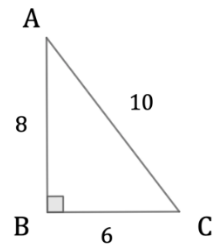
Geometry and Trigonometry Module 7 The Unit Circle and Trigonometric Functions

Section 7.1 Trigonometric Ratios for Right Triangles

Practice Problems 7.1

For Problem 1-9, use the given triangle to solve the problem.



For Problem 1-6, write the trigonometric ratio.

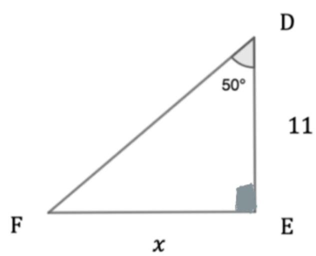
1. $\sin A$
2. $\sin C$
3. $\cos A$
4. $\cos C$
5. $\tan A$
6. $\tan C$

For Problem 7-9, fill in the blank.

7. $\sin A = \cos$ ________
8. $\cos A = \sin$ ________
9. $\tan A$ is the ______________________ of $\tan C$

For Problem 10-14, solve for x in the triangle given. Give an approximation to the nearest tenths place.

10.



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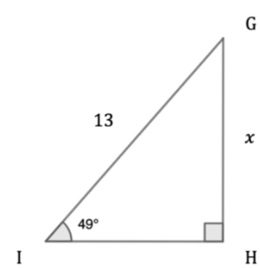

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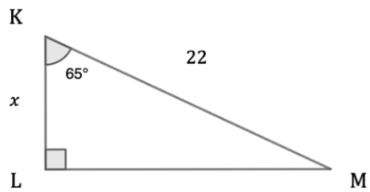

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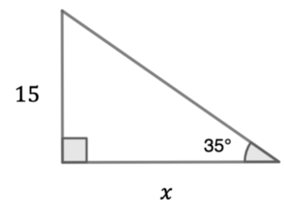

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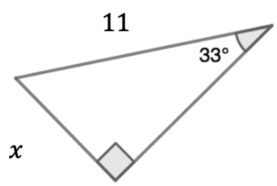
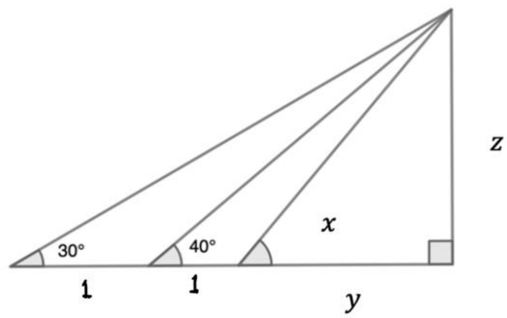


For Problem 15, solve the word problem.

15. Kayleigh is looking at a flagpole. She is 5’ tall and is 25’ from the flagpole. Her clinometer tells her there is a 42° angle from her eye level to the top of the flagpole. Draw a diagram of the scenario and use trigonometric ratios to find the approximate height of the flagpole.

For Problem 16-19, use the diagram below to solve the problem.



For Problem 16-19, fill in the blank(s).

16. $\tan 30° = \frac{}{y+2}$

17. $\tan 40° = \frac{z}{ + }$

18. $(y + 2)(\tan 30°) = $ ________

19. $(y + 1)(\tan 40°) = $ ________

For Problem 20, solve for y.

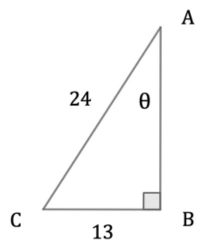
20. $(y + 2)(\tan 30°) = (y + 1)(\tan 40°)$

Section 7.2 Inverse Trigonometric Functions

Practice Problem 7.2

For Problem 1-4, find the missing angle in the triangle given. Round your answer to the tenths place.

1.



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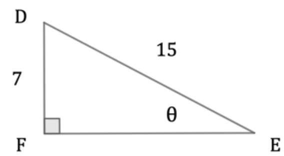

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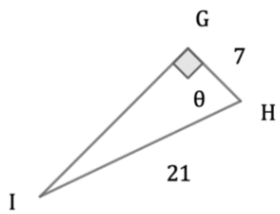

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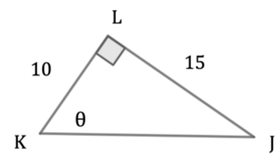


For Problem 5-8, use the triangles from Problem 1-4 to solve the problem.

5. What is the degree of ∠C in ΔABC?

6. What is the degree of ∠D in ΔDEF?

7. What is the degree of ∠I in ΔGHI?

8. What is the degree of ∠J in ΔJKL?

For Problem 9-12, solve the word problem.

9. Zach is building a ramp from his treehouse so he can slide down it and land at the kitchen door when Amy calls him in for dinner. He builds the treehouse only 5' from the ground because his sister Caeley does not like to go any higher. The base of the tree is 18' from the kitchen door. Draw a diagram to illustrate the situation.

10. How long does the slide have to be to go from the treehouse, above the base of the tree, to the kitchen door?

11. Caeley says if the slide is more than a 20° angle the trip to the door will be too far and dangerous for her. What is the angle of the slide? Is it safe for her?

12. Garrett and Blaiden misunderstand Caeley and make the ramp angle exactly 20° degrees. How high will Zach have to build the treehouse from the ground?

For Problem 13-16, solve the word problem.

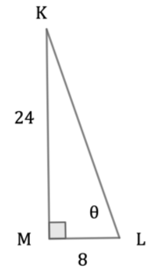
13. An 18’ high ladder rests against a wall. The base of the ladder is 5’ from the wall. What angle does the ladder make with the ground? Round to the nearest tenth.

14. How high up the wall is the 18’ ladder?

15. If the ladder slides out 2’, what is the new angle the ladder makes to the ground?

16. What is the new height of the ladder up the wall?

For Problem 17-19, use the diagram below to solve the problem.

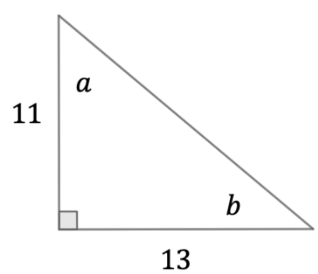


17. Solve for θ.

18. What is the measure of ∠K?

19. What is the length of side $\overline{KL}$?

For Problem 20, use the diagram below to solve the problem.



20. Which angle is larger: a or b?

Section 7.3 Special Right Triangle Ratios

Practice Problems 7.3

For Problem 1-3, follow the directions and fill in the table.

1. Sketch an equilateral triangle (three equal sides). Label each angle. Draw the altitude. You should now have two triangles. Label the new angle measurements. Label one side of your triangle 2 units.

Now, write the ratio using the lengths you previously found. Label each of the measures of the two triangles and complete the table for the 30° angle.

Next, use your calculator to find the decimal approximation for each exact value. Record to the hundredths digit.

Finally, find each ratio using the trigonometric function keys on the calculator. What do you notice? Check the calculator settings. Is the calculation mode exact or approximate? Try both settings.

	Sine $30°$	**Cosine $30°$**	**Tangent $30°$**
Exact Value of Ratio			
Decimal Approximation of Exact Value			
Value by the Trigonometric Function Key			

2. Complete the table for the 60° angle from Problem 1.

60° Angle	**Sine 60°**	**Cosine 60°**	**Tangent 60°**
Exact Value of Ratio			
Decimal Approximation of Exact Value			
Value by Trigonometric Function Keys			

3. Sketch a right triangle that is isosceles. Label each acute angle. Label one leg 1 unit and calculate the exact length of the other two sides. Complete the table for the acute angles and write the exact ratios in the first row.

Then, use your calculator to find the decimal approximations for the second row. Record to the hundredths digit. Next, use the trigonometric function keys on your calculator to complete the third row.

	Sine 45°	**Cosine 45°**	**Tangent 45°**
Exact Value of Ratio			
Decimal Approximation of Exact Value			
Value by the Trigonometric Function Key			

For Problem 4 and 5, solve the multiple-choice problem.

4. Find the length of the hypotenuse of an isosceles right triangle given the length of each leg.

a) 4 b) 5

c) 7 d) $\sqrt{3}$

5. Find the length of the leg given the length of the hypotenuse of an isosceles right triangle.

a) $2\sqrt{2}$ b) $3\sqrt{2}$

c) 5 d) 1

For Problem 6-8, solve the word problem.

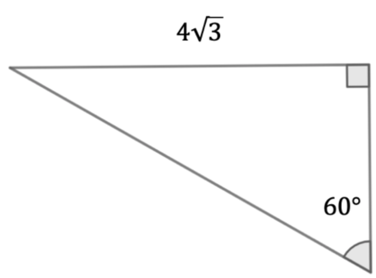
6. Find the length of the short leg in a $30° - 60° - 90°$ triangle given the hypotenuse is 4.2.

7. Find the length of the long leg of a $30° - 60° - 90°$ triangle given the hypotenuse is 8.

8. If the long leg of a $30° - 60° - 90°$ triangle is $9\sqrt{3}$, how long is the hypotenuse?

For Problem 9 and 10, label the missing angle and sides in the triangle.

9.



