to conceptualize but the red arrows may help.





As we substitute smaller and smaller values in for x, the fraction $\frac{1}{x}$ gets larger: $\frac{1}{8}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1$; the values of y increase. On the graph, as x gets smaller and smaller, it moves to the left; as y gets larger, it moves up. As x is decreasing, y is increasing; as x is moving to the left, y is moving up in the third quadrant (as the red arrows on the graph to the left indicate). The values of y will never be 0. This can be difficult to conceptualize but the red arrows may help.

Do the red lines (not the arrows) in the graph above ever touch the x-axis or y-axis?

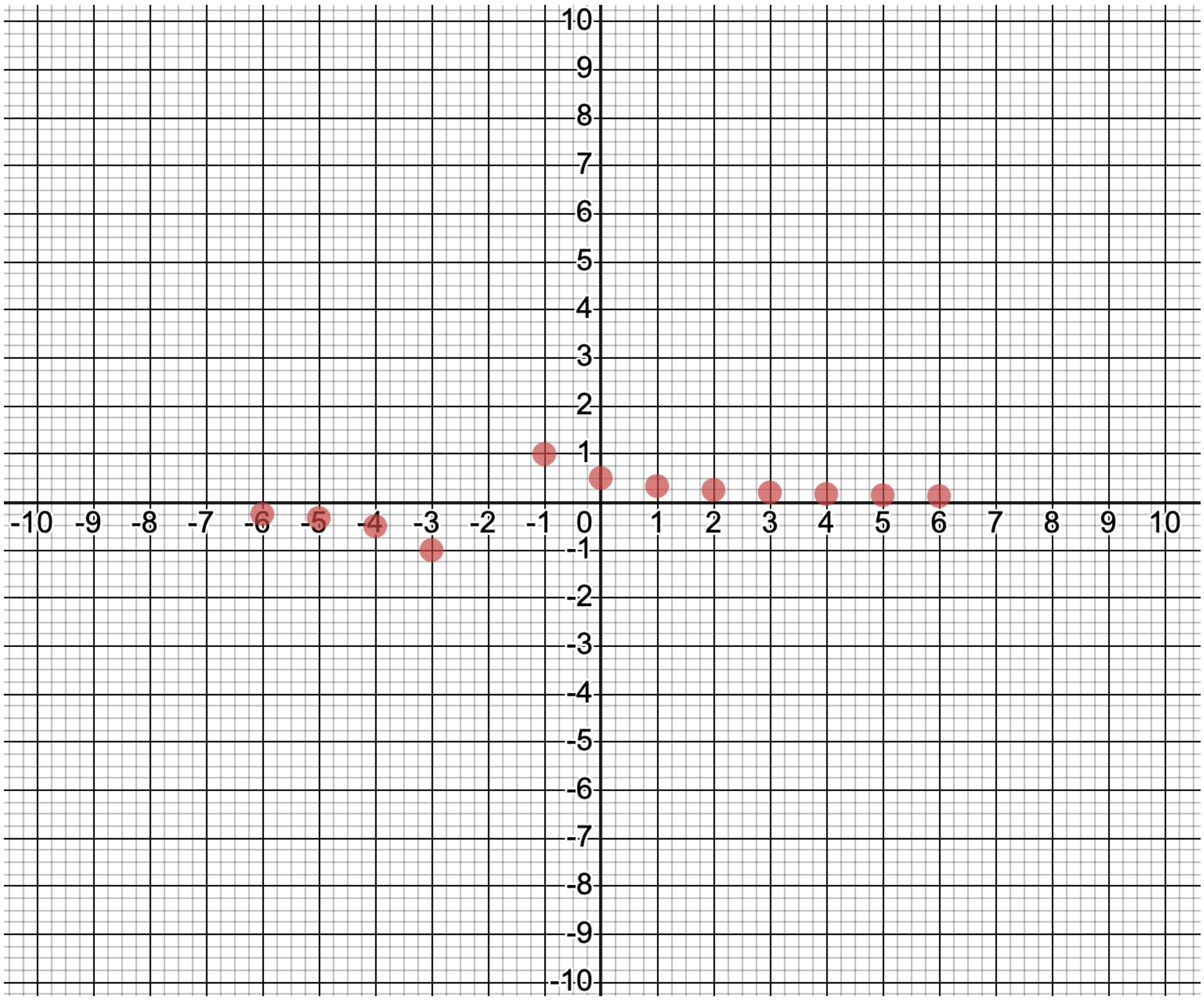
They do not. The x-axis and y-axis are called asymptotes. The values of x approach 0, but cannot be 0, as 0 in the denominator is undefined. The values of y will never be 0 because there must be a 0 in the numerator for that to happen; however, in this case, there is not a 0 in the numerator, there is a 1.

Looking Ahead 3.4

Example 1: The table below has been completed for x and y for the function $y = \frac{1}{x+2}$. What about the table stands out to you? Explain why.

