Module 1: Linear Relationships

Section 1.1 Solving Multi-Step Equations

Looking Back 1.1

In *Pre-Algebra*, we learned how to solve equations starting with one-step equations and building up to multi-step equations with variables on both sides. Mastering skills in *Algebra 1* is essential to being successful in higher mathematics, and at the top of the list of these skills is solving equations.

Looking Ahead 1.1

When solving equations or inequalities with variables on both sides, the easiest way is to combine all the like terms on each side of the equation or inequality before starting to actually "undo" the equation or inequality.

1. Simplify the parenthesis and exponents on each side of the equation.
2. Combine like terms on each side of the equation.
3. Move variables to one side of the equation and numerals to the other side of the equation.
4. Solve for the variable by isolating the variable.
5. Check your solution.

Example 1: Solve $3x - 6 + 7x + 18 = 12x - 3 + 6x + 23$.

Example 2: Solve $3(x+6)+5(2x-3)=-3(x+3)+4x$.

Let us review what we know about a "$-$" sign in front of parentheses.
$-a$ means the opposite of a
$-(x+3)$ means the opposite of the binomial, $x+3$, or $-x-3$.

It might be easier to see if you use the distributive property.
Put a "1" in after the negative sign: $-1(x+3)$.
When we distribute, we have $(-1)(x)+(-1)(3)$ or $-1x+(-3)$ or $-x-3$.

To summarize, a negative sign in front of parenthesis changes the sign of each term in the parenthesis.

Example 3: Solve $5(1-5x)-(-8x-2)=4x-8x+33$.

Section 1.2 Solving Equations with Exponents and Radicals

Looking Back 1.2

In *Pre-Algebra*, we learned that $(\sqrt[n]{x})^n$ is equal to x or that $\sqrt[n]{(x)^n}$ is equal to x, which means exponents and roots are inverses of each other. That fact helps us to solve equations that have exponents or roots in them.

All the rules we reviewed in Section 1 still apply. However, if the exponent or root is a quantity, then we treat it as one number.

Looking Ahead 1.2

Using inverse operations, let us try a few examples.

Example 1:	Solve for x in $x^2 = 81$.

The inverse of x-squared is the square root of x. To solve, we take the square root of both sides of the equation. Remember, there are two solutions: a positive number and a negative number.

Example 2:	Solve for x in $x^3 = -27$.

The inverse of x-cubed is the cube root of x. We also know that because x^3 is negative, our solution must be negative. There is only one solution.

Example 3:	Solve for y in $\sqrt[4]{y} = 5$.

Example 4:	Solve for x in $(x+2)^2 = 9$.

Remember, the square root of anything squared is itself.

Example 5:	Solve for x in $\sqrt[5]{(x+6)} = 2$.

Remember, the fifth root of anything to the fifth power is itself.

Example 6:	Solve for a in $\sqrt{-16+10a} + 5 = 13$.

Subtract the 5 from both sides first because it is outside of the radical sign.

Example 7:	Solve for x in $(x-3)^3+4=12$.

Example 8:	Solve for x in $6\sqrt{2x-4}=24$.

Section 1.3 Solving Distance-Rate and Mixture Problems

Looking Back 1.3

We have solved a lot of different equations and word problems with differing degrees of difficulty. With word problems, even setting up the equations can be difficult. In this section, we will look at two types of word problems: distance-rate problems and mixture problems. These can be confusing at first, so to help us in solving these types of problems we will use a table to help organize the information.

Looking Ahead 1.3

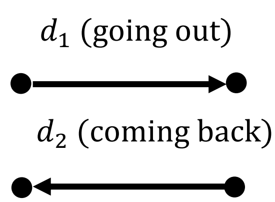
For the distance-rate problems, we use the formula $rate \cdot time = distance$. There are generally three types of distance-rate equations.

One type of distance-rate problem involves two objects moving towards each other and meeting or moving in the opposite direction and traveling a certain distance apart.

$$d_1 + d_2 = d_T$$

This can be used to find the total distance (d_T).

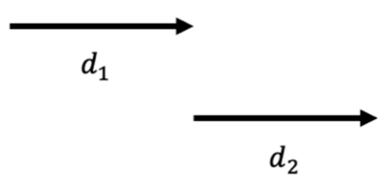
A second type of distance-rate problem involves two objects moving out to a certain point and traveling the same distance to return to the start point.



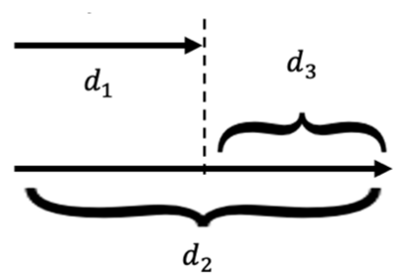
$$d_1 = d_2$$

This can be used for finding rates or time when the distances are the same.

A third type of distance-rate problem involves two objects moving in the same direction but starting at different times or moving in the same direction at different rates. This is often seen in situations in which one object catches up with or overtakes another object.



$$d_1 + d_2 = d_T$$



$$d_2 - d_1 = d_3$$

These can be used to find total distance or differences in distance traveled.

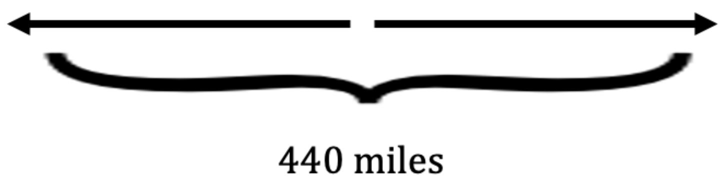
Another thing that you can do is construct a table like the one below, which helps you to organize the information in the problem.

$r \cdot t = d$	Rate	Time	Distance
Object 1			
Object 2			

Example 1: Fletcher and Jennifer left Englewood for Spring Break. Fletcher traveled west at a rate of 60 mph. Jennifer traveled east at a rate of 50 mph. At what time will they be 440 miles apart?

Start by making a sketch of the situation and setting up a table.

The sketch looks like this:



$r \cdot t = d$	Rate	Time	Distance
Fletcher			
Jennifer			

Example 2: Jesse and Taylor also went on Spring Break, but they were spending their Spring Break working at a church camp for underprivileged children. Taylor left for the camp driving the church bus full of equipment and other workers. Jesse, late as usual, left 2 hours after Taylor, but was traveling 20 mph faster than Taylor because she was driving a car. Find the rate of each driver if Jesse caught up with Taylor after 3 hours.

$r \cdot t = d$	Rate	Time	Distance
Taylor			
Jesse			

Example 3: A rescue team needs to get to the site of an accident that is 160 miles away as quickly as possible. They drive to the nearest airport at a rate of 50 mph. From there, they take a helicopter that travels at a rate of 60 mph. The total trip takes 3 hours. How long did they travel in the helicopter?