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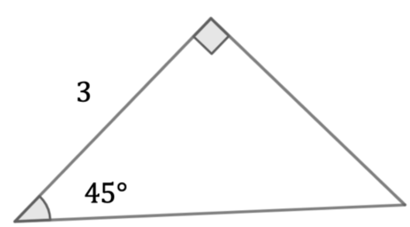
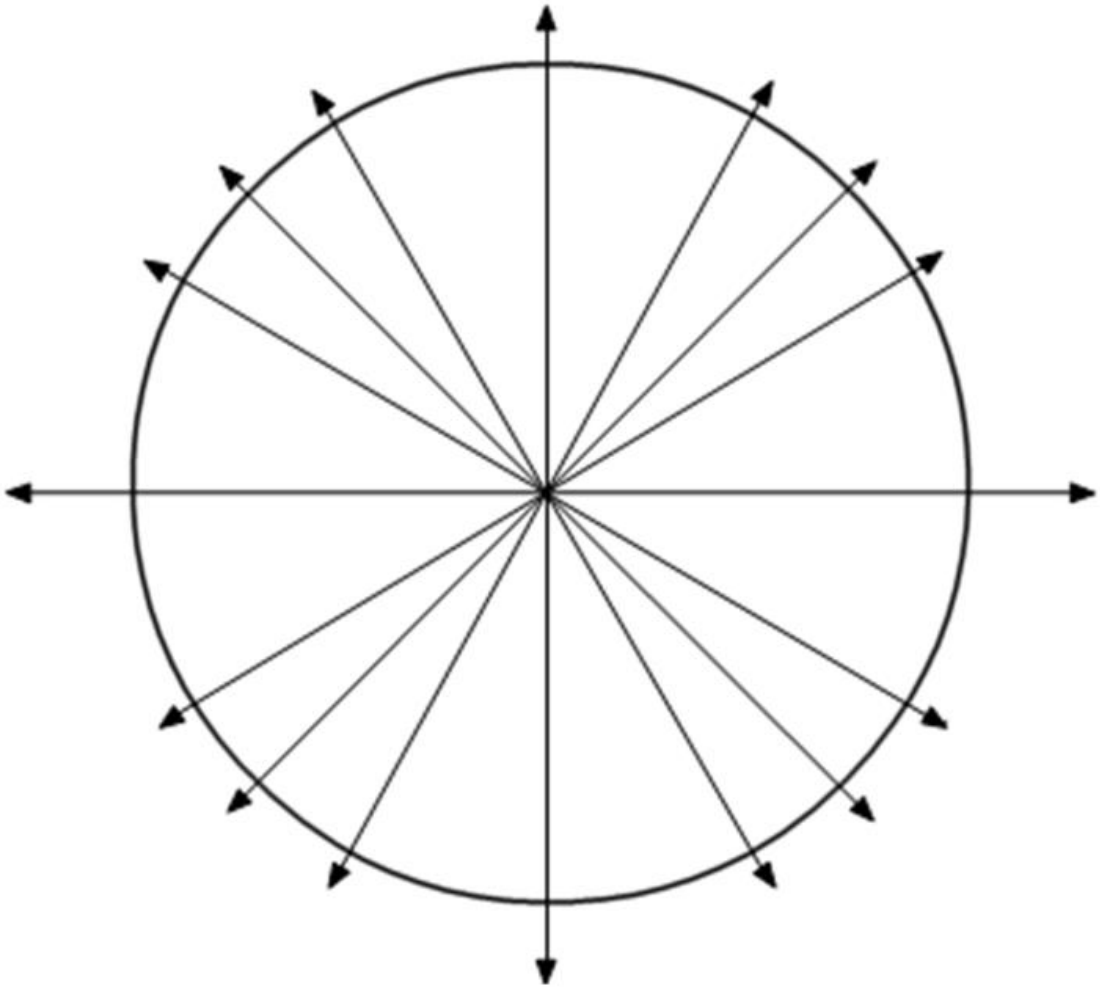


Section 7.4 The Unit Circle and the Wrapping Function

Practice Problems 7.4



Today, instead of doing practice problems, study the unit circle you completed in the lesson notes, and then complete the blank one above when ready. Try to remember these by identifying patterns. You will use three different colors to label the following measures where each radius intercepts the unit circle:

1. Degrees
2. Radians
3. Ordered Pairs (Exact Real Numbers)
4. Ordered Pairs (Decimal Approximations to the hundredths digit)

Section 7.5 The Unit Circle and the Smarties® Simulation

Practice Problem 7.5

Complete the table for the radian measure in a circle given the degrees. Write the fractions in terms of π and write the decimals to the ten-thousandths digit.

Degree Measure	**Radian (Fraction)**	**Radian (Decimal)**
$0°$	0	0
$30°$		
$45°$		
$60°$		
$90°$	$\frac{\pi}{2}$	1.570
$120°$		
$135°$		
$150°$		
$180°$		
$210°$		
$225°$		
$240°$		
$270°$		
$300°$		
$315°$		
$330°$		
$360°$		

<u>Section 7.6 Reference Angles, Radians, and Degrees</u>

<u>Practice Problems 7.6</u>

For Problem 1-4, convert the radians to degrees.

1. $\frac{3\pi}{2}$

2. $\frac{\pi}{2}$

3. $\frac{\pi}{12}$

4. $\frac{1}{6}\pi$

For Problem 5-8, convert the degrees to radians.

5. $66°$

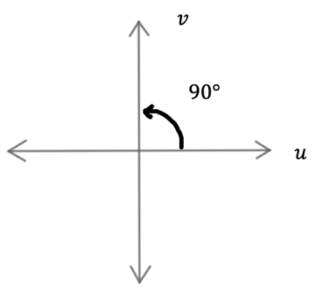
6. $360°$

7. $255°$

8. $90°$

For Problem 9-14, find the standard angle and the reference angle in each of the diagrams.

9.



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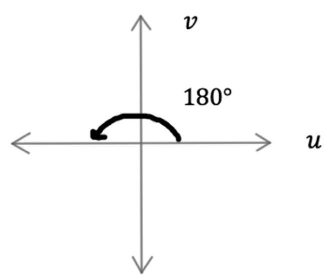

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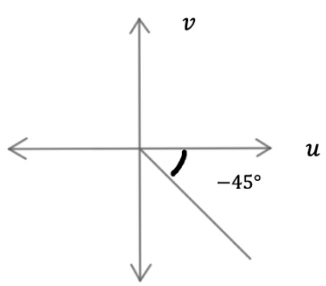

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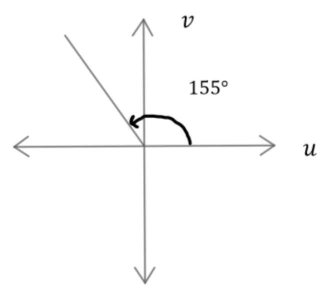

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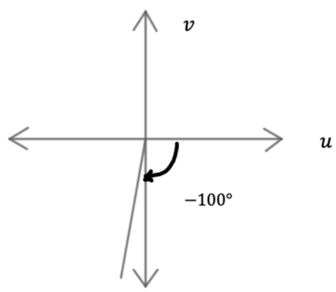

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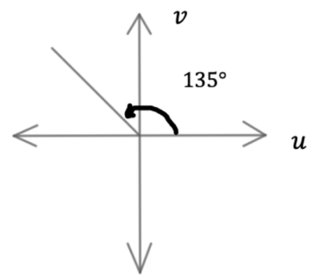


For Problem 15 and 16, follow the instructions and solve the problem.

15. Draw a perpendicular line from the terminal side of $135°$ to the u-axis in Problem 14.

16. If the segment for the terminal side is $5\sqrt{2}$, what is the measure of each of the legs of the right triangle formed?

For Problem 17-20, name another angle measure for these standard position angles moving in a clockwise rotation.

17. $140°$

18. $25°$

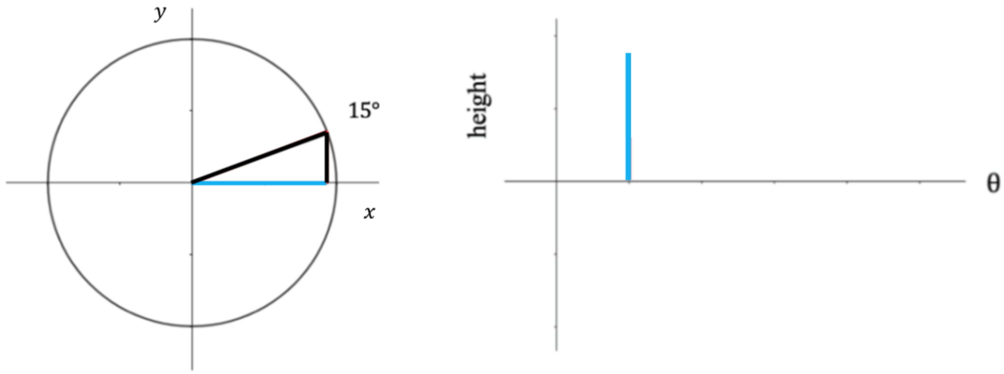
19. $270°$

20. $360°$

Section 7.7 The Sine and Cosine Functions

Practice Problems 7.7

Use the unit circle and coordinate grid below to simulate the graph of the cosine curve.



1. Use a protractor to mark every angle on the unit circle in $15°$ increments. Draw the radius from the origin to the point where the radius intercepts the arc of the circle. Then draw a line from this point of interception perpendicular to the x-axis. The first one is done for you at $15°$. This a triangle for an angle rotation of $15°$ in standard position.

2. Label the x-axis on the coordinate grid from $0°$ to $360°$ in intervals of $15°$ to represent these angles of rotation in the unit circle (θ). Use a toothpick to measure the radius of the unit circle (in 'toothpicks'). Transfer this height to the y-axis of the coordinate grid, marking a height of 1 radius above the y-axis and -1 radius below the y-axis.