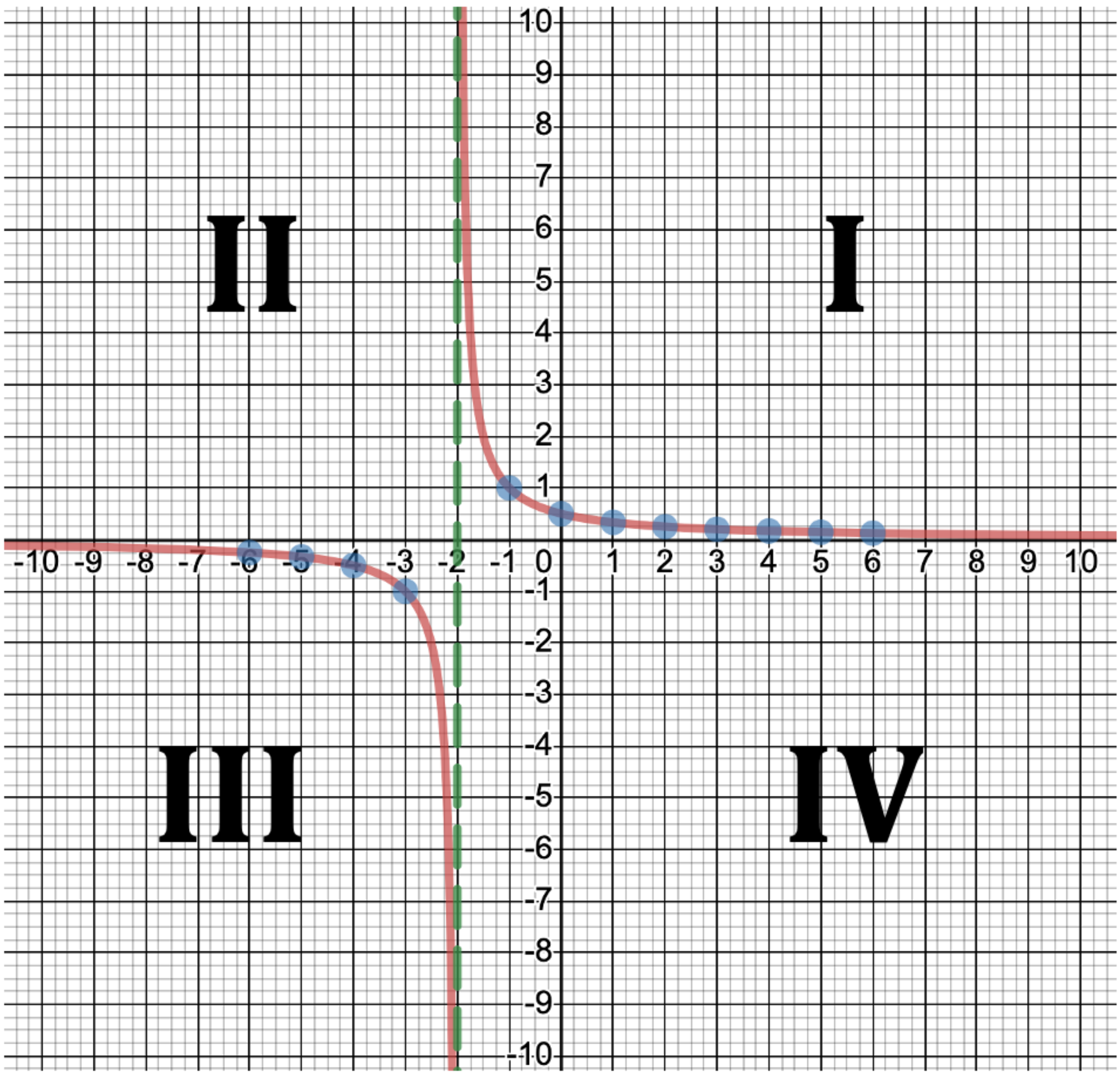
x	y	
-6	$-\frac{1}{4}$	$\frac{1}{-6+2} = \frac{1}{-4} = -\frac{1}{4}$
-5	$-\frac{1}{3}$	$\frac{1}{-5+2} = \frac{1}{-3} = -\frac{1}{3}$
-4	$-\frac{1}{2}$	$\frac{1}{-4+2} = \frac{1}{-2} = -\frac{1}{2}$
-3	-1	$\frac{1}{-3+2} = \frac{1}{-1} = -1$
-2	Undefined	$\frac{1}{-2+2} = \frac{1}{0} = \text{Undefined}$
-1	1	$\frac{1}{-1+2} = \frac{1}{1} = 1$
0	$\frac{1}{2}$	$\frac{1}{0+2} = \frac{1}{2}$
1	$\frac{1}{3}$	$\frac{1}{1+2} = \frac{1}{3}$
2	$\frac{1}{4}$	$\frac{1}{2+2} = \frac{1}{4}$
3	$\frac{1}{5}$	$\frac{1}{3+2} = \frac{1}{5}$
4	$\frac{1}{6}$	$\frac{1}{4+2} = \frac{1}{6}$
5	$\frac{1}{7}$	$\frac{1}{5+2} = \frac{1}{7}$
6	$\frac{1}{8}$	$\frac{1}{6+2} = \frac{1}{8}$