$r \cdot t = d$	Rate	Time	Distance
By car			
By helicopter			

Mixture problems basically involve blending two items to get a new blend. It could be peanuts and cashews to get a nut mixture. It could be combining two solutions of different strengths to get a new solution at a different strength (percent). For each of the final mixtures or solutions, you multiply the amount by the price per unit or percent to get the total value. A table can help with a mixture problem as well.

	Amount	Percent/Cost Per Unit	Total
First Item			
Second Item			
Final Mixture			

Example 4: The Nut Shoppe sells a nut-mixture of peanuts and cashews. How many pounds of peanuts that cost $1.75 a pound must be mixed with 7 pounds of cashews that cost $4.00 a pound to get a mixture that costs $2.75 a pound?

	Amount	Cost Per Unit	Total Value
Peanuts			
Cashews			
Mixture			

$$\text{Total Value of Peanuts} + \text{Total Value of Cashews} = \text{Total Value of the Mixture}$$

Example 5: For some lab tests, Susan needs 10 liters of 15% solution of peroxide. The supplier only sells 10% and 30% solution of peroxide. The cost to have them make a 15% solution is pretty expensive, so Susan decides to make her own. How much of the 10% and 30% solution does she need?

	Amount (L.)	Percent Per Unit	Total Liters
10% Peroxide			
30% Peroxide			
Final Mixture			

Section 1.4 Solving Literal Equations

Looking Back 1.4

We have solved a lot of equations. We have rearranged a standard form equation ($Ax + By = C$) to a slope-intercept form equation ($y = \frac{-A}{B}x + \frac{C}{B}$). Because A, B, and C are parameters and x and y represent variables, we cannot solve for a numerical value for x and y. All we can do is solve for y in terms of x or x in terms of y.

In *Pre-Algebra*, we learned about the math triangle. That is a simplified form of solving literal equations. For example, the distance formula ($r \cdot t = d$) could be written as $t = \frac{d}{r}$ or $r = \frac{d}{t}$.

Example 1: Suppose you are going to travel 400 miles and you are trying to decide when to leave to make it to a wedding on time. It depends on who is driving: Grandma goes 45 mph; Marilyn likes to go fast, she goes 70 mph; Sue goes 60 mph. Tell the time it takes each to get to the wedding. Because you are solving for t for each traveler, you can use the literal equation in which you solved for t in terms of d and r.

Looking Ahead 1.4

Literal equations are equations with several variables. You do not solve literal equations for a numerical value; you solve for one variable in terms of another. That means when you solve it, the one variable you are solving for is on one side by itself; all the other variables are on the other side.

Example 2: Solve the formula $P = IRT$ for T.

Example 3: Solve the equation $2x - 3y = 8$ for y.

Example 4: Solve the equation $A = h(b + c)$ for b.

Example 5: Solve the formula below for r. What is the radius of a cylinder when the height is 10 cm and the volume is 300 cm^3?

$$V = \pi r^2 h$$

Section 1.5 Direct Variation

Looking Back 1.5

Linear equations are so fundamental to algebra that once they are explored they must be analyzed. The remainder of the module will be devoted to an in-depth analysis of linear equations.

If 8 bottles of water cost $4.40, then 1 bottle of water is $\$4.40 \div 8 = \$0.55$ each. The ratio is shown below:

$$\frac{\$4.40}{8} = \frac{\$0.55}{1}$$

$$\$4.40(1) = \$0.55(8)$$

$$\$4.40 = \$4.40$$

When one quantity is directly proportional to another quantity, as above, the ratio of the quantities is the same (constant). To find the cost of a certain number of bottles of water, the number of bottles would be multiplied by the cost of each bottle.

Number of Bottles of Water	Total Cost of Bottles of Water
1	$\$0.55 \cdot 1 = \$0.55$
2	$\$0.55 \cdot 2 = \$1.10$
3	$\$0.55 \cdot 3 = \$1.65$
…	…
8	$\$0.55 \cdot 8 = \$4.40$
x	$\$0.55 \cdot x = \$0.55x$

The expression for the cost of any given number of bottles of water is $\$0.55x$.

If y represents the total cost, then y is equal to that expression and the equation is $y = \$0.55x$.

The equation $\frac{y}{x} = \$0.55$ is the same as $\frac{y}{x} = \frac{\$0.55}{1}$. In the equation, y corresponds to $0.55 and x corresponds to 1. The variable y stands for the cost in dollars, which corresponds to $0.55, and x is the number of bottles, which corresponds to 1.

The ratio $\frac{y}{x}\left(\frac{\text{Total Cost}}{\text{Bottles}}\right)$ is equal to $\frac{\$0.55}{1}\left(\frac{\text{Total Cost}}{\text{Bottles}}\right)$ or $0.55 per bottle.

This is called the unit rate. The unit rate here is the cost of one item. Now that you know the unit rate, you can calculate the cost of any number of bottles of water.

A Boeing 767 passenger aircraft once had to make an emergency landing because the pilot used metric units to chart the flight, but the ground crew that navigated the plane used English units to chart the flight and the plane ran out of gas while in the air. That is why scientists and mathematicians use measurements within the same system so everyone knows just what the numbers mean. This could have been a very costly mistake as passengers (people) cannot be replaced!

Looking Ahead 1.5

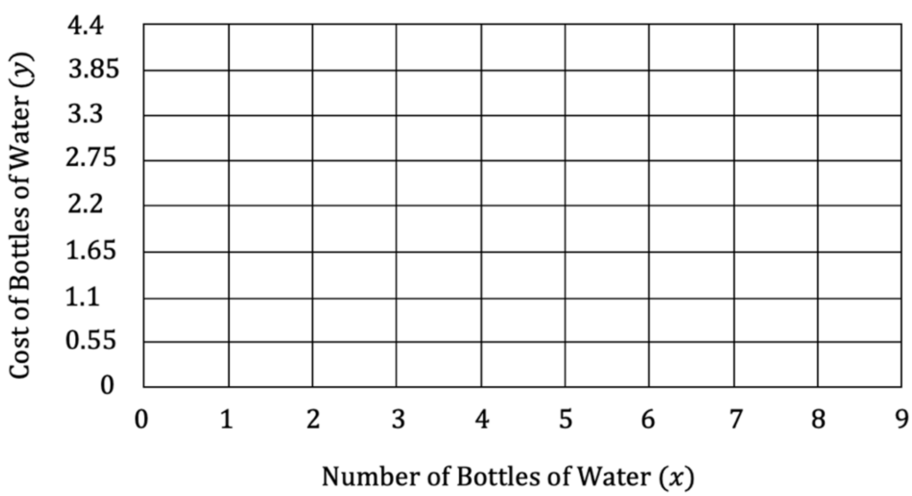
Example 1: Solve for y in the proportion $\frac{y}{x} = \frac{\$0.55}{1}$.