3. Remember, since the cosine function is the ratio of the adjacent side over the hypotenuse of the triangle at the $15°$ angle, this would be the length of the leg of the triangle on the x-axis ($x_{15°}$) over the radius length (shown above in blue). That is $\cos 15° = \frac{x_{15°}}{1}$ or $\cos 15° = x_{15°}$. Using the toothpick, measure the horizontal length of $x_{15°}$ and transfer it to the coordinate grid. Place it vertically at the $15°$ mark. This is the cosine ratio of the $15°$ angle and is drawn in blue on the unit circle and graph.

4. Draw a second triangle at $30°$ and, using the toothpick, measure and transfer the length $x_{30°}$ to the coordinate grid. The perpendicular line from the 30° angle arc intercept is to the x-axis on the unit circle. The horizontal distance from the origin to the x-intercept is the horizontal length.

5. Draw all the triangles for the unit circle. Notice the horizontal lengths are all on the x-axis of the unit circle. The horizontal lengths are positive for Quadrants I and IV and will be above the x-axis. The horizontal lengths are negative for Quadrants II and III and will be below the x-axis.

6. Place the start point at $(0°, 1)$ and the end point at $(360°, 1)$ on the coordinate grid. Draw a smooth curve between those points connecting the ends of the positive and negative lengths of all the lines for one full cycle of the cosine curve.

Answer the following questions regarding the cosine function simulation.

a) What do x and y represent on the curve? Fill in (x, y) in terms of the coordinate grid.

b) What is the equation that models the curve of the graph?

c) What is the maximum height of the graph? At what point(s) does this occur? What is the minimum height of the graph? At what point does this occur?

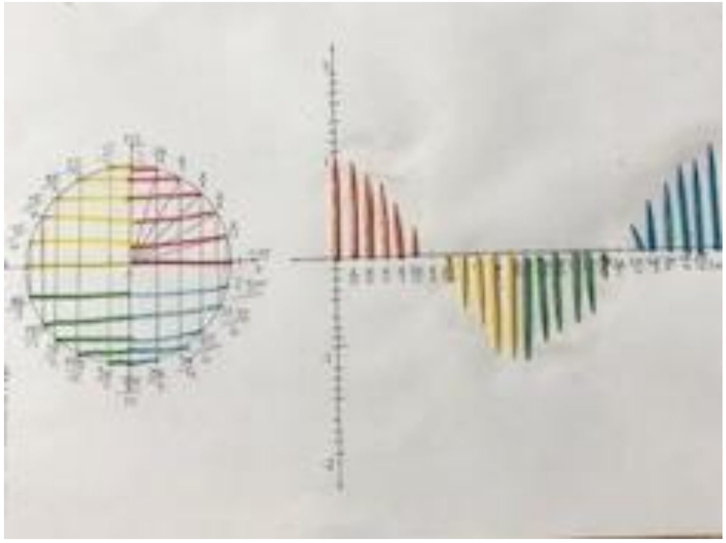
d) The mid-line is the horizontal axis that is halfway between the maximum and minimum values of the function. What is the equation for the mid-line?

e) The amplitude in a periodic function is half the distance between the highest and lowest point on the graph. This is also the distance from the mid-line to the maximum or minimum point. What is the amplitude?

f) What is the domain of the cosine function? What is the range of the cosine function?

g) What are the x-intercepts on the unit circle? What points represent these on the coordinate curve? What are the y-intercepts on the unit circle? What do they represent on the coordinate curve?

Shown in toothpicks below is the completed graph of $y = \cos\theta$.



Section 7.8 The Tangent Function and Inverse Graphs

Practice Problems 7.8

For Problem 1 and 2, complete the table given.

1. Complete the table for the values of sin θ when θ is between 0 and π. Use the trigonometric key on the calculator; set it to 'radian' mode and 'decimal approximation' and round to the nearest hundredth.

θ	sin θ
0	
$\frac{\pi}{12}$	
$\frac{\pi}{6}$	
$\frac{\pi}{4}$	
$\frac{\pi}{3}$	
$\frac{5\pi}{12}$	
$\frac{\pi}{2}$	
$\frac{7\pi}{12}$	
$\frac{2\pi}{3}$	
$\frac{3\pi}{4}$	
$\frac{5\pi}{6}$	
$\frac{11\pi}{12}$	
π	0

2. Since the sine function is cyclic, complete the table for sin θ when $\pi \leq \theta \leq 2\pi$ using patterns. Check your answers using the graphing calculator. Round to the nearest hundredth.

θ	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$
sin θ								

θ	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
sin θ					

For Problem 3-10, solve the problem given.

3. The shortest repeating cycle is the period of the function. What is the period of the sine function?

4. Tell whether this statement is true or false: The maximum value of $y = \sin x$ occurs when $x = \frac{\pi}{2} + 2\pi n$ for any integers n.

5. Where do the minimum values of $y = \sin x$ occur?

6. Switch the values of θ and sin θ in the table from Problem 1 to graph the inverse of the sine function: $y = \sin^{-1} x$. Is the inverse a function? If not, draw lines where the maximum and minimum output values must occur for this to be a function.

7. What is the domain of $y = \sin^{-1} x$?

8. Is $\sin^{-1} x$ equal to $(\sin x)^{-1}$?

9. What is the linear radian measure for the sine function after two revolutions around the unit circle?

10. What is one-fourth of the period of $y = \sin x$?

For Problem 11 and 12, complete the table given.

11. Complete the table for the values of $\cos\theta$ when θ is between 0 and π. Using the trigonometric key on the calculator, set it to 'radian' mode and 'decimal approximation' and round to the nearest hundredth.

θ	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$\cos\theta$												

12. Since the cosine function is cyclic, complete the table for $\cos\theta$ when $\pi \le \theta \le 2\pi$ using patterns. Check your answers using the graphing calculator.

θ	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
$\cos\theta$													

For Problem 13-20, solve the problem given.

13. The shortest repeating cycle is the period of the function. What is the period of the cosine function?

14. Where does the maximum value of $y = \cos x$ occur?

15. Does the minimum value of $y = \cos x$ occur when $x = 2\pi n + 2\pi$ for any integer n?

16. Switch the values of the θ and $\cos\theta$ in the table to graph the inverse of the cosine function: $y = \cos^{-1} x$. Is the inverse a function? If not, draw lines where the maximum and minimum output values must occur for this to be a function.

17. What is the domain of $y = \cos^{-1} x$?

18. Evaluate $\cos^{-1}(-0.5) = y$ based on the definition that $y = \cos^{-1} x$ if and only if $x = \cos y$ and $0 \leq y \leq \pi$.

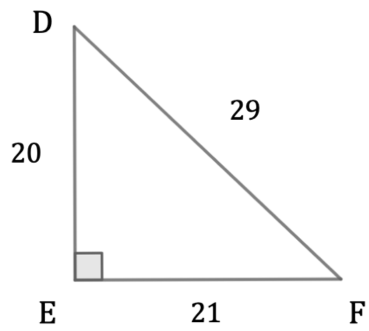
19. Evaluate $\cos^{-1}(1.3) = y$ based on the definition in in Problem 18.

20. Given $\cos \theta = 0.26$, what are the possible quadrants θ lies in. Does this agree with the table in Problem 11?

Section 7.9 The Reciprocal Trigonometric Functions

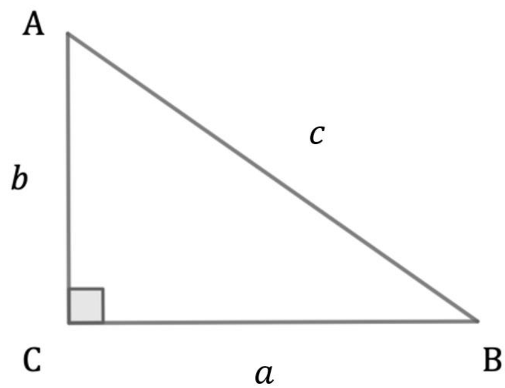
Practice Problems 7.9

For Problem 1-14, use the right triangle below to find the given ratio.



1. sin D
2. cos D
3. tan D
4. sin F
5. cos F
6. tan F
7. sin E
8. csc D
9. sec D
10. cot D
11. csc F
12. sec F
13. cot F
14. What is the reciprocal of the cotangent ratio?

For Problem 15-20, use the triangle below (from the lesson notes) to find the ratio.



15. sin B

16. csc B

17. cos B

18. sec B

19. tan B

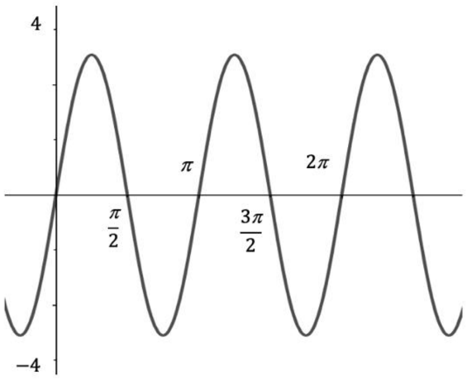
20. cot B

Section 7.10 Amplitude and Period

Practice Problems 7.10

For Problem 1-4, find the amplitude and period of the graph and write the equation.

1.