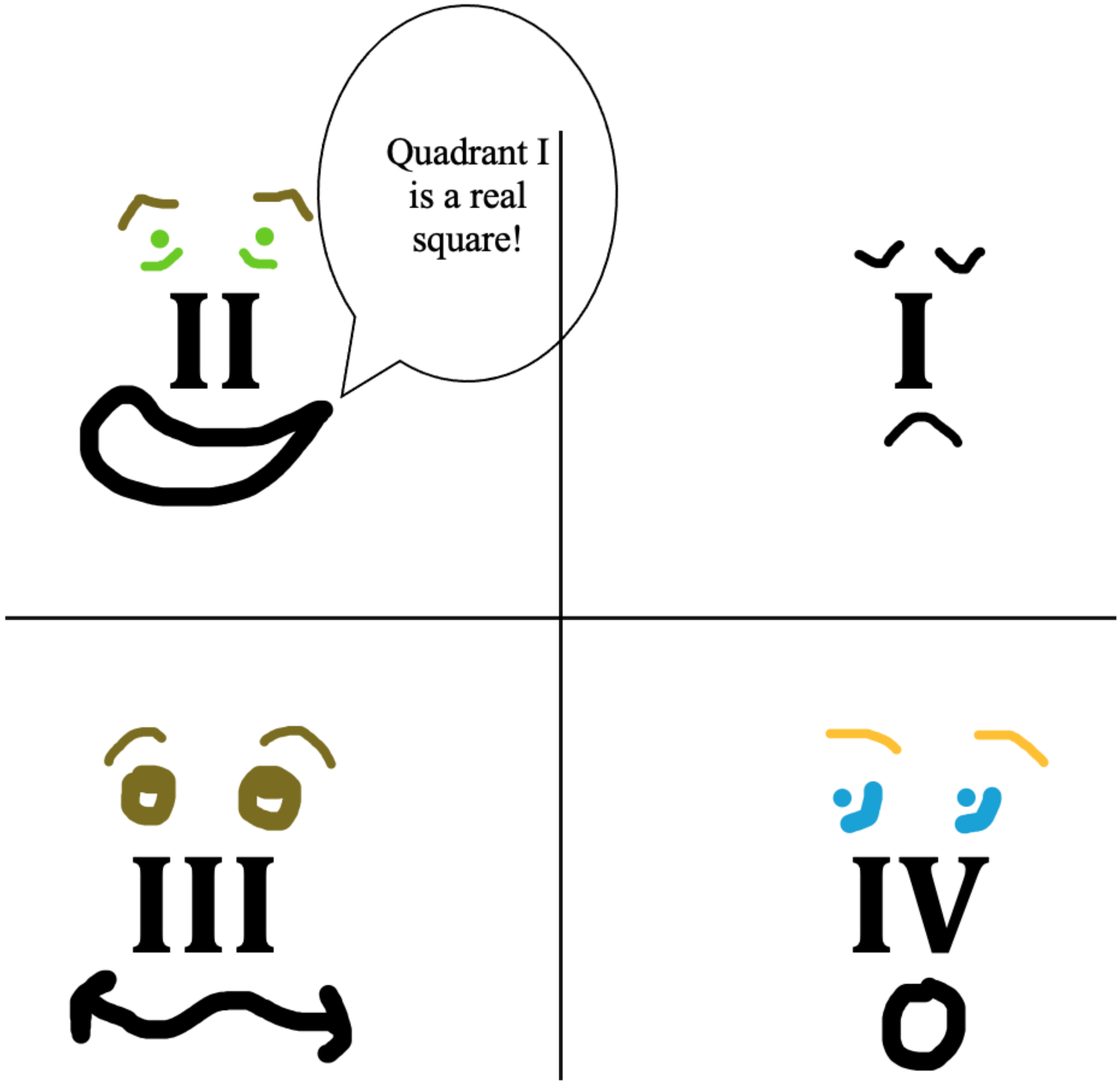
Example 2: The graph of the coordinates of $y = \frac{1}{x+2}$ is shown below. Draw dashed lines for the asymptotes and sketch the final graph.



Example 3: Fill in the blanks for the graph below:

The function is undefined at $x =$ ____________. There is a dashed line at $x = -2$. This is called a(n) _________________________________, which we will explore further in the next section. Because the graph is an inverse variation, make the shape similar to the shape of the parent function of an ___________________________ ___________________________ or $y = \frac{1}{x}$. The dots connect where the x increases. The dots connect where x decreases. The graph will come close to the asymptote $x = -2$ but will never _____________________ it. At $x = -2$, the equation and graph are _________________________________.




Quadrant I is a real square!
II
I
III
IV

Section 3.5 Horizontal and Vertical Asymptotes

Looking Back 3.5

Inverse variations are of the form $xy = k$ in which $k \neq 0$. In the equation $y = \frac{k}{x}$, the quantities represented by x and y are inversely proportional, and k is the constant of variation.