In the water bottle example above, $0.55 is called the constant of proportionality. The variable used to represent the constant of proportionality is k; $y = kx$ is called a direct variation. This is because y varies directly with x. As x scales up by a scale factor, y scales up by the same scale factor, or as x scales down by a scale factor, y scales down by the same scale factor.

In a direct variation, the ratio of the two variables is constant. Therefore, the two variables are directly proportional. In the equation $y = kx$, x and y are the variables (they vary or change) and k is the constant (the numerical value that stays the same for a given equation).

The graph can also be investigated. Let x represent the independent variable, the number of bottles of water, and let y represent the dependent variable, the total cost of the bottles of water.

Example 2: Complete the graph that represents the table for the total cost of bottles of water based on number of bottles of water purchased.



Section 1.6 Direct Variation Equations In the Real World

Looking Back 1.6

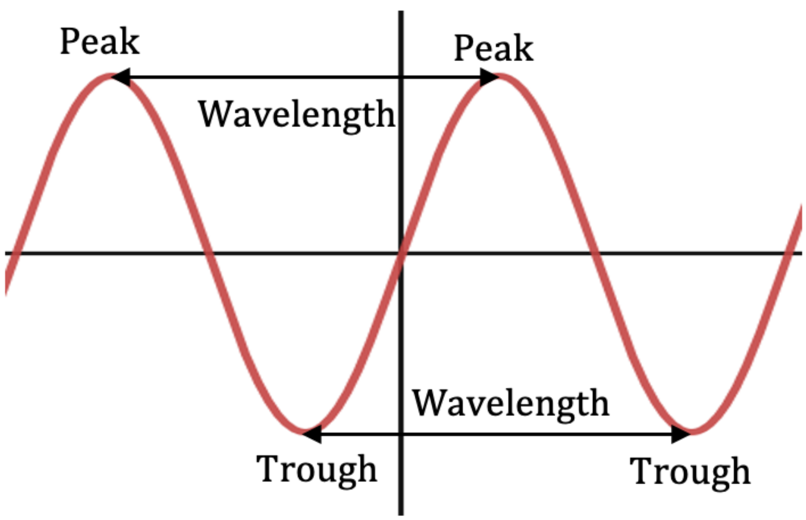
Long ago there lived a mathematician named Pythagoras. Pythagoras and his brotherhood of Pythagoreans believed numbers were *all* and that numbers *ruled the universe*. The brotherhood adhered to rituals including rote prayers and the mystical use of symbols. Legend has it that after discovering the famous Pythagorean Theorem, 100 oxen were sacrificed. In truth, this order in numbers does not come from the numbers themselves but rather from God, God who created the world in six days and gave us 24.14 hours of day from the first day.

In arithmetic sequences, patterns develop and can be seen in the principles of science applied to God's universe.

Looking Ahead 1.6

This activity involves waves. If you have access to a slinky and you have a partner, hold one end while your partner moves the other end up and down and you will generate transverse waves.

The frequency of a wavelength is the number of cycles per second. The unit of measure of frequency in one second is called a Hertz. The wavelength is the repeat distance of a wave from trough to trough (low point to low point) or peak to peak (high point to high point). This is sometimes called crest to crest. A transversal wave is made by moving the slinky up and down as opposed to longitudinal waves made by moving the slinky back and forth.



The symbol for wavelength is λ (the Greek symbol lambda). The symbol for frequency (measured in Hertz-Hz) is f.

Let us consider the formula for distance: $\text{distace} = \text{rate} \times \text{time}$ ($D = r \cdot t$). If the rate is the speed then $\text{speed} = \frac{\text{distance}}{\text{time}}$. In science the formula is sometimes given as $\text{speed} = \frac{\text{distance traveled}}{\text{time traveled}}$.

There is a difference between speed and velocity. Speed tells us how quickly an object moves but velocity tells us how quickly an object moves and in what direction it moves. The formula for velocity of a wave is $v = f\lambda$, velocity equals frequency times wavelengths.

Example 1: In the slinky experiment above, Barb and Donna use a slinky with a 1-inch diameter of zinc coils to measure wavelength and frequency. Next, they measure wavelength and frequency of a slinky with a 3-inch diameter zinc coil. What are the speeds of Barb and Donnas' slinky and what do Barb and Donna conclude from the experiment? The wavelength is measured in meters and the frequency is measured in $\mathrm{m/sec}$ or Hz.

(zinc coils)	$\boldsymbol{\lambda}(\mathbf{m})$	$\boldsymbol{f}(\mathbf{Hz})$	$\boldsymbol{V}(\mathbf{m/sec})$
1-in diameter	1.75	2.0	3.5
1-in diameter	0.90	3.9	3.5
(zinc coils)			
3-in diameter	0.95	2.2	2.1
3-in diameter	1.82	1.2	2.1

Example 2: Donna (from Example 1) said she had a copper coil slinky that was 1-inch in diameter as well. Barb said if they did the same experiment, the velocity should be 3.5-m/sec like the zinc coil with a 1-in diameter.

They measured the wavelength and frequency, then multiplied them to get the velocity and put the results in a table. What changed Donna and Barbs' conclusion?

(Copper Coil)	$\boldsymbol{\lambda}(\mathbf{m})$	$\boldsymbol{f}(\mathbf{Hz})$	$\boldsymbol{V}(\mathbf{m/sec})$
1-in diameter	1.19	2.1	2.5
1-in diameter	0.60	4.2	2.5

There is another mathematician to discuss here. His name is Eudoxos of Cridus. Eudoxos lived in the same century as Pythagoras. He contributed greatly to proportional theory; because we are studying direct variations, which are directly proportional, and will soon study inverse variations, which are inversely proportional, it is appropriate to appreciate his contribution to this field now.

Eudoxos' work on ratios was foundational to *Volume V* of *Euclid's Book of Elements*. Furthermore, he was a Greek astronomer and philosopher as well and attended Plato's school. Eudoxos adhered to Plato's philosophy that planets rotated around the earth (not the sun). Eudoxos was the greatest mathematician of antiquity at that point in time, but was later surpassed by Archimedes.

We do not have any of Eudoxos' actual writing in existence today, but many of his geometric proofs have survived through Euclid's writings and others.

