Pre-Calculus and Calculus Module 7 Derivatives and Differentiation

Section 7.1 Experimental Rates of Change

Practice Problems 7.1

For today's Practice Problems section, you will perform the Tootsie Roll Pop® experiment. Follow the directions from the Lesson Notes and answer the questions below (which are from the Looking Ahead section).

Time (s.)	Circumference c (cm.)	Radius r (cm.)	Volume V (cm.^3)	ΔV (cm.^3)	$\frac{\Delta V}{\Delta t}$ ($\frac{\text{cm.}^3}{\text{s.}}$)
0					
30					
60					
90					
120					
150					
180					
210					
240					
270					
300					
330					
360					

1. Complete the radius column (**Radius r (cm.)**) of the table. (Divide the circumference by 2π and round to the nearest thousandth.)

2. Draw a time versus radius graph for the data. What do you expect this graph to look like?

3. Find the slope of the tangent line of the time versus radius graph. What does this slope represent?

4. Complete the volume column (**Volume V (cm.^3)**) of the table. (Use the formula for volume of a sphere ($V = \frac{4}{3}\pi r^3$) and round to the nearest ten-thousandth.)

5. Draw a time versus volume graph for the data. What do you expect this graph to look like?

6. Complete the change in volume column (**ΔV (cm.^{3})**) of the table. (Subtract the volume at each 30-second interval from the previous 30-second interval of volume.)

7. Complete the change in volume over the change in time column ($\frac{\Delta V}{\Delta t}$ ($\frac{cm.^3}{s.}$)) of the table. (Divide ΔV by Δt, which is 30 seconds for each interval, and round to the nearest ten-thousandth.)

8. Draw a time versus change in volume graph for the data. What do you expect this graph to look like?

9. Find the average of the change in volume over the change in time column ($\frac{\Delta V}{\Delta t}$ ($\frac{cm.^3}{s.}$)). (Add up all the $\frac{\Delta V}{\Delta t}$ values and divide by the total number of values, which is 12.)

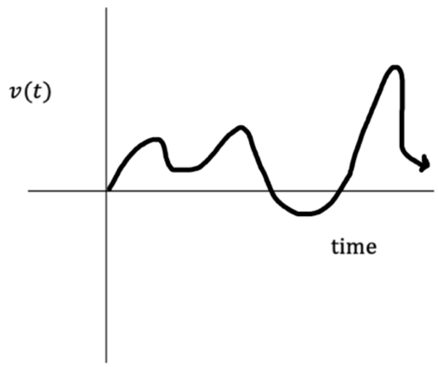
10. What is the average value of the rate of change of the volume of the Tootsie Pop®? How does this compare to the slope of the time versus volume graph?

Section 7.2 Acceleration Versus Velocity Graphs

Practice Problems 7.2

For Problem 1-4, use the given information and graph to solve the problem.

The graph $v(t)$ represents time versus velocity of an object.



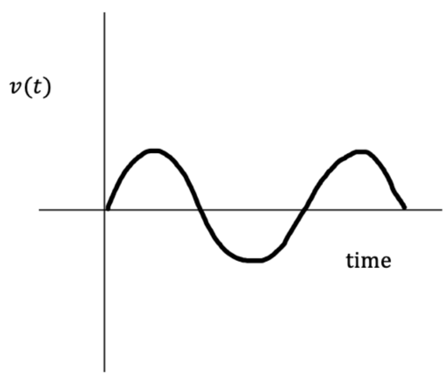
1. Use a yellow colored-pencil to mark the place on the graph where the velocity is greatest.

2. Use a red colored-pencil to draw the speed graph.

3. Use a blue colored-pencil to draw the place on the graph where the speed is greatest.

4. Why are the greatest speed and greatest (most positive) velocity at the same place?

For Problem 5-8, use the given information and graph to solve the problem.



5. Use a yellow colored-pencil to mark the place on the graph where the velocity is greatest.

6. Use a red colored-pencil to draw the speed graph.

7. Use a blue colored-pencil to draw a place on the graph where the speed is the greatest.

8. Why are the greatest speed and the greatest velocity at the different places.

For Problem 9-16, see if you can discover any patterns given the functions and derivatives; use the patterns to solve the problem.

9. An airplane accelerates down a runway at 4.1 m/s^2 for 34.4 seconds and then lifts off the ground. Determine the distance traveled before takeoff.

10. A race car accelerates uniformly from 18.2 m/s to 51.5 m/s in 2.98 seconds. Determine the acceleration of the car.

11. The largest medical delivery planes accelerate at 3.1 m/s^2 and have a takeoff speed of 92.3 m/s. What is the minimum length of the runway needed for these planes to take off?