The graph of $y = \frac{1}{x}$ approaches both axes, x-axis and y-axis, but does not cross or touch either axis. The line that the graph approaches is called an asymptote. The axes are asymptotes.

In Example 2 and Example 3 of the previous section, the function $y = \frac{1}{x+2}$ still had a vertical asymptote on the x-axis ($y = 0$), but the horizontal asymptote changed to $x = -2$ because of the shift (transformation) from the parent function $y = \frac{1}{x}$.

Remember, the x-axis is the horizontal line $y = 0$ because all the y-values on the x-axis are 0. The y-axis is on the vertical line $x = 0$ because all the x-values on the y-axis are 0. In the previous Practice Problems section for Problem 10 and Problem 12, you graphed $y = \frac{1}{x-3}$. The graph had a vertical asymptote at $x = 3$ and a horizontal asymptote at $y = 0$.

Looking Ahead 3.5

Fill in the blanks:

$y = \frac{1}{x}$	$y = \frac{1}{x+2}$	$y = \frac{1}{x-3}$
Asymptotes:	Asymptotes:	Asymptotes:
$x =$ ____________	$x =$ ____________	$x =$ ____________
$y =$ ____________	$y =$ ____________	$y =$ ____________

What is similar? They each have an asymptote of $y =$ ____________. What is the difference? The vertical asymptotes are 0, -2, and 3. Where do you think these asymptotes come from? They are numbers that make the denominator ______________________ and make the function ______________________ at that point.

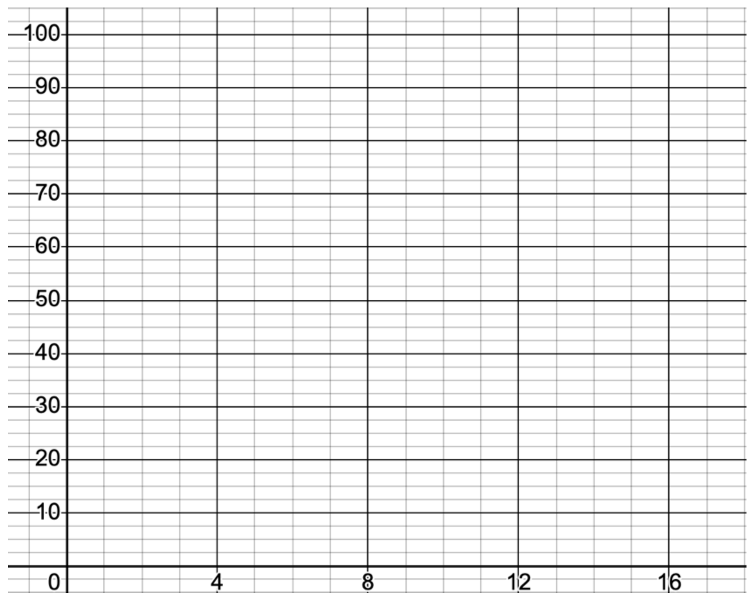
$y = \frac{1}{x}$	$y = \frac{1}{x+2}$	$y = \frac{1}{x-3}$
$y = \frac{1}{0}$	$y = \frac{1}{-2+2}$	$y = \frac{1}{3-3}$
Undefined at	Undefined at	Undefined at
$x =$ ____________	$x =$ ____________	$x =$ ____________

How does this affect the graph? At each of these points, there is a vertical ______________________________.

Example 1: If the elderly residents of a retirement community had to pay a group fee of $125.00 to visit the Audubon Society, then the individual fee could be derived from the function $y = \frac{125}{x}$. What would the individual trip fee be if lunch were included with each individual contributing an additional $6.00? How would this change the graph? The fee would still be the total amount divided by the number of individuals but with an individual $6.00 lunch fee included?

Let us call the first function y_1 and the second function y_2; $y_1 = \frac{125}{x}$ and $y_2 = \frac{125}{x} + 6$.

x	$\frac{125}{x}$	$\frac{125}{x} + 6$
16	$7.81	$13.82
14	$8.93	$14.93
12	$10.40	$16.42
10	$12.50	$18.50
8	$15.63	$21.63
6	$20.83	$26.83
4	$31.25	$37.25
2	$62.50	$68.50
0	Undefined	Undefined



a) What is the least amount of money a resident would pay if more than 125 residents attended?

b) What line does y_2 seem to be approaching?

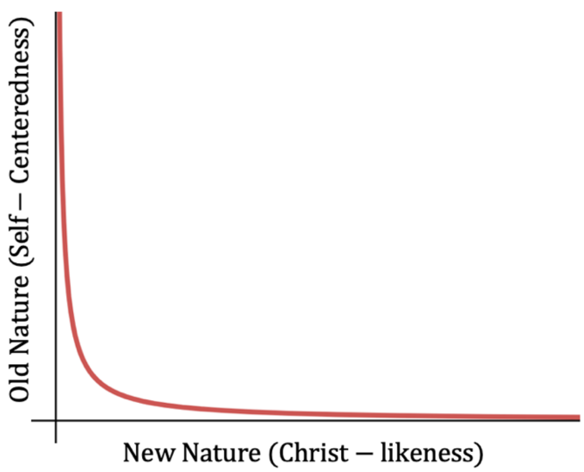
c) What is the horizontal and vertical asymptotes of $y = \frac{125}{x} + 6$?

d) Where do you find the constant in the equation? What does the constant represent?