Just as the Bible before the printing press, the copies of the Torah (or Pentateuch) authored by Moses are not in writing today in Moses' hand. However, we do have copies penned by the scribes and those appointed by God and preserved through other writers. There are 24,000 extant (still in existence) copies of scripture that can be found in museums around the world today. The next highest amount of copies from antiquity is Homer's *Illiad* and *Odyssey*, which has only 150 extant copies.

Homer was blind. He did not write down his tales of Greek mythology. He told the stories orally (by mouth). Then they were copied down by others and passed down through history and are still studied in schools today. These, however, are myths. The stories in the Bible are true and much evidence of the internal, external, and bibliographic texts are available for verification.

Section 1.7 Arithmetic Sequences

Looking Back 1.7

Linear equations have a constant slope. This constant rate of change is a common difference. Look at the table from Section 1.5. The constant ratio of total cost of bottles of water to number of bottles of water is as follows:

$$\frac{\$0.55}{1} = \frac{\$1.10}{2} = \frac{\$1.65}{3} = \frac{\$2.20}{4}$$

The common difference of total cost of bottles of water to number of bottles of water is as follows:

$$\$0.55 - 0 = \$1.10 - \$0.55 = \$1.65 - \$1.10 = \$2.20 - \$1.65$$

	Number of Bottles of Water (B)	Total Cost of Bottles of Water (C)	
	0	0	
$+1$			$+0.55$
	1	$\$0.55$	
$+1$			$+0.55$
	2	$\$1.10$	
$+1$			$+0.55$
	3	$\$1.65$	
$+1$			$+0.55$
	4	$\$2.20$	

If B represents the number of bottles of water and C represents the total cost of bottles of water, the equation for the total cost of water given the number of bottles purchased is $C = \$0.55B$. If we want to graph this, we would let x equal B (the independent variable) and y equal C (the dependent variable). The equation in terms of x and y is $y = \$0.55x$. Do not let the variable names cause confusion. The important thing is to understand what the variables represent.

In the table above, $\$0.55$ is the slope of the equation $y = mx$ (generally), and $y = \$0.55x$ (specifically). The slope is the change in y over the change in x; in this case, $\frac{\$0.55}{1}$. Because the line passes through the origin, $(0, 0)$, this is also a direct variation, and y varies directly with x. In the equation $y = kx$, the k, or constant of variation, is $\$0.55$. Therefore, $\frac{y}{x} = \$0.55$ (specifically); $m = \$0.55$ and $k = \$0.55$. It is both the slope and the constant of variation.

Looking Ahead 1.7

Now, let us take it a step further. The slope $m = \frac{y}{x}$ or the constant of proportionality $k = \frac{y}{x}$ here is the common difference between any two numbers following one another on the right side of the table. The slope or constant of proportionality is being added to the previous number to get the next number for y since the number of bottles is increasing by 1 for x. We call this an arithmetic sequence. A sequence is an ordered list of numbers that form some sort of pattern. It is arithmetic when something is being added each time.

In the example given, the sequence is: $\$0$, $\$0.55$, $\$1.10$, $\$1.65$, $\$2.20$, … Each term of the sequence has a term number and a term value. Above is the value of each term. The value of the initial term (term zero) is $\$0$; the value of the first term (term one) is $\$0.55$. The terms are numbered 0, 1, 2, 3, 4, …

From these two things (term number and term value) come tables, graphs, equations, and dependent and independent variables. The ratio or rate of change is also known as the first common difference. Linear equations are to the first power (degree one) and their first common difference is constant. That is part of the definition of a linear equation- the rate of change is constant.

Value of the Term ($a(n)$)	\$0	\$0.55	\$1.10	\$1.65	\$2.20
Term Number (n)	0	1	2	3	4

If n represents the term number, then $a(n)$ represents the value of the term number in the arithmetic sequence. So, when $n = 3$, then $a(3) = \$1.65$. When $a(n) = \$2.20$, then $n = 4$ because the fourth term in the sequence has a value of \$2.20. Therefore, n is the term number of the sequence and $a(n)$ is the value of the term at that point in the sequence.

Example 1: Answer the questions below using the table above for bottles of water.

a) If $n = 5$, what is $a(5)$?

b) If $a(n) = \$1.10$, what is n?

c) If $n = 5$, what is $a(n - 3)$?

d) What is the arithmetic sequence for $a(n)$?

Example 2: Use the table below to answer the questions below.

$\boldsymbol{n}$	0	1	2	3	4	5
$\boldsymbol{a(n)}$	6	9	12	15	18	21

a) What is $a(4)$?

b) What is n if $a(n) = 12$?

c) What is the common difference?

d) What is the constant rate of change?

e) What is $a(6)$?

f) What is the arithmetic sequence for $a(n)$?

Section 1.8 Recursive Formulas for Arithmetic Sequences

Looking Back 1.8

In the previous section, an arithmetic sequence was defined as a list of numbers that form a pattern in which the difference between each pair of values of consecutive terms is the same. A table can be created from a sequence in which the input value is the term number, or order in the sequence list, and the output value is the value of the term itself.

If n is the term number, $a(n)$ represents the value of the n^{th} term in the arithmetic sequence. This applies to any sequence, including multiplicative sequences that will be studied in Module 8. For functions, these input and output values are called the domain and range of the function.

A function rule that relates each term of the sequence after the first one to the one before it is called a recursive formula. Using a recursive formula, any number in a sequence can be found as long as the number before it is known. Just apply that rule to the number to find the next number. The word recursive means to occur or appear again, especially after an interval. So, a recursive formula means the rule is being applied over and over for each term of the sequence.

In arithmetic sequences, something is being added each time. If a negative number is being added, then subtraction occurs. Addition and subtraction are inverse operations so that is recursive as well.

Looking Ahead 1.8

Consider this sequence: 4, 10, 16, 22, 28, 34, … The common difference of the terms can be used to write an arithmetic sequence.

Let n be the term number in the sequence
Let $a(n)$ be the value of the n^{th} term in the sequence
Let d be the common difference

Therefore:
The value of term 1 is: $a(1) = 4$
The value of term 2 is: $a(2) = 10$
The value of term 3 is: $a(3) = 16$
The value of term 4 is: $a(4) = 22$
The value of term 5 is: $a(5) = 28$
The value of term 6 is: $a(6) = 34$

The value of any term in the arithmetic sequence is the value of the previous term plus the common difference. If n is the term, then $n - 1$ is the term before it. So, the value of the previous term is $a(n-1)$. To add the common difference of 6, write $a(n-1) + 6$. The recursive formula for any number, n, in this sequence is the number before it, $a(n-1)$, plus 6. This can be written: $a(n) = a(n-1) + 6$.

Notice, $n - 1$ is the previous term in the sequence (the one before any term) but $a(n-1)$ is the value of that previous term. The term is the input. If the value of $a(7)$ is to be found, then $a(n) = a(n-1) + 6$ is the recursive formula that must be used.