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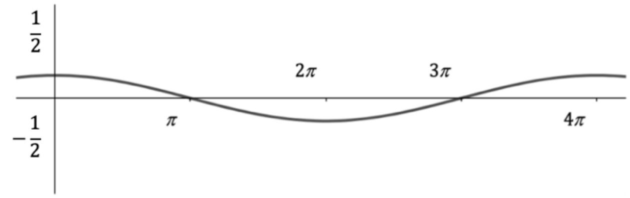

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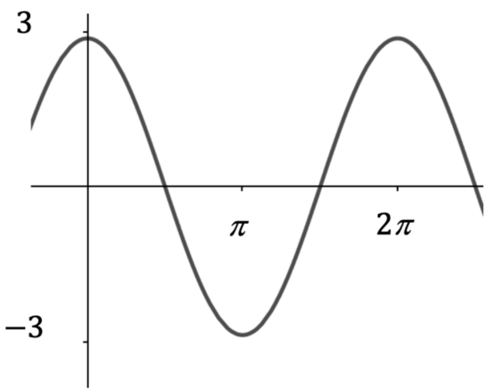

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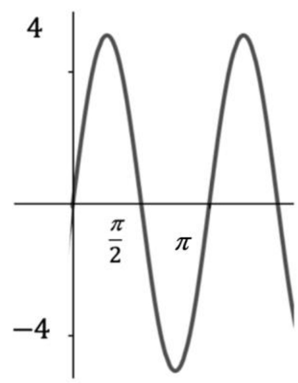


For Problem 5-10, find the amplitude and period of the equation.

5. $y = -\sin x$

6. $y = \frac{1}{3}\sin\frac{2\pi}{3}x$

7. $y = \frac{1}{4}\sin\frac{\pi}{4}x$

8. $y = \cos 2\pi x$

9. $y = 5\cos 3\pi x$

10. $y = \frac{1}{2}\cos\frac{1}{2}\pi x$

For Problem 11 and 12, answer the multiple-choice question.

11. Which graph has the most frequent oscillations over a period of 2π?

a) $y = \sin\frac{1}{2}x$

b) $y = \sin 2x$

c) $y = -\sin x$

d) $y = 4\sin 4x$

12. Which graph has the least frequent oscillations over a period of 2π?

a) $y = \sin\frac{1}{2}x$

b) $y = \sin 2x$

c) $y = -\sin x$

d) $y = 4\sin 4x$

For Problem 13-16, write an equation given the amplitude and period. Let a and b be greater than 0.

13. $y = a\cos bx$
Amplitude: $\frac{3}{2}$
Period: π

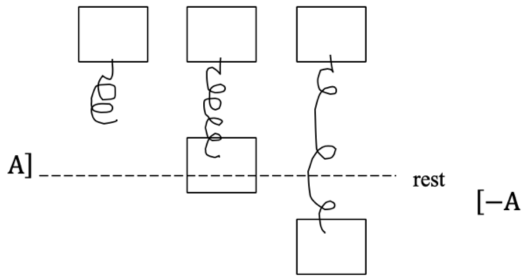
14. $y = a\sin bx$
Amplitude: $\frac{1}{3}$
Period: $\frac{\pi}{3}$

15. $y = a\cos bx$
Amplitude: 1
Period: $\frac{2\pi}{3}$

16. $y = a\sin bx$
Amplitude: 4
Period: 2π

For Problem 17 and 18, use the information given to solve the problem.

17. The function $y = \mathrm{A}\cos kt$ models the simple motion of a spring where A is the initial displacement in feet, k is the constant of elasticity for the spring, t is the time in seconds, and y is the vertical displacement of the spring. Find the amplitude and period for a spring with $\mathrm{A} = \frac{1}{4}$ and $k = 2$.

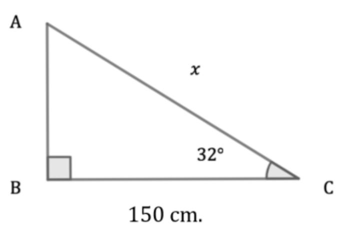


18. Use what you know about special triangles to complete the table for the trigonometric functions and their inverses.

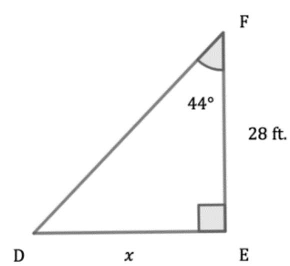
θ	**0**	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin θ	0		$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$			-1	
cos θ	1	$\frac{\sqrt{3}}{2}$		$\frac{1}{2}$	0			
tan θ	0	$\frac{\sqrt{3}}{3}$			Undef.			
csc θ	Undef.		$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$				Undef.
sec θ	1		$\sqrt{2}$				Undef.	1
cot θ		$\sqrt{3}$		$\frac{\sqrt{3}}{3}$		Undef.		

For Problem 19-20, solve for x in the figure given. Round to the nearest tenth.

19.



20.

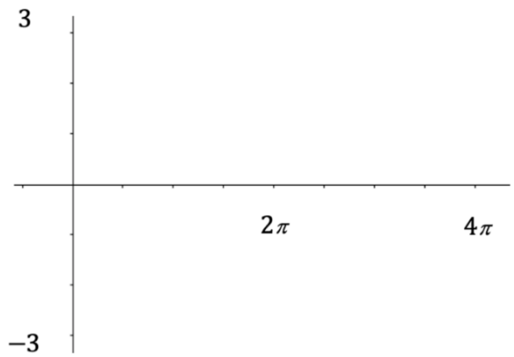


Section 7.11 Graphing Trigonometric Transformations

Practice Problems 7.11

For Problem 1, use the table from the lesson notes as a guide to complete the graph given.

1. Plot the graph of $\sec x$ on the graph.



For Problem 2 and 3, solve the word problem.

2. What is the quotient property for tangent in terms of the trigonometric functions?

3. Based on the quotient property in Problem 2, why does the $\tan x$ have asymptotes where it does?

For Problem 4-7, find the amplitude, period, and maximum and minimum values for the equation.

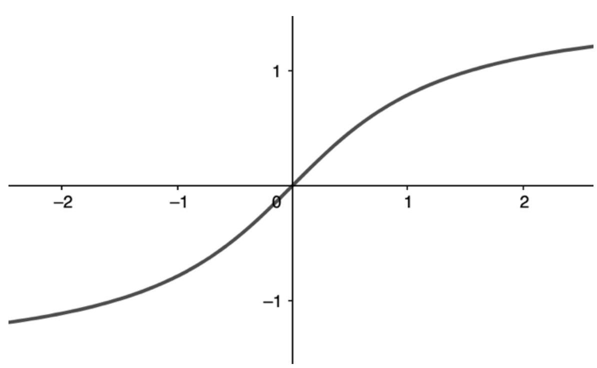
4. $y = -3\cos x$

5. $y = \frac{1}{4}\sin 2\,\pi x$

6. $y = 4\sin 4x$

7. $y = 5\sin\frac{\pi}{4}$

For Problem 8, use the graph below to answer the questions.



a) What is the domain of the function?

b) What is the range of the function?

c) How do you know it is a function?

For Problem 9-14, follow the instructions to solve the problem.

9. If $0 \leq \theta \leq \frac{\pi}{2}$, what must be true of the signs below? Tell whether they are positive or negative.

a) $\sin\theta$

b) $\cos\theta$

c) $\tan\theta$

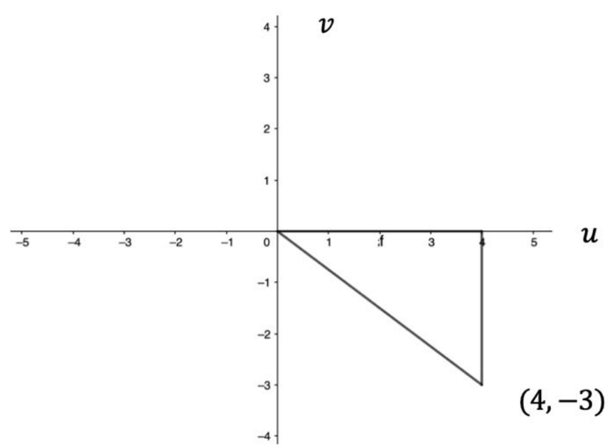
10. Write the value of each trigonometric function as a reciprocal function.

a) $\csc\theta$

b) $\sec\theta$

c) $\cot\theta$

11. The figure below shows θ in standard position with a terminal side that contains the point $(4, -3)$.



a) Draw the reference angle θ_{Ref} with an arrow.

b) Find the six trigonometric functions of θ.

12. If $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$, find the other four trigonometric functions.

13. Find the decimal approximation for the value of the function given and round to the nearest thousandth.
a) $\cos(125°)$

b) $\csc(-21°)$

c) $\cot(241°)$

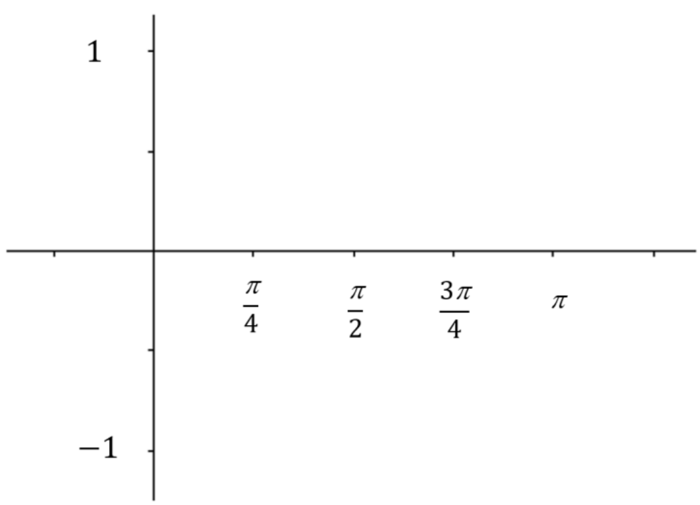
14. Tell whether the statement is true or false.
a) $\cos(125°) = -\sin(55°)$

b) $\csc(-21°) = \frac{1}{\sin(21°)}$

c) $\cot(241°) = \frac{1}{\tan(241°)}$

For Problem 15-20, fill in the blanks and graph the transformation function for one cycle using the critical points.