Example 2:	Find the horizontal and vertical asymptotes for $y = \frac{6}{x-3} - 2$.

The expression $\frac{6}{x-3} - 2$ is called a rational expression. The equation $y = \frac{6}{x-3} - 2$ is called a rational equation. Remember a rational number may be written as a quotient $\frac{a}{b}$ where b is not equal to zero.

John 3:30 (ESV) says, "He must increase, but I must decrease." This inverse relationship describes our old nature- our self-centeredness, and our new nature- our Christ-likeness. The closer we get to God, and the more time we spend in Bible Study, prayer, and serving others, the more we will be like Him. As our Christ-likeness- our new nature increases, then our self-centeredness- our old nature, decreases. The graph would look as follows:



2 Corinthians 5:17 says, "Therefore, if any man is in Christ, he is a new creature; the old things passed away, behold all things have become new."

Section 3.6 Rational Expressions

Looking Back 3.6

We learned in *Pre-Algebra* that a rational number is any number that can be written as a fraction. A rational expression is also a fraction, but the numerator, the denominator, or both contain expressions that are polynomials. In short, a rational expression is a fraction with variables to an integer power. The integer exponent is a non-negative integer.

We know from the previous sections that when there is an expression in the denominator, the fraction will sometimes be undefined when certain values are substituted for the variable and the denominator becomes 0. We have found the asymptotes for undefined rational expressions.

Some examples of rational expressions would be as follows:

$$\frac{x^2-3x+2}{y+3}, \text{ or } \frac{x+y}{x^3y+3xy^2}, \text{ or } \frac{-4}{6x-2}, \text{ or } \frac{1}{x}, \frac{t}{3}, \text{ or } \frac{4}{5}$$

The same rules that apply to fractions apply to rational expressions. In this and the next few sections, we will be applying those rules in new ways.

Looking Ahead 3.6

Example 1:	Evaluate $\frac{-4}{6x-2}$ when $x = -2$.

It is helpful to be able to identify when a rational expression would be undefined. To do this, we set the denominator equal to 0 and solve for x.

Example 2:	Find the values of x that make $\frac{7x+2}{3x^2-12}$ undefined.

The values that make a rational expression defined are the domain of the rational expression.

Example 3:	Find the values of x that make $y = \frac{x^2-3x+2}{2x^2-32}$ undefined.

Example 4:	Simplify the following rational expressions:

a) $\frac{3x}{4x}$

b) $\frac{12r^2}{4r}$

c) $\frac{6y}{6y}$

Section 3.7 Simplifying Rational Expressions

Looking Back 3.7

Rational expressions are like fractions. However, the fractions are polynomials (monomials: one term; binomials: two terms; trinomials: three terms) in both the numerator and the denominator. You will learn more about polynomials in the next module.

A monomial is a real number (such as 4) or a variable (such as m) or the product of a real number and a variable (such as $4m$). The polynomial is a sum of monomials. Remember, subtraction is the same as adding a negative so that is how a difference is written as a sum.

The exponents of polynomials are whole numbers: $0, 1, 2, 3, ...$ The exponents of polynomials cannot be negative; for example, x^{-1} must be written as "$\frac{1}{x}$" (a negative cannot be in the numerator of a rational expression). The variable x^{-1} is not a monomial because the exponent is -1, which is not a whole number. If you remember, whole numbers begin with 0 and continue with positive values. The expression $\sqrt{x}$ is not a monomial because $\sqrt{x} = x^{\frac{1}{2}}$ and the exponent is $\frac{1}{2}$, which is not an integer.

If you want to add, subtract, multiply, or divide polynomials, the rules for exponents apply. You can review exponent rules in *Pre-Algebra*. We will look at a few examples in this section. More difficult examples will be viewed in Module 4 on polynomials.

Looking Ahead 3.7

Example 1: Simplify $\frac{21a^2b}{7a^3b}$.

Example 2: Simplify $\frac{x-1}{1-x}$.

BEWARE!!! Many people incorrectly think you can simplify $\frac{x-1}{x-1}$ term by term ($\frac{\cancel{x}-\cancel{1}}{\cancel{x}-\cancel{1}}$), but actually you cannot multiply or divide across an addition or subtraction sign. You can only multiply or divide terms that are being multiplied or divided, not added or subtracted. They must be like quantities in order to cancel. The binomial in the numerator cancels with the binomial in the denominator.

The following works because the binomial on the top cancels with the binomial on the bottom:

$$\frac{(x-2)^1}{(x-2)^1} = \frac{1}{1} = 1$$

Again, many people incorrectly think you can cancel term by term ($\frac{\cancel{x}-\cancel{2}}{\cancel{x}-\cancel{2}}$), but actually you cannot multiply or divide across an addition or subtraction sign because this would result in $\frac{1-1}{1-1} = \frac{0}{0}$, which is indeterminate.