Example 1: Answer the questions below.

a) What is $a(7)$?

b) What is $a(n-3)$ given $n = 4$?

c) How would the recursive formula change if the common difference is -2?

d) What is the recursive formula for any common difference, d?

Example 2: Find the recursive formula for the arithmetic sequence below. What is the value of the fifth term?

$$27, 24, 21, 18, \ldots$$

Example 3: The chemical symbol for water is H_2O. We learned previously that one molecule of water is made up of 2 atoms of hydrogen and 1 atom of oxygen. Find the pattern to determine the number of atoms in 20 molecules of water. What is the recursive formula for the arithmetic sequence? Why is it difficult to use here?

Section 1.9 Explicit Formula for Arithmetic Sequences

Looking Back 1.9

A recursive formula can be used to find any value of a term in the sequence as long as the term before it is known. This can get quite cumbersome. If the value of the 103^{rd} term is the one to be found, then the value of the 102^{nd} term must be known. That is a long list of numbers. There is an easier way to find the value of any term. It can be done using an explicit formula. (Explicit means clearly and precisely expressed.) For example, $y = 5x + 8$ is explicit in which y is the dependent variable and is dependent on the independent variable x.

Any term of a sequence can be found without knowing the previous term of the sequence. In the explicit formula, as in the recursive formula, the function rule relates each term number to the term value. The explicit formula takes us to the output directly for any given input.

Looking Ahead 1.9

Looking at the sequence that follows, a table can be made:

Term or Term Number	**Term Value**
1	10
2	15
3	20
4	25
5	30
6	35

The common difference is 5. The rule is to add 5 to the previous number. By the 3^{rd} number, 5 has been added two times to the initial number, and by the 4^{th} number, 5 has been added three times to the initial number. There is a pattern here that will help us to write an explicit formula.

Term or Term Number	**Term Value**		
1	10	10	$10 + 5(0)$
2	15	$10 + 5$	$10 + 5(1)$
3	20	$10 + 5 + 5$	$10 + 5(2)$
4	25	$10 + 5 + 5 + 5$	$10 + 5(3)$
5	30	$10 + 5 + 5 + 5 + 5$	$10 + 5(4)$
6	35	$10 + 5 + 5 + 5 + 5 + 5$	$10 + 5(5)$

Because repeated addition is multiplication, by the 6^{th} term, 5 has been added five times to the initial number.

Let us see how this relates to the term number if the term is n:
When $n = 1$, the common difference is multiplied by 0.
When $n = 2$, the common difference is multiplied by 1.
When $n = 3$, the common difference is multiplied by 2.

Whenever d (the common difference) is multiplied by 1 less than the term number $(n - 1)$, we have an explicit formula. The general explicit formula for arithmetic sequences is given below:

$$a(n) = a(1) + (n - 1)d$$

value of the n^{th} term value of the 1st term previous term common difference

Example 1: Using the table at the beginning of the Looking Ahead portion (shown below), find $a(7)$ using the explicit formula.

Term or Term Number	**Term Value**
1	10
2	15
3	20
4	25
5	30
6	35

Example 2: Find the explicit formula for the sequence below. Find d and $a(1)$.

$$22, 29, 36, 43, 50$$

The common difference is ______________________________, $d =$_______________

The first term is ______________________________, $a(1) =$____________________

The explicit formula for this example is ____

Example 3:	Find $a(3)$ using the explicit formula in Example 2.

Example 4:	Find $a(100)$ using the explicit formula in Example 2.

Example 5:	Write the explicit formula for the sequence below. Then find the 10th term using the explicit formula.

$$9, 6, 3, 0, -3, -6$$

Section 1.10 Connecting Recursive and Explicit for Arithmetic Sequences

Looking Back 1.10

The recursive formula is helpful when programming a calculator or computer to generate a list.

The explicit formula is very handy for finding the value of any term in a sequence without writing the entire list of terms in the sequence. The previous number to any term may not be known and is not necessary when using the explicit formula. All that is needed is the first term, the common difference, and the number of the needed term.

Given enough information, it is possible to convert one to the other.

n is the term we want $n-1$ is the previous term

The recursive formula for the arithmetic sequences is $a(\boldsymbol{n}) = a(n-1) + d$.

The explicit formula for the arithmetic sequence is $a(\boldsymbol{n}) = a(1) + (n-1)d$.

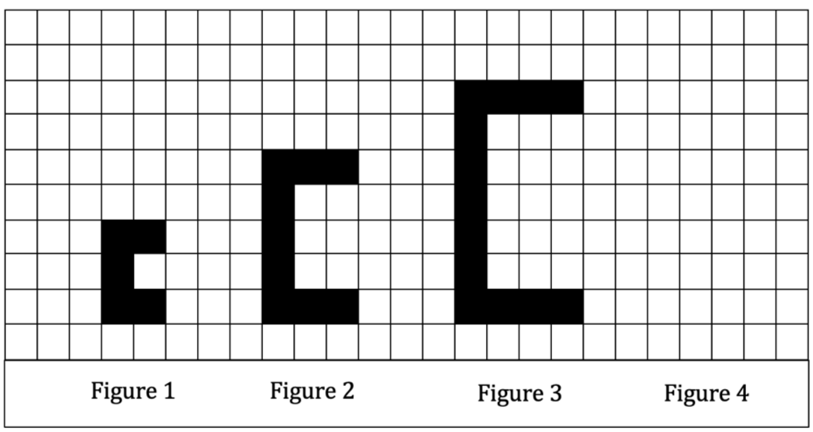
$a(1)$ is the first (initial) term d is the common difference

If the recursive formula is $a(n) = a(n-1) + 5$, then we know the common difference is 5. If we know $a(1) = 3$, then we can write the explicit formula, which is $a(n) = 3 + (n-1)5$.

If the explicit formula is $a(n) = -5 + (n-1)3$, then we know the common difference is 3. Also then, we can write the recursive formula, which is $a(n) = a(n-1) + 3$.