15. $y = \sin 2x$



$a =$

Amplitude =

Period =

Interval Space =

Start =

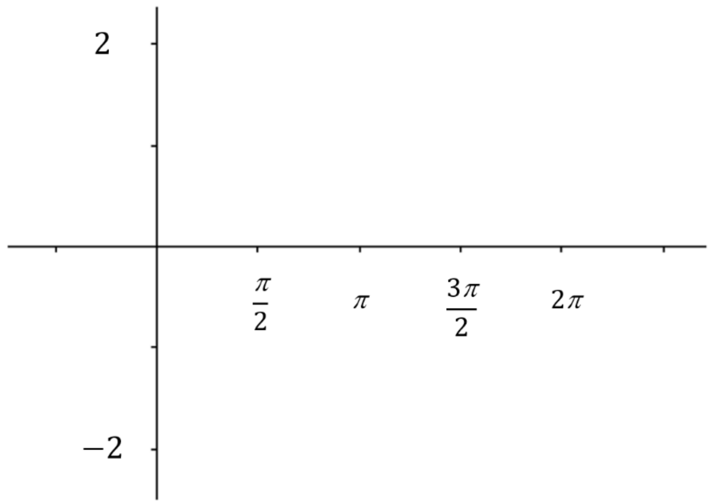
$\frac{1}{4}$ cycle =

$\frac{1}{2}$ cycle =

$\frac{3}{4}$ cycle =

End =

16. $y = -2\cos x$



$a =$

Amplitude =

Period =

Interval Space =

Start =

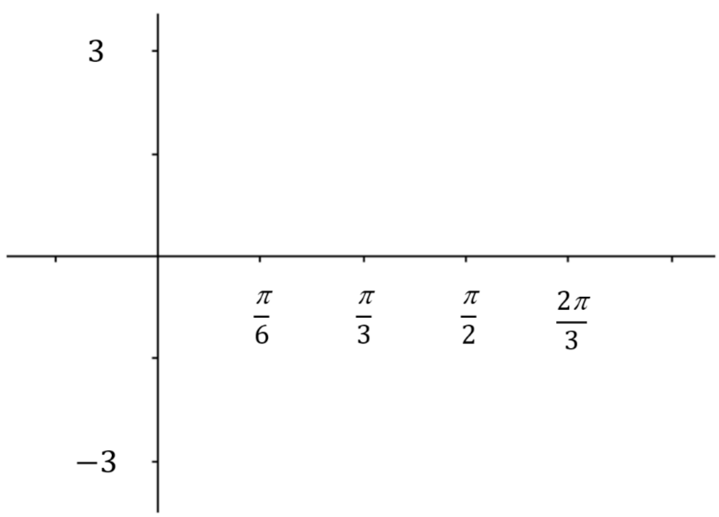
$\frac{1}{4}$ cycle =

$\frac{1}{2}$ cycle =

$\frac{3}{4}$ cycle =

End =

17. $y = 3\tan 3x$



$a =$

Amplitude =

Period =

Interval Space =

Start =

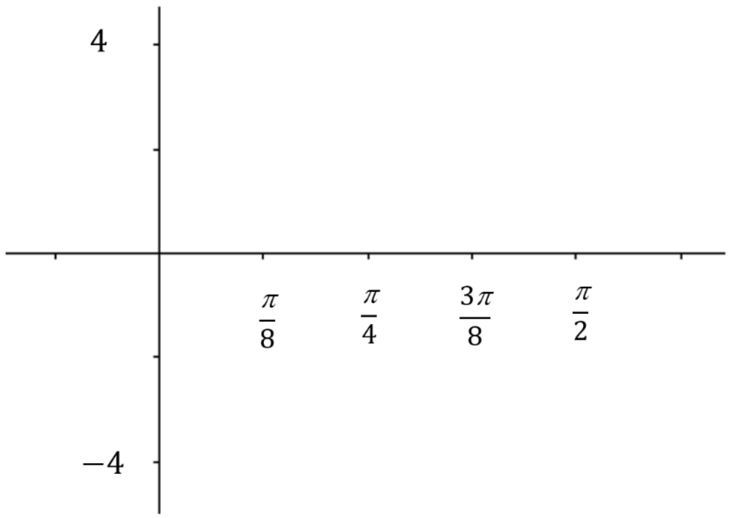
$\frac{1}{4}$ cycle =

$\frac{1}{2}$ cycle =

$\frac{3}{4}$ cycle =

End =

18. $y = 4\csc 4x$



$a =$

Amplitude =

Period =

Interval Space =

Start =

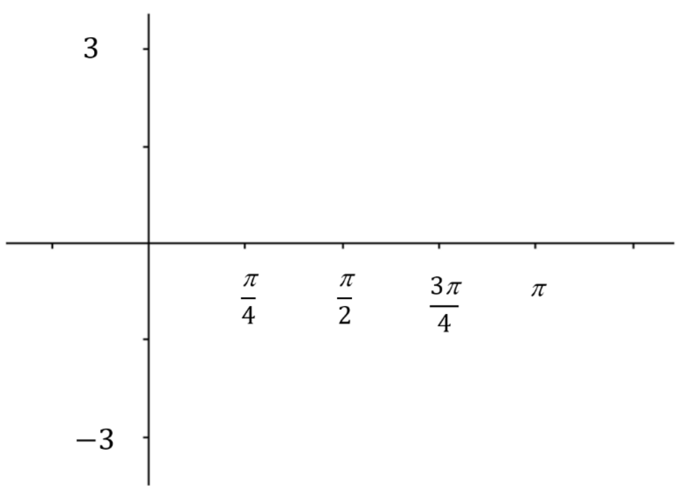
$\frac{1}{4}$ cycle =

$\frac{1}{2}$ cycle =

$\frac{3}{4}$ cycle =

End =

19. $y = 3\sec 2x$



$a =$

Amplitude =

Period =

Interval Space =

Start =

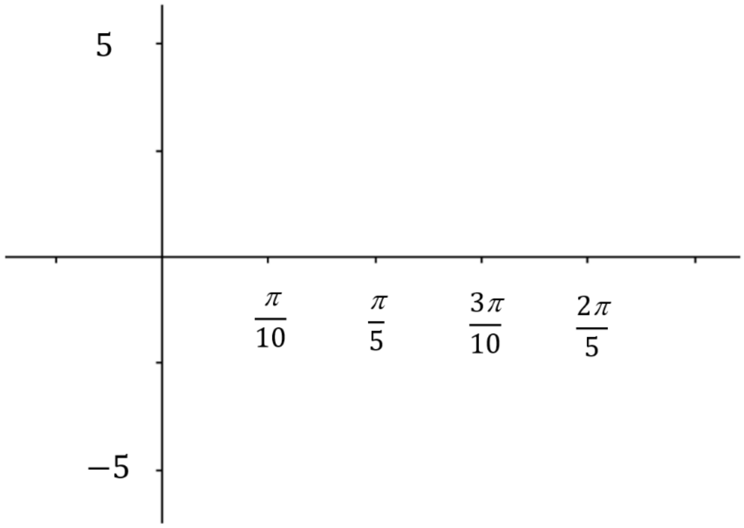
$\frac{1}{4}$ cycle =

$\frac{1}{2}$ cycle =

$\frac{3}{4}$ cycle =

End =

20. $y = 5\cot 5x$



$a =$

Amplitude =

Period =

Interval Space =

Start =

$\frac{1}{4}$ cycle =

$\frac{1}{2}$ cycle =

$\frac{3}{4}$ cycle =

End =

Section 7.12 Phase Shifts and Vertical Shifts

Practice Problems 7.12

For Problem 1-4, graph the trigonometric function.

1. $y = \tan x$

Inflection Point: __________

Period: __________

Interval Spacing: __________

Start of y: __________

$\frac{1}{4}$ point: __________

$\frac{1}{2}$ point: __________

$\frac{3}{4}$ point: __________

End point: __________

2. $y = \csc 2x$

a: __________

Maximum or Minimum: __________

Period: __________

Interval Spacing: __________

Start of y: __________

$\frac{1}{4}$ point: __________

$\frac{1}{2}$ point: __________

$\frac{3}{4}$ point: __________

End point: __________

3. $y = 4\sec 3x$

a: __________

Maximum or Minimum: __________

Period: __________

Interval Spacing: __________

Start of y: __________

$\frac{1}{4}$ point: __________

$\frac{1}{2}$ point: __________

$\frac{3}{4}$ point: __________

End point: __________

4. $y = -\cot x$

a: __________

Maximum or Minimum: __________

Period: __________

Interval Spacing: __________

Start of y: __________

$\frac{1}{4}$ point: __________

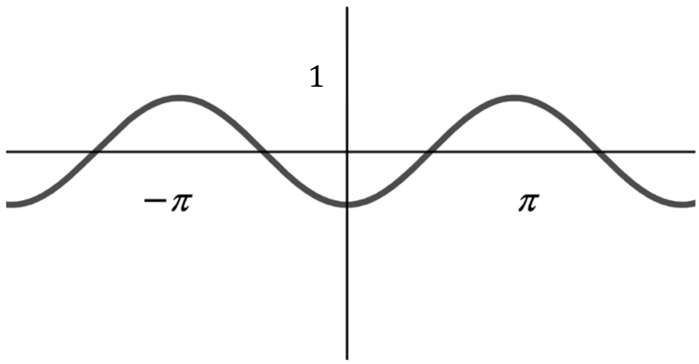
$\frac{1}{2}$ point: __________

$\frac{3}{4}$ point: __________

End point: __________

For Problem 5 and 6, fill in the blanks and write a reasonable equation for the graph given.