Let us substitute 4 in for x to prove that $\frac{x-2}{x-2}$ is not the same as $\frac{x}{x} - \frac{2}{2}$.

Example 3: Simplify $\frac{3n-3}{6n-12}$.

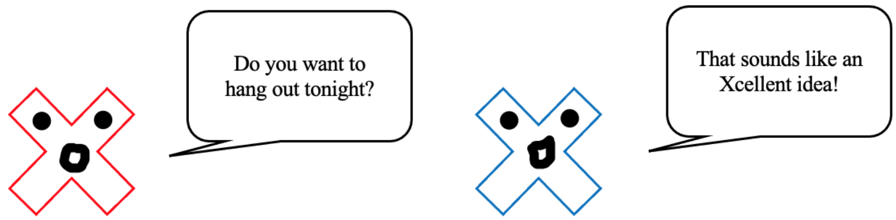
Section 3.8 Adding Rational Expressions

Looking Back 3.8

Because rational expressions are fractions with polynomials in the numerator and denominator, they follow the rules of fractions. If there is a common denominator, then the numerators may be added in order to be simplified.

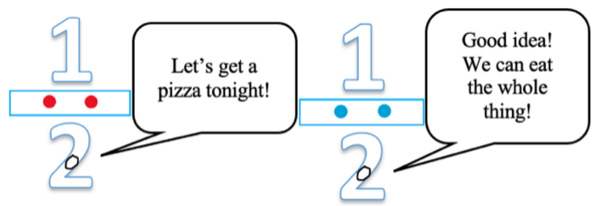
$$\text{If } \frac{1}{2}+\frac{1}{2}=\frac{2}{2}, \text{ then } \frac{x}{2}+\frac{x}{2}=\frac{2x}{2}=x.$$

The xs above both have coefficients of 1 and because the bases are the same, the coefficients are added. Numbers can be added to numbers, but variables may only be added together if they have the same base and same exponent.



Looking Ahead 3.8

Example 1: Add the rational expressions $\frac{x}{4}+\frac{y}{4}$. What is the common denominator?



Example 2: Add the rational expressions $\frac{x^2y}{5}+\frac{2xy^2}{5}+\frac{xy}{2}+\frac{x^2y}{5}$. Find the common denominator first.

Example 3: Add the rational expressions $\frac{3x+2y}{4} + \frac{5y}{8}$. What is the common denominator?

Example 4: Add the rational expressions $\frac{2}{3y} + \frac{2}{9x}$. Find the common denominator first.

Example 5: Suppose our solution to a problem was $\frac{6x+6y}{9xy}$. Could we simplify our solution?

Section 3.9 Subtracting Rational Expressions

Looking Back 3.9

The same rules for adding and subtracting numerical fractions apply to rational expressions. If the denominator is not a common denominator, then the Least Common Denominator must be found. The Least Common Denominator is found by converting the fractions to fractions with the same denominator, and then subtracting the numerators. However, the Least Common Denominator does not have to be used, though it will make your work simpler. If you do not use the Least Common Denominator, but rather a common denominator that is larger, you will have to simplify that in the end.

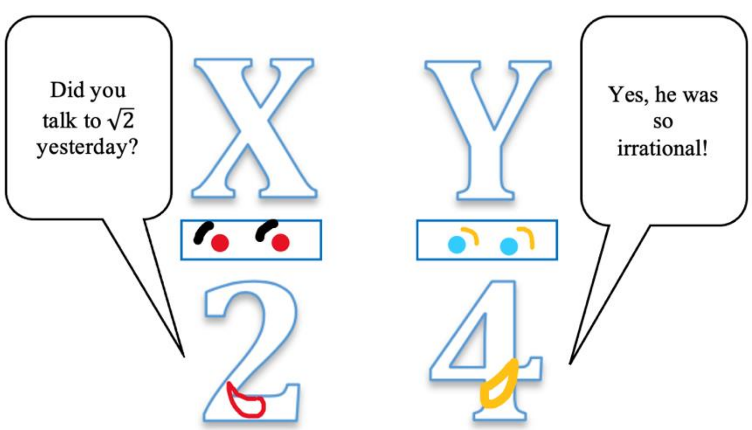
Subtracting rational expressions is more difficult than adding rational expressions and therefore, makes it easier to make mistakes when doing calculations; you solved one of these problems in the previous Practice Problems section, but in this section, subtraction will be the focus.

Looking Ahead 3.9

Example 1:	Is $\frac{3x+2}{4} - \frac{3x+5}{4}$ equal to $\frac{3x+2-3x+5}{4}$ equal to $\frac{7}{4}$?

Example 2:	Subtract the rational expression $\frac{3ab}{4a} - \frac{3ab}{4b}$. What is the common denominator?