Looking Ahead 1.10

Look at the pattern of C blocks below:



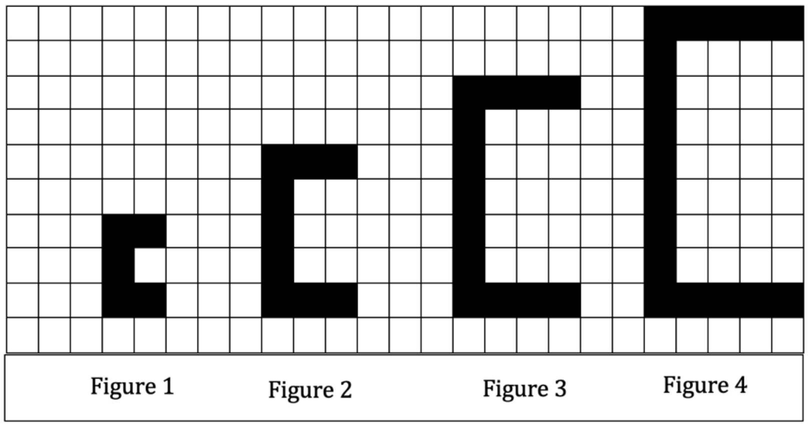
a) Can you see a pattern along the sides of the C?

b) Do you see patterns in the total number of squares in each C?

c) Can you draw a Figure 4?

Let us make a table in which x is the figure number and y is the total number of squares in the figure and find the common difference.

x	y
1	
2	
3	
4	

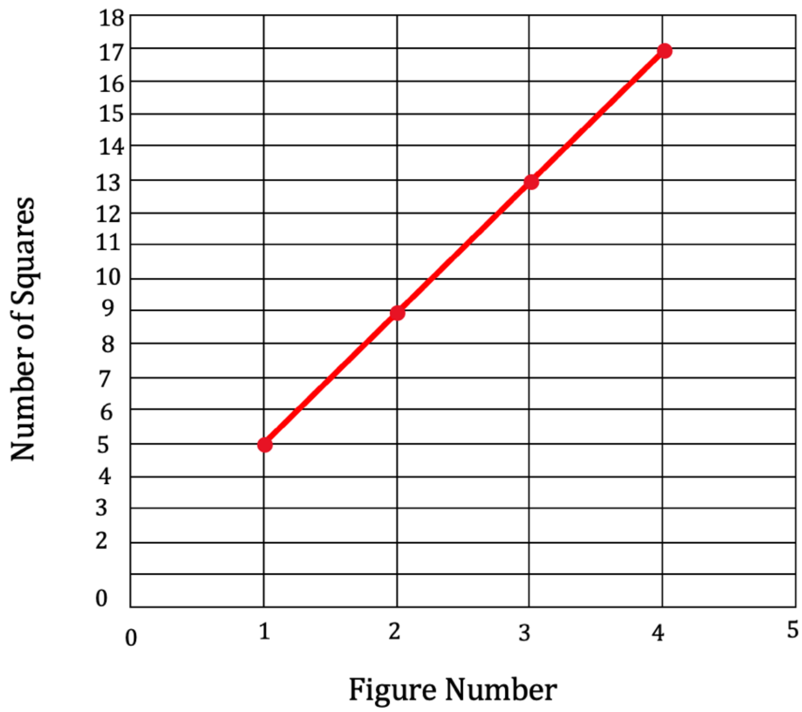


We have noticed patterns that can be made into rules.
The initial term is our start value. In these figures, it is labeled as Figure 1. Therefore, your start value is $a(1)$, our first term. In these figures, $a(1) = 5$. This will be used in the explicit formula.
Our recursive formula for the figures is $a(n) = a(n-1) + 4$. In these figures, $d = 4$.

The figure number represents the term number. The total number of squares is the term value.

Term Number	Figure Number	1	2	3	4
Term Value	Number of Squares	5	9	13	17

Let us graph this table.



a) What is the slope of the line?

b) What is the relationship between the slope and the common difference of the graph?

c) Use the distributive property to simplify the formula for the arithmetic sequence.

$$a(n) = a(1) + (n - 1) \cdot 4$$

Notice the equation in c) is of the slope-intercept form $y = mx + b$ in which y may be substituted for $a(n)$ and x may be substituted for n. The linear equation for the arithmetic sequence is $y = 4x + 1$.

The Practice Problems section for this Lesson Notes are games that can be played to generate tables, graphs, and explicit and linear formulas used to answer questions. Hopefully you recognize that arithmetic sequences generate linear graphs.

Section 1.11 Linearity and Calorie Burning

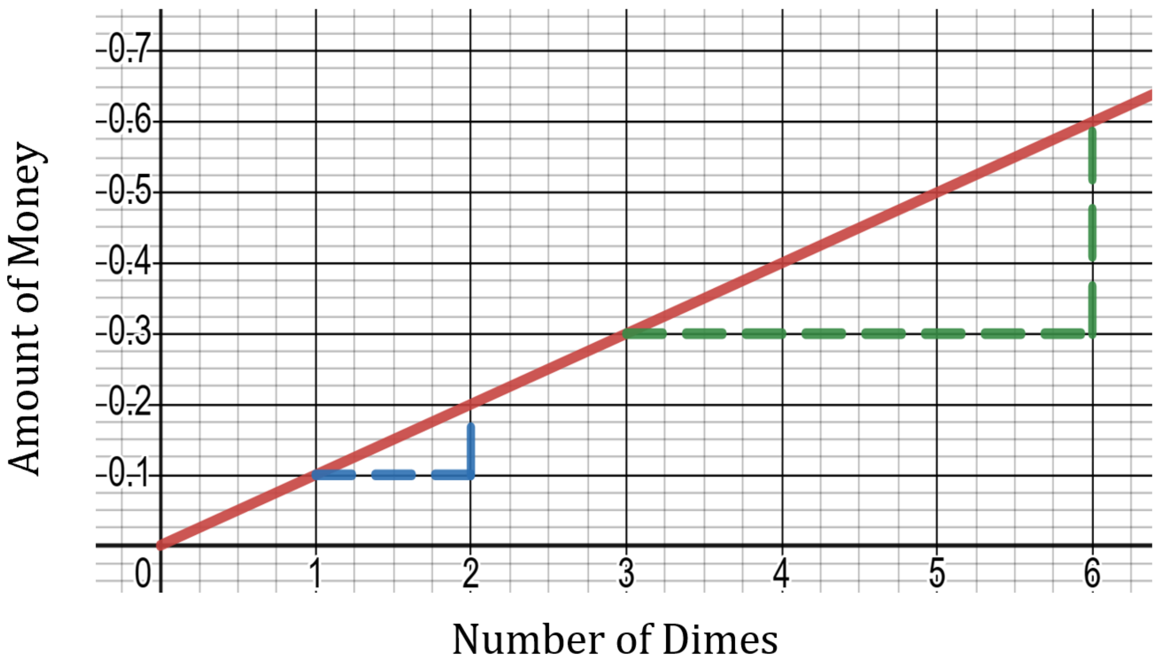
Looking Back 1.11

Slope has been called rise over run and represented by the change in y over the change in x. When an equation is linear, the slope between any two points is constant. In the graph below, the slope is 0.10.

The smaller slope triangle has a slope of $\frac{0.1}{1} = 0.1$.