5.



Amplitude: __________

Period: __________

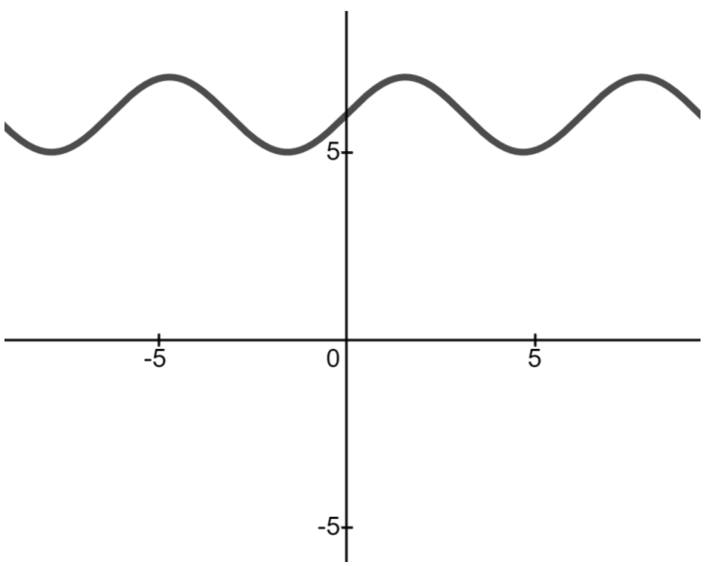
Phase Shift: __________

Vertical Shift: __________

Mid-line: __________

$b = 1$

6.



Amplitude: __________

Period: __________

Phase Shift: __________

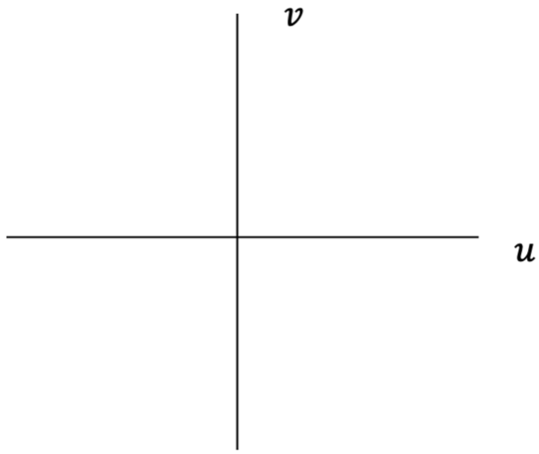
Vertical Shift: __________

Mid-line: __________

$b = 1$

For Problem 7-11, follow the instructions to solve the problem.

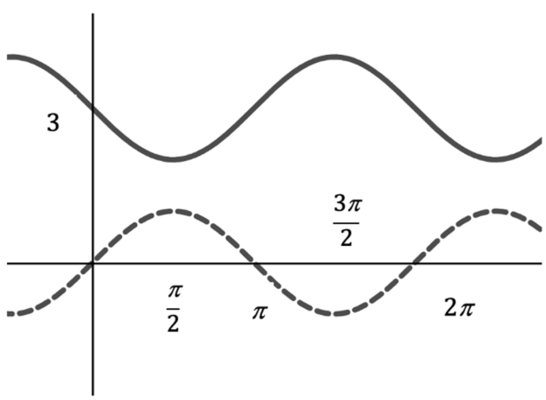
7. Sketch an angle of $120°$ in standard position and mark the reference angle, θ_{Ref}, with an arrow and label its measure. Write the six trigonometric functions of $120°$.



8. True or False: Coterminal angles are two angles in standard position whose angles differ by $360°$, or $\theta + 360n$, where n is the number of rotations.

9. Draw a $180°$ angle in standard position with point (u, v) on the terminal side. Draw a circle centered at the origin that goes through (u, v) and explain why $\cos 180° = -1$.

10. The graph of $f(\theta) = \sin\theta$ is the dashed line and $g(\theta)$ is the solid line. Function g is a horizontal and vertical translation of function f. Write an equation for $g(\theta)$.



11. Find the amplitude, period, phase shift, and vertical shift for each of the following functions:

a) $y = \frac{1}{3}\cos(x - \frac{\pi}{2})$

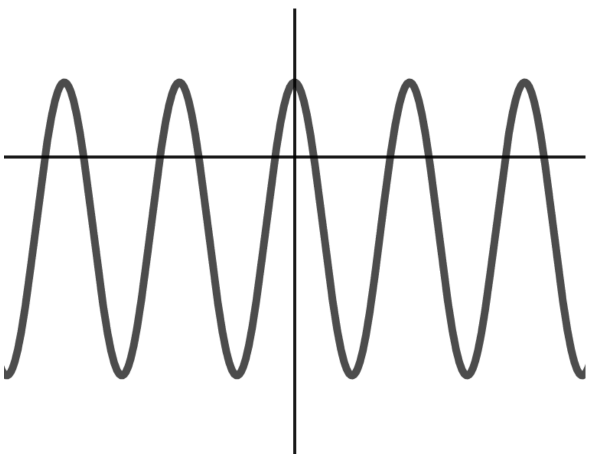
b) $y = \sin(2x + \frac{\pi}{3})$

c) $y = -\cos(x - \frac{\pi}{6}) - 5$

d) $y = -1 + 3\cos(3x + \frac{\pi}{2})$

For Problem 12-15, match the graph with it function. Explain your reasoning.

12.

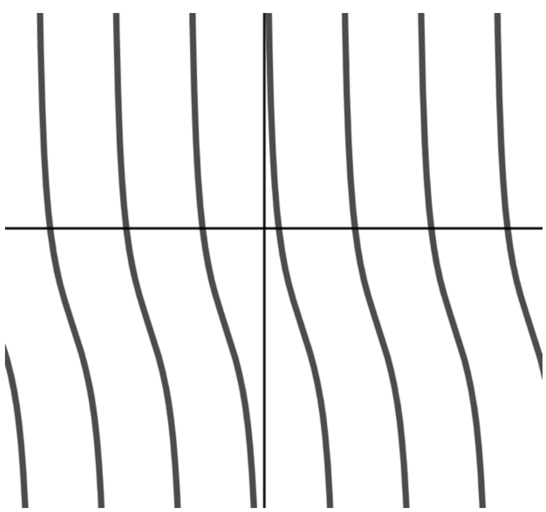


$y = 2\cos(2x - \pi)$

$y = 2\cos(4x - 2\pi) - 1$

$y = \cos(2x - \pi) - 3$

13.

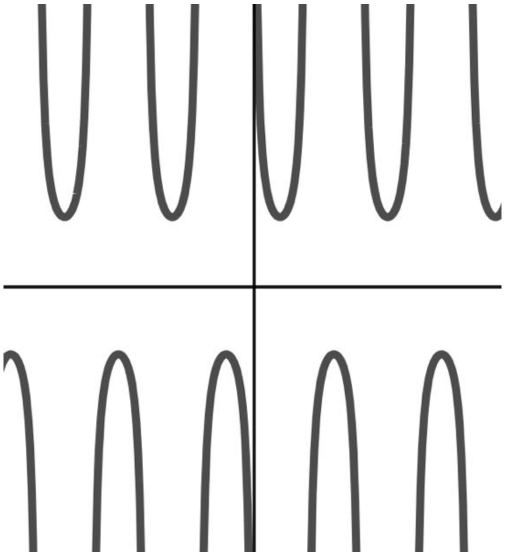


$y = 3\cot(x) - 4$

$y = 3\tan(x + \frac{\pi}{4})$

$y = \frac{1}{2}\cot(x - \frac{\pi}{4})$

14.

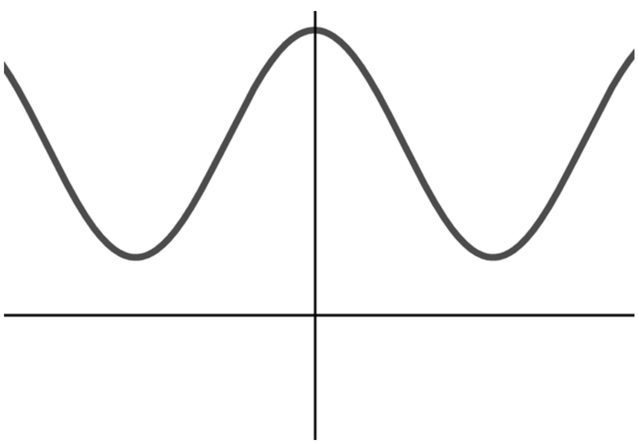


$y = \csc(x - \frac{\pi}{2})$

$y = \csc(x + \pi) + 4$

$y = \csc(4x)$

15.



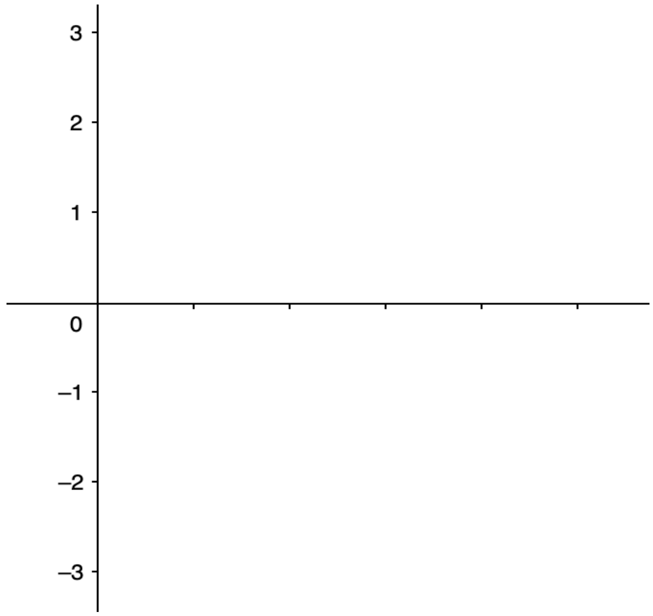
$y = -3\cos(2x)$

$y = 2\cos(x + 3)$

$y = 2\cos(x) + 3$

For Problem 16-20, use the tables to complete the critical points for the transformed function and draw a graph. Label each axis as accurately as possible. Draw the mid-line as a dashed line.