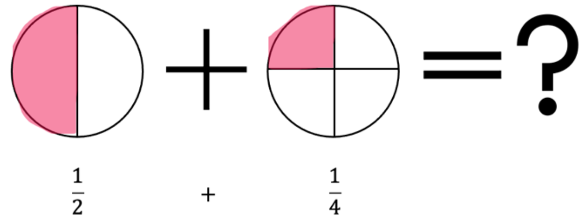
Example 3:	Subtract the rational expression $\frac{2}{d+3} - \frac{2}{d-2}$. Find the common denominator first.

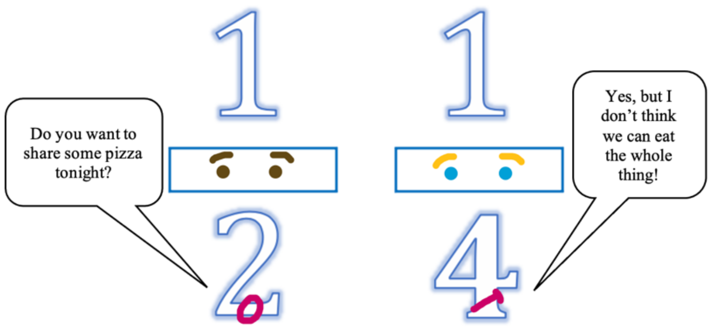


Section 3.10 Multiplying Rational Expressions

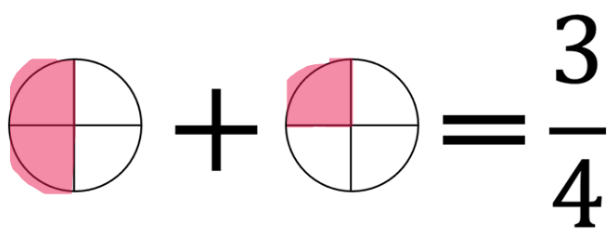
Looking Back 3.10

Again, because rational expressions are made up of variables and constants, the rules for numbers apply to rational expressions. In *Pre-Algebra*, we learned some rules about exponents and numbers. You may want to review those rules before beginning this section. These rules also apply for fractions: for addition and subtraction of fractions, the denominators must be the same! We cannot add $\frac{1}{2}$ to $\frac{1}{4}$ until we find a common denominator; they are different parts of the whole.

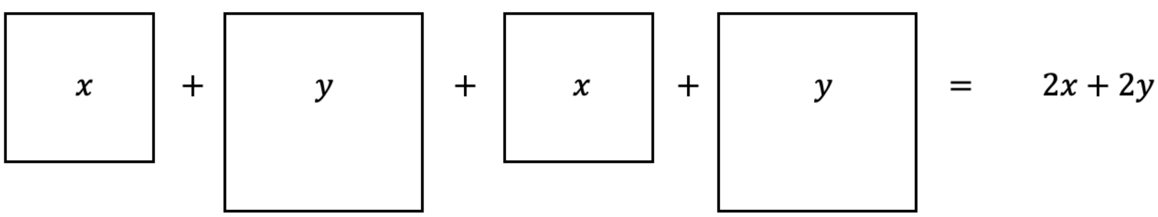




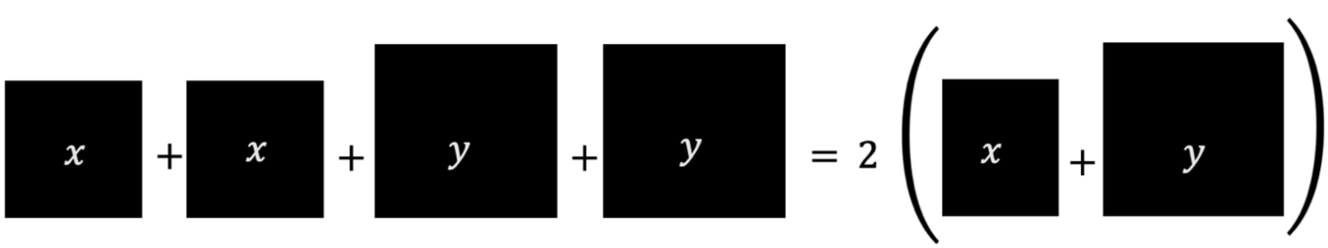
The fractions above are different parts of the whole. However, the $\frac{1}{2}$ can be cut into two $\frac{1}{4}$ pieces, then the parts are the same size and can be added together. Below is a graphic that shows an example of finding the common denominator.



Moreover, with variables, xs can be added to xs and ys can be added to ys, but xs cannot be added to ys.



We can factor $2x + 2y$ to $2(x + y)$, which is represented as follows:



However, we can multiply x and y and we will see the picture of what that looks like in the upcoming polynomials module. Variables can be multiplied just like halves and fourths.

$$\frac{1}{2}x \cdot \frac{3}{4}x = \frac{x}{2} \cdot \frac{3x}{4} = \frac{x(3x)}{2(4)} = \frac{3x^2}{8}$$

The numerators are multiplied, and the denominators are multiplied. Let us look again at the product of the two rational expressions $\frac{x}{3}$ and $\frac{y}{2}$:

$$\frac{x}{3} \cdot \frac{y}{2} = \frac{xy}{6}$$

Again, the numerators are multiplied by one another and the denominators are multiplied by one another.