The larger slope triangle has a slope of $\frac{0.3}{3} = 0.1$.

The rate of change is \$0.10. This makes sense. As you add each dime more, the amount of money increases by \$0.10.



Looking Ahead 1.10

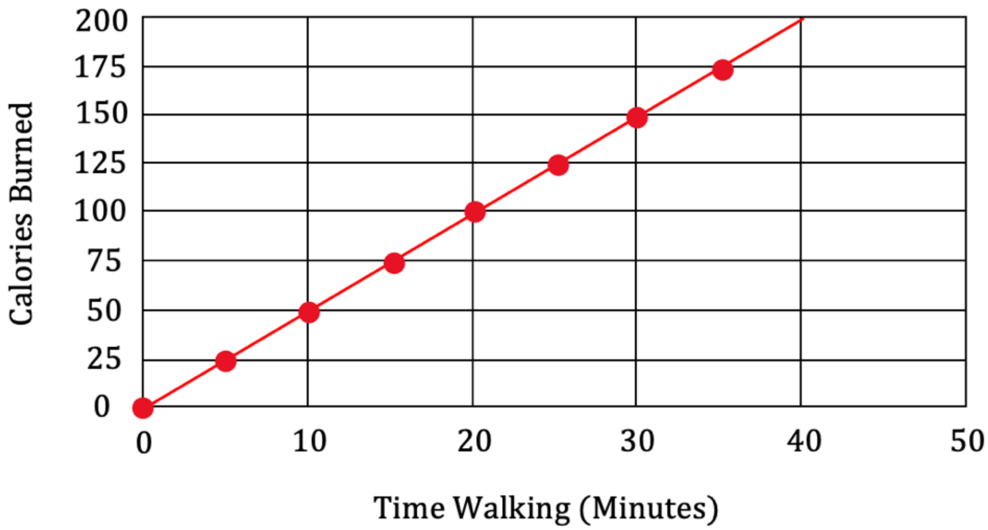
One might think that walking burns 5 calories per minute. Calories are units of energy. Energy and calories come from food intake. One calorie is the approximate amount of energy needed to raise the temperature of one gram of water by one degree.

Therefore, during exercise, energy is expended, and calories are burned. The human body, however, uses calories at varying rates. These rates depend on a person's size and weight, and their physical condition (whether they are sedate or active). Age, gender, climate, and other conditions are factors as well. During physical activity, estimates of calorie costs can be made.

Example 1: Below is a table that shows the calories burned for a 125-pound person walking. The time is in minutes. Graph the data in the table and determine if the graph is mostly linear or perfectly linear.

Time (Minutes)	Calories Burned
5	25
10	50
15	76
20	101
25	126
30	151
35	176
40	202

The slope is approximately $\frac{25}{5} = 5$ calories burned per minute.



Is the line perfectly linear? What would be an equation that could represent the line?

Example 2: How many calories would a 140-pound person burn in a 10-minute walk if they walk at 3 mph? (It takes this person 20 minutes to walk a mile, so their pace is 20 min/mile.) Use the formula below to solve the problem.

$$\text{Kilograms} \times \text{Energy Expended } \times \text{Time(in minutes)} = \text{Calories Burned}$$

Calories are measured in the unit $\frac{\text{calories}}{\text{minute}\cdot\text{kilogram}} = \frac{\text{calories}}{\text{min}\cdot\text{kg}}$. This is the energy expended.

Use the table below and the conversion factor $1 \text{ lb.} = 0.453392 \text{ kg}$ to find the number of calories they would burn.

$$140 \text{ lb.} \cdot \frac{0.453492 \text{ kg.}}{1 \text{ lb.}} = 63.9 \text{ kg.}$$

Activity	$\frac{\textbf{Calories}}{\textbf{min} \cdot \textbf{kg}}$
Dancing	0.08
Running (5 mph) (12 min/mile)	0.12
Running (6 mph) (10 min/mile)	0.13
Mowing Lawn	0.80
Raking Leaves	0.70
Walking (3 mph) (20 min/mile)	0.06
Walking (4 mph) (15 min/mile)	0.08
Vacuuming	0.05
Cleaning House	0.06

Example 3: How many calories would a 140-pound person burn in a 30-minute walk if they walked at 4 mph? (It takes this person 15 minutes to walk a mile so their pace is 15 min/mile.)

Section 1.12 Trend Lines

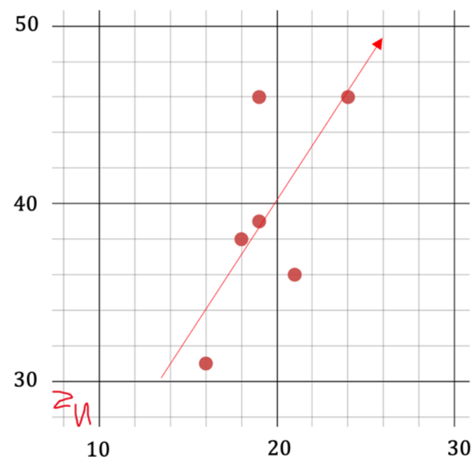
Looking Back 1.12

As you saw in the previous section, data does not always fit perfectly on a line even if it is mostly linear. However, data can follow a trend line or predictable pattern. For this reason, a trend line is often used to model experimental data. In *Pre-Algebra*, this was called the "line of best fit."

The graph of the data below is a scatterplot. A trend line can be drawn near as many points as possible even crossing as many points as possible. This line shows a trend: the general direction of the data if there are about the same number of points above and below the line. If the trend line appears to "fit" the data, it is called the line of best fit.

When plotted, real-world data often follows a pattern. In a later module, non-linear patterns will also be studied. If lines model the data, then equations can be derived, the data can be analyzed, and predictions can be made.

x	y
21	36
18	38
24	46
19	39
19	46
16	31



Notice, the graph does not start at $(0, 0)$. The squiggly line at the origin on the x-axis shows that the data has been shifted left, and the squiggly line at the origin on the y-axis shows that the data has been shifted down. The x-range is 10-30 and the y-range is 30-50.

Looking Ahead 1.12

The above trend line runs through two of the points and has two points above it and two points below it. This is best-case scenario. The trend line will not always go through the same amount of points with an equal number of points above and below the line; sometimes, there will be more points above or below the trend lines; sometimes, no points will be on the trend line. However, we want to get it as close to equal as possible to best represent the distribution.

Example 1: Find the equation of the trend line above using the points $(22, 43)$ and $(24, 46)$, which are on the line. Write your equation in Standard Form.

Find the slope using the points given and the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$.