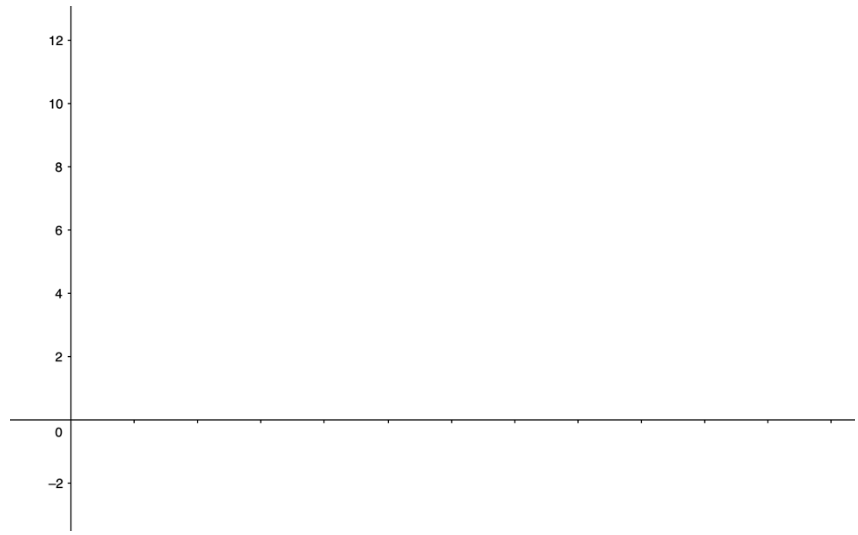
16. $y = 2\sin(x - \frac{\pi}{2})$



$+\frac{\pi}{2}$	x	$\sin x$	$\cdot 2$

x	y

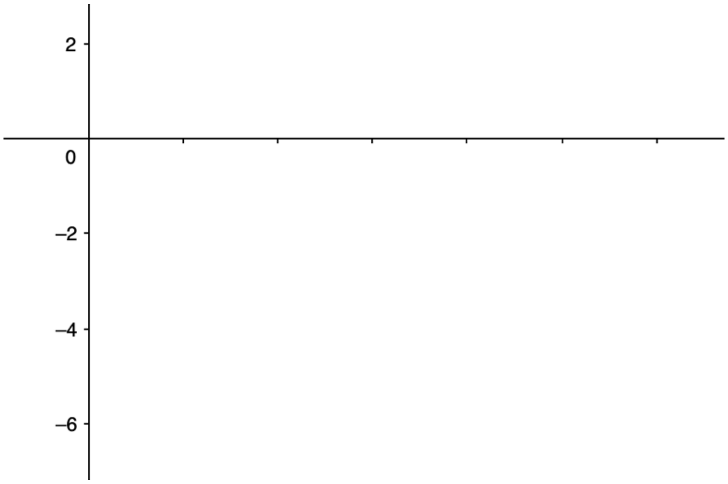
17. $y = 4\cos 3\,(\theta + 10) + 7$



-10	$\div 3$	θ	$\cos\theta$	$\cdot 4$	$+7$

θ	$\cos\theta$

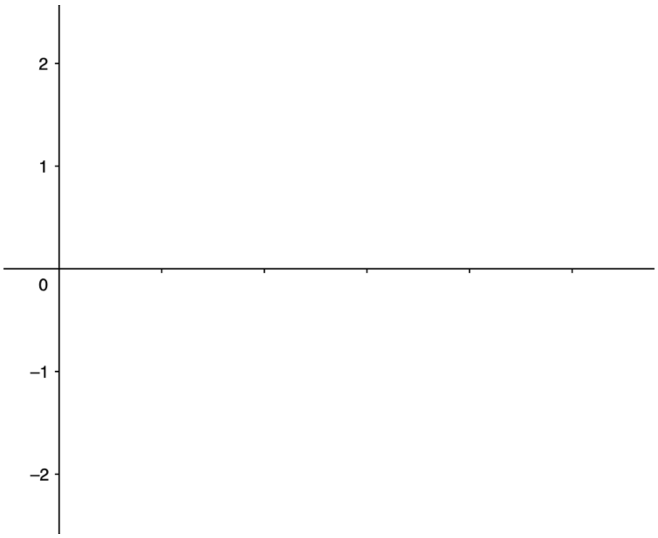
18. $y = 3\sin 2\left(x - \frac{\pi}{4}\right) - 2$



$+\frac{\pi}{4}$	$\div 2$	x	$\sin x$	$\cdot 3$	-2

x	$\sin x$

19. What two types of transformations occur from the parent function of $y = \sin x$ to obtain the function $y = 2\sin(x - \frac{\pi}{2})$? Complete the table and graph $y = \sin x$ in one color and $y = 2\sin(x - \frac{\pi}{2})$ in another color to check.

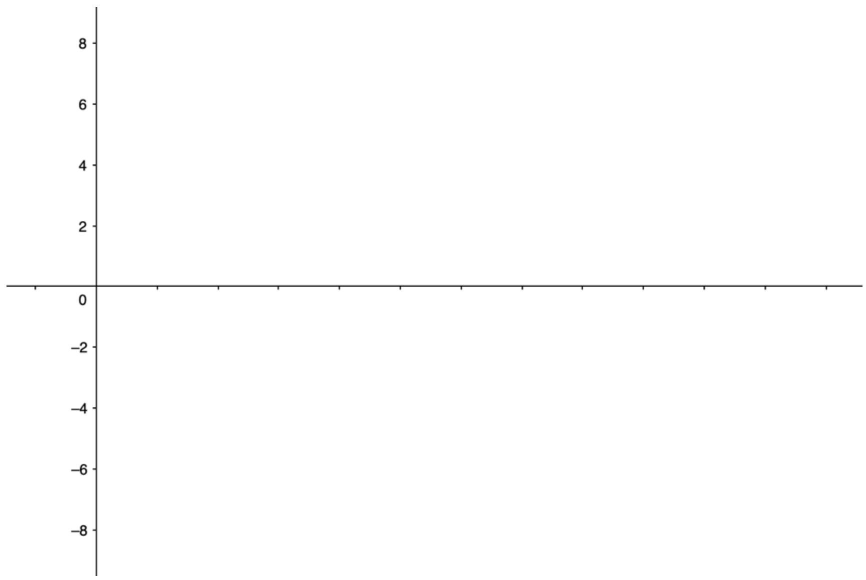


$$y = 2\sin(x - \frac{\pi}{2})$$

	x	$\sin x$	

x	$\sin x$

20. Write and graph the equation of a cosine function that has an amplitude of 5, horizontal shift right of $\frac{\pi}{4}$, a vertical shift down of 3, and a period of 4π.



$$y = 5\cos\frac{1}{2}\left(x - \frac{\pi}{4}\right) - 3$$

$-\frac{\pi}{4}$	$\cdot 2$	x	$\cos x$	$\cdot 5$	-3

x	$\cos x$

Section 7.13 Trigonometric Functions in the Real-World

Practice Problems 7.13

In the experiment from the lesson notes, place the folder behind the part of the diagram labeled "Air." Place the laser at each 10° angle on either side of 0° through the Jell-O® (again, wear goggles and do not point the laser at your eyes). You will see the laser light come out of the Jell-O® onto the angles of the paper marked "Air."

For Problem 1-4, solve the word problem.

1. At each angle of incidence, does the refracted ray move closer to or farther away from the normal? Does the refracted ray move inward or outward?

2. Use Snell's Law and the 10° angle of incidence to find the index of refraction for Jell-O.®

3. Total internal reflection occurs when a ray of light within a medium is reflected back into the medium (Jell-O® in our experiment). At what angle of incidence did you first observe total internal reflection?

4. As the index of refraction increases, what happens to θ (the angle of refraction)? The index of refraction for a diamond is 2.4 and the critical angle for total internal reflection occurs at 25°. Use this information to explain why diamonds sparkle.

The linear velocity (v) of a point on a rotating object is the ratio of the distance the point travels on the circular path to the units of time: $v = \frac{s}{t}$, where s is the arc length.

The angular velocity (ω) of a point on a rotating object is the ratio of the number of radians, revolutions, or degrees through which the point turns to the units of time: $\omega = \frac{\theta}{t}$.

Angular velocity of an object moving around a circle is the rate of change of the measures of the arc between an object and its starting position with respect to time.

For Problem 5-9, use the given information to solve the problem.

The Navy Pier Ferris wheel in Chicago rotates to heights of 196 feet. A person enters a seat and is locked in the Centennial wheel. One ride is twelve minutes and is two revolutions.

5. a) What is the linear speed of a person riding on the Ferris wheel? Find the ratio of $2\pi r$ (the circumference) to the time of one revolution. The speed will be the distance traveled in one unit of time measured in feet per second. Since $s = r\theta$, then $\frac{s}{t} = \frac{r\,\theta}{t}$ and $\frac{s}{t} = r \cdot \frac{\theta}{t}$.

b) What is the angular speed of the rider's rotation? Find the ratio of the radians of one revolution to the time of one revolution.

c) What is the distance the Ferris wheel travels in one revolution?