<u>Looking Ahead 3.10</u>

Example 1:	Find the product of the rational expressions below.

$$\frac{6}{m^2} \cdot \frac{-4}{m^3}$$

Example 2:	Find the product of the rational expressions below.

$$\frac{2x+3}{4x^2+2x-6}(2x-2)$$

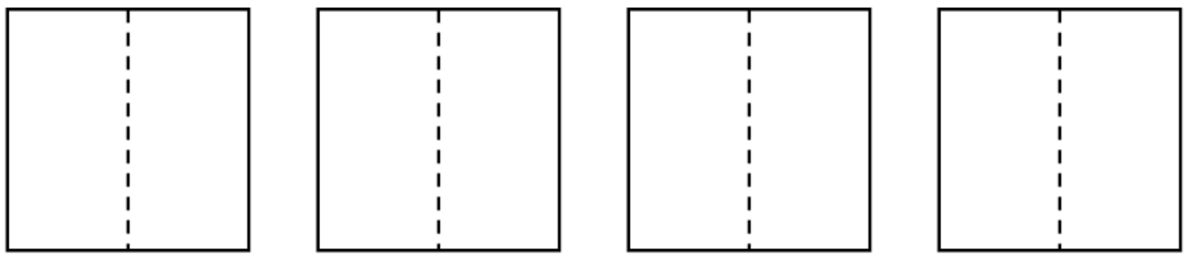
Section 3.11 Dividing Rational Expressions

Looking Back 3.11

Division of rational expressions, like multiplication of rational expressions, uses the same methods of simplifying as real numbers. The rules of exponents apply.

$$4 \div \frac{1}{2} = 8$$

The expression $4 \div \frac{1}{2}$ means 4 divided into $\frac{1}{2}$-pieces; there are 8 one-half pieces in 4 total objects.



The total number of pieces is doubled when each piece is cut in half. So, dividing 4 by $\frac{1}{2}$ is the same as multiplying 4 by $\frac{2}{1}$. Make the number 4 a fraction by putting it over 1. Change division to multiplication and take the reciprocal of $\frac{1}{2}$, which is $\frac{2}{1}$ (which is the same as 2).

$$4 \div \frac{1}{2} = \frac{4}{1} \cdot \frac{2}{1} = \frac{8}{1} = 8$$

In order to divide rational expressions, convert the division problem to a multiplication problem and take the reciprocal of the second expression (following the division sign), then follow the steps from the previous section to multiply and simplify the expression.

Looking Ahead 3.11

Example 1:	Divide the rational expressions below.

$$\frac{t^2 + t}{3t} \div \frac{1}{15t^3}$$

Example 2: Divide the rational expressions below and simplify.

$$\frac{6z+12}{5z} \div \frac{z+2}{10z^2}$$

Example 3: Divide the complex rational expressions and simplify.

$$\frac{\frac{-4}{4s^2+4}}{\frac{-8}{(s^2+1)(s^2+3)}}$$

Section 3.12 Simplifying Multi-Step Expressions

Looking Back 3.12

Sometimes rational expressions may involve multiple operations; in that case, the grouping symbols of multiplication or division, or exponents or parentheses are done first. This is followed by any addition and subtraction not in the grouping symbols. All multiplication and division can be done without finding common denominators. Before addition and subtraction, common denominators must be found first.

Looking Ahead 3.12

Example 1: Simplify the expression below.

$$\frac{x^2}{x+3} - \frac{x^2}{4x} \cdot (x+3)$$

Example 2: Simplify the expression below.

$$\frac{2+x}{x} + \frac{x-2}{y} \cdot \frac{4}{xy}$$

Example 3: Simplify the expression below.

$$\frac{y-2}{y} \div \frac{y^2}{4} + \frac{6-x}{x}$$

Mathematics can get very complicated, especially when fractions and negative signs are involved. You must be careful to complete each step in the proper order. You must also be careful to check your work along the way and when you are finished. Lastly, make sure the solution makes logical sense.

Life can get very complicated also. God tells us to be careful in what we do and that we follow the rules He has given us for our own good. Deuteronomy 8:1a (NASB) says: "All the ______________________________ that I am commanding you today you shall be ______________________ to do, that you may live and ______________________________ ..."

Section 3.13 Solving Rational Equations

Looking Back 3.13

To simplify means to write an expression in lowest terms and divide out all common factors. To solve means to find a solution. When finding a solution, there will be an equal sign and the solution is the value of the variable that makes the equation true. Again, the Least Common Denominator must be found because you must add or subtract the rational expressions first in order to combine them.

Looking Ahead 3.13

Example 1:	Solve the equation below.

$$\frac{7}{12} - \frac{1}{2m} = \frac{2}{3m}$$

Example 2: Using one size of pipe, a worker can fill an industrial tub in 4 hours. A pipe of another size can be used to fill the same industrial tub in 2 hours. If the two pipes are used to fill the tub at the same time, how long will it take to fill the tub?

Example 3: Solve the rational expression below.

$$\frac{6}{x+2} = \frac{6}{x-2}$$