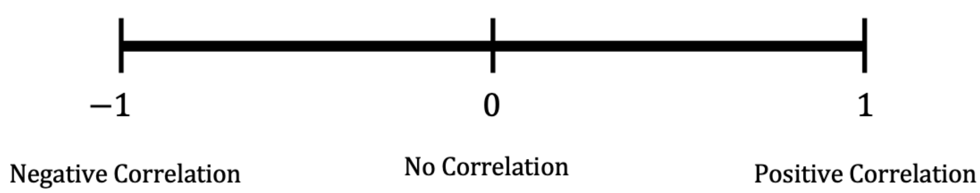
Use the point $(20, 40)$ to write an equation for the trend line in point-slope form $(y - y_1) = m(x - x_1)$, and convert it to slope-intercept form.

Write an equation for the trend line in slope-intercept form $(y = mx + b)$. Substitute m, x, and y into the equation and solve for b. Write an equation for the trend line using point $(20, 40)$.

Example 2: If the output is 45 for the above equation, what would you expect the input to be? Use the graph and the equation for the trend line or line of best fit to find this expected input.

Example 3: If the input is 16 for the above equation, what would you expect the output to be? Use the graph and the equation for the trend line or line of best fit to find this expected output.

A line of best fit can be found using a graphing calculator. The graphing calculator will also give you what is called a line of regression, and r (which is called the coefficient of correlation). It will also tell how closely the line models the data. The closer r is to $+1$ or -1, the closer the data fits the model (line of best fit).



You will learn more about correlations in the next section. In Algebra 2, you will calculate the coefficient of correlation and use the graphing calculator to find the line of best fit for given data.

Section 1.13 Coefficient of Correlation

Looking Back 1.13

We have learned about the coefficient of correlation in Pre-Algebra. We will study it again in this section to further analyze linear data.

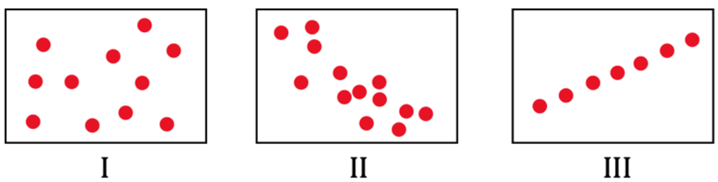
If coordinate points that are a part of a scatterplot have a relationship, they are said to have a correlation. If the relationship yields a coefficient of correlation close to $+1$ or -1, there is a connection. Perhaps as x increases, y increases, or as the independent variable decreases, the dependent variable decreases. (The means to calculate the coefficient of correlation will be done by hand and with a graphing calculator in Algebra 2.)

A casual relationship is present when the change in one variable causes a change in another variable, which means the causation shows a strong correlation between the two variables. However, that does not mean that if there is a correlation there is always a causation. From the previous Practice Problems section, we learned that higher saturated fat in a sandwich causes there to be more calories in a sandwich.

For now, knowing that $r = 1$ or $r = -1$ is a strong correlation and $r = 0$ is no correlation is enough. Let us look at what the graphs of these correlations look like.

Looking Ahead 1.13

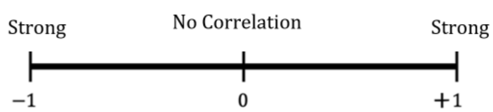
Look at the three graphs below. Which would you say has a strong correlation, a weak correlation, and no correlation? (Remember, the coefficient of correlation is represented by r.)



I: There is no correlation. The data is scattered all over and no relationship between points can be determined. In this case, $r = 0$ or r is close to 0.

II: There is a weak correlation. The data shows a general negative direction, but it would be difficult to find a line of best fit that has the same amount of points above and below the line. It seems that as x is increasing, y is decreasing.

III: There is a strong correlation. The data shows a general positive direction, and a trend line could be drawn so there are approximately the same amount of data points above and below the line. It seems that as x is increasing, y is increasing.



As the line moves from 0 to $+1$ or -1, the correlation goes from a weak correlation to a strong correlation.

A correlation coefficient of $r = -1$ shows a strong negative correlation. The data lie on the trend line that decreases as x increases.

A correlation coefficient of $r = 1$ shows a strong positive correlation. That data lie on the trend line that increases as x increases.

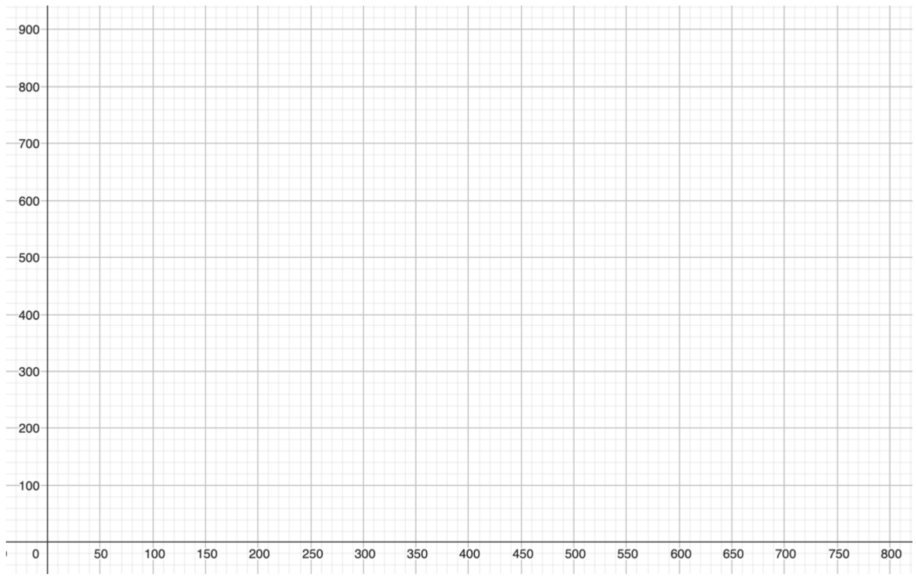
The calculator uses a method called linear regression, which you will use in Algebra 2, to find a line of best fit, which is a precise trend line. Causation can show trends and the lines of best fit can be used to analyze data as you did with the sandwiches in the previous Practice Problems section.

When a graph or an equation is used to approximate an unknown value between two values that are known, this is called interpolation.

When a graph or an equation is used to predict an unknown value outside of the output of known values, this is called extrapolation.

Example 1: Graph the data below to see if there is a positive or negative correlation and if it is strong or weak. Use interpolation to find the value of the output if the input is 400.

x	y
40	200
300	400
360	500
480	600
530	650
580	700
750	800
780	900



Example 2: Graph the data below to see if there is a positive or negative correlation and if it is strong or weak. Use extrapolation to find the input if the output is 700.

x	y
2	630
10	570
39	440
56	310
75	300
80	180
83	200
90	1,600