Each minute hand of the Big Ben clock in England is 4.2 meters long and weighs 100 kg. (approximately 220 pounds).

6. a) What is the speed of the tip of the minute hand?

b) What is the angular speed of the minute hand's rotation?

c) What is the distance the minute hand travels in 30 seconds?

If you know the latitude and longitude of two cities, the formula to find the distance between them is as follows:

$$D = \frac{x \cdot r}{180} \cos^{-1}(\sin \emptyset_A \sin \theta_A + \cos \emptyset_B \cos \theta_B \cos(\theta_A - \theta_B))$$

r = radius of the earth in miles

$\emptyset_A$ = latitude of city A

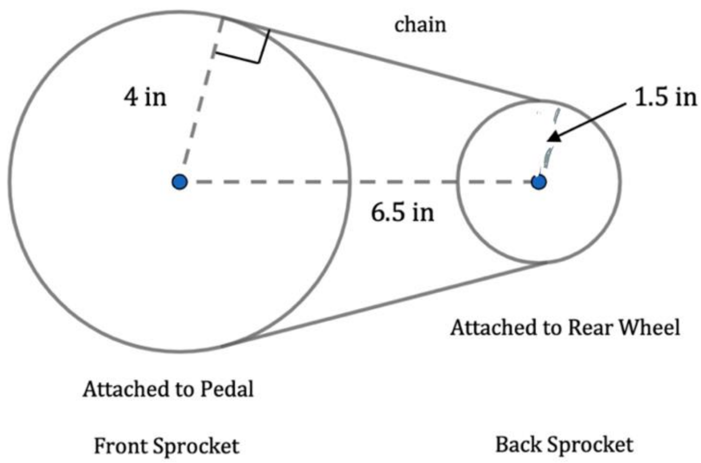
$\emptyset_B$ = latitude of city B

θ_A = longitude of city A

θ_B = longitude of city B

7. Find the distance between Ft. Lauderdale, Florida $(26.1224°\text{ N}, 80.1373°\text{ W})$ and Paris, France $(48.8566°\text{ N}, 2.3522°\text{ E})$.

8. Assume a small bicycle has a large front sprocket that is attached to the pedals at the center and a small back sprocket that is attached to the rear wheel at the center. The sprockets are connected by a chain. The rotating sprockets connected by the pedal have the same angular velocity.



a) If a cyclist turns the pedals at 6 rad/s, is this the linear velocity or the angular velocity?

b) What is the linear velocity of all points on the chain of the bicycle? (This is the same as the linear velocity of the front sprocket.)

c) Is the linear velocity of the back sprocket the same as the linear velocity of the front sprocket or different?

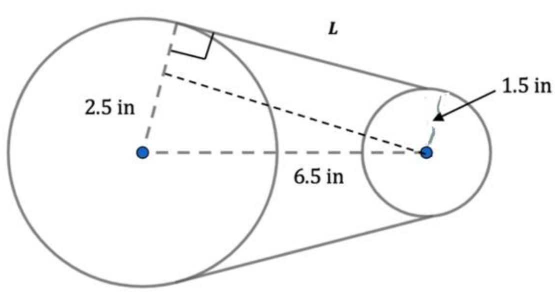
d) Is the angular velocity of the back sprocket at the center the same as the angular velocity of the rim of the back sprocket or different?

e) What is the angular velocity of the back sprocket?

f) If the rear wheel has a spoke with a diameter of 28 inches, how fast is the bicycle going in miles per hour?

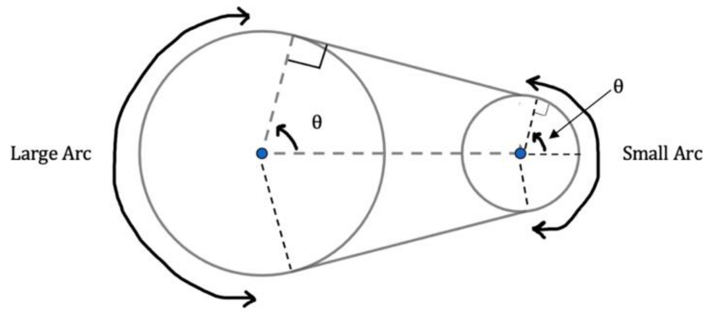
g) Find the length of the pulley belt, which is the chain of the bicycle. To do this, find the length of each straight segment between the front sprocket and back sprocket. They are the same length. Then find the arc length around the large back sprocket and the arc length around the small front sprocket and find the sum of all four lengths.

I.



The length of the segment between the two sprockets (L) is equal to the long leg of the right triangle. Find the length of L.

II.



The angle θ is part of a right triangle. The adjacent side of the angle is 2.5 inches and the hypotenuse is 6.5 inches. Therefore, use inverse cosine to find θ, and $s = r\theta$ for the length of the circular arc. Subtract double this from the circumference of the large circle to get the length of the large arc.

III. Find the length of the small arc using the same method you used in part II.

IV. Find the length of the chain by adding all the parts together.

Section 7.14 Module Review

For Problem 1-4, complete the chart to determine the sign value of the trigonometric function given the angle magnitude and the quadrant location.

	Angle Magnitude	Quadrant Location	Cosine	Sine	Tangent
1.	$0 < \theta < \frac{\pi}{2}$	I	$+$		$+$
2.	$\frac{\pi}{2} < \theta < \pi$	II	$-$		$-$
3.	$\pi < \theta < \frac{3\pi}{2}$	III		$-$	$+$
4.	$\frac{3\pi}{2} < \theta < 2\pi$	IV	$+$	$-$	

For Problem 5-8, tell whether the statement is true or false.

5. $\cos 35° = \cos(-35°)$

6. $\sin(-\frac{3\pi}{2}) = -1$

7. If θ is in Quadrant II, then tan θ is positive.

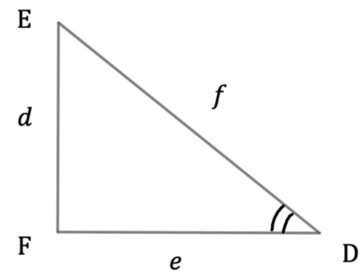
8. The values of sin θ for $0 < \theta < \pi$ are positive.

For Problem 9 and 10, solve the word problem.

9. Firefighters must place a ladder against a building at $70°$ to the ground. The building is $29'$ high. Should they use a 30-foot ladder or 40-foot ladder to reach the top of the building?

10. If a hand on a clock tower is rotated one revolution counterclockwise, what is the degree of the angle generated by a $\frac{1}{8}^{\text{th}}$ revolution of the clock hand counterclockwise?

For Problem 11-13, given the triangle DEF, find the trigonometric ratio.



11. csc D

12. sec D

13. cot D

For Problem 14-17, use the function $y = 2\sin 2x$ to solve the problem.

14. What is the amplitude of the function?

15. What is the period of the function?

16. Sketch the graph of the trigonometric function.

For Problem 17-20, use $y = 4\sin(\frac{1}{2}x - \frac{\pi}{4}) + 3$ to solve the problem.

17. What is the amplitude of the function?

18. What is the period of the function?

19. What is the horizontal phase shift of the function?

20. What is the vertical displacement of the function?

Section 7.15 Module Test

For Problem 1-4, complete the table to determine the sign value of the trigonometric function given the angle of magnitude and the quadrant location.

	Angle of Magnitude	**Quadrant Location**	**Cosine**	**Sine**	**Tangent**
1.	$0 < \theta < 90°$	I		$+$	$+$
2.	$90° < \theta < 180°$	II		$+$	$-$
3.	$180° < \theta < 270°$	III	$-$	$-$	
4.	$270° < \theta < 360°$	IV		$-$	$-$

For Problem 5-8, tell whether the problem is true or false.

5. $\sin 155° = \sin(-155°)$

6. $\cos(\frac{3\pi}{2}) = 0$

7. As θ increases from 0 to $\frac{\pi}{2}$, $f(\theta) = \cos\theta$ also increases.

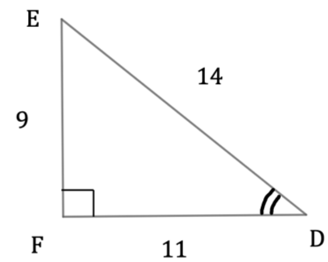
8. The $\tan(-\frac{3\pi}{2})$ is positive.

For Problem 9 and 10, solve the word problem.

9. A skateboarder can choose a $7'$ ramp at an angle of $35°$ or a $10'$ ramp at an angle of $25°$ to do a stunt. The skateboarder must reach a height of $4'$ from the ground by the end of the ramp to do the trick. Which ramp should the skateboarder use?

10. The hand of a clock in a tower is rotated $\frac{1}{12}$th of a revolution clockwise. What is the degree and sign of the rotation of the clock hand?

For Problem 11-13, given the triangle DEF, find the trigonometric ratio.



11. $\csc \mathrm{D}$

12. $\sec \mathrm{D}$

13. $\cot \mathrm{D}$

For Problem 14-16, use the function $y = -2\cos x$ to solve the problem.

14. What is the amplitude of the function?

15. What is the period of the function?

16. Sketch the graph of the trigonometric function.

For Problem 17-20, use the function $y = \frac{1}{2}\sin(3x + \pi) - 2$ to solve the problem.

17. What is the amplitude of the function?

18. What is the period of the function?

19. What is the horizontal phase shift of the function?

20. What is the vertical displacement of the function?