12. You drive home from Los Angeles to New York City in four days for a total of 2,789 miles. What is your average speed in miles per hour?

13. You are at a stoplight. When the light turns green, you accelerate at 10 m/s^2. After 7 seconds, how far did you travel?

14. A racecar accelerated at 25 m/s^2 around a 3,200-meter raceway. How long did the racecar accelerate for?

(Hint: Use the formula $t = \sqrt{\frac{2d}{a}}$.)

15. Garland's remote-control car is traveling at 4 m/s. He then accelerates it at 1.5 m/s^2 for the next 3 seconds. Garland maintains a constant velocity for the next 7 seconds. What is the total distance the remote-control car traveled in these 10 seconds? Draw the time versus velocity graph to calculate the distance.

16. Alexa is driving at 20 m/s and begins to accelerate at $-1.0\ \text{m/s}^2$ until she comes to a complete stop. Use equations to calculate the distance traveled during Alexa's deceleration. Check your calculations with a time versus velocity graph.

For Problem 17-20, find the average rate of change for the given function over the given interval.

17. $y = 2x^2 + 2x + 2$ over the interval $[0, 1]$

18. $y = 2x^2 - 1$ over the interval $[-2, -1]$

19. $y = x^2 + 1$ over the interval $[-2, 0]$

20. $y = -x^2 - 1$ over the interval $[-2, 1]$

Section 7.3 The Power Rule

Practice Problems 7.3

For Problem 1-20, find the derivative of the function.

1. $f(x) = x^6$

2. $f(x) = -2x^5$

3. $f(x) = 10x$

4. $f(x) = -5x$

5. $f(x) = 5x^5$

6. $f(x) = -8x^3$

7. $f(x) = 3x^{-3}$

8. $f(x) = \frac{3}{x^3}$ (Hint: $\frac{1}{x^3} = x^{-3}$)

9. $f(x) = \frac{2}{x^3}$

10. $f(x) = -\frac{6}{x^2}$

11. $f(x) = -\frac{1}{x^4}$

12. $f(x) = -\frac{5}{x^4}$

13. $f(x) = -\frac{1}{x^6}$

14. $f(x) = 7\sqrt{x}$ (Hint: $\sqrt{x} = x^{\frac{1}{2}}$)

15. $f(x) = 2\sqrt[3]{x^2}$

16. $f(x) = -3\sqrt{x^3}$

17. $f(x) = 3\sqrt{x}$

18. $f(x) = 4\sqrt{x}$

19. $f(x) = -6\sqrt[4]{x^3}$

20. $f(x) = \sqrt[4]{x^7}$

Section 7.4 The Sum Rule

Practice Problems 7.4

For Problem 1-20, find the derivative of the function.

1. $f(x) = -4^5 + 3x^2$

2. $g(x) = -10x^3 + 3x^2 - 4$

3. $h(x) = 22x + 7$

4. $y(t) = -0.3t^2 - 1.2t$

5. $y(t) = 6t^3 - 6t^2 + 6t$

6. $y(t) = 10t^2 - 10t$

7. $f(x) = 3x^3 - 2x^2$

8. $g(x) = -x^5 - x^4 + 3$

9. $f(x) = x^6 + 2x^5$

10. $f(x) = 5x^3 + \frac{3}{x}$

11. $f(x) = \frac{2}{x} + 5\sqrt{x}$

12. $f(x) = 3x^4 - x^3 + x$

13. $f(x) = \sqrt[3]{x} + 1$

14. $f(x) = 1 - 6\sqrt[4]{x^3}$

15. $f(x) = 2x^2 + 5\sqrt[3]{x}$

16. $f(x) = 4x^5 + \frac{4}{x^2} - 2\sqrt{x^3}$

17. $f(x) = 3x^3 + 2x^2 + x$

18. $f(x) = 3 + x^2 + x^3$

19. $f(x) = 2x - 4$

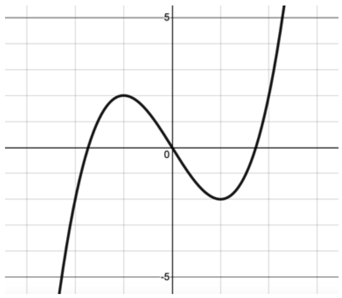
20. $f(x) = 1 + x^2 - 5x^4$

Section 7.5 Finding the Derivative Graphically

Practice Problems 7.5

For Problem 1-10, match the function graph with its derivative graph.

1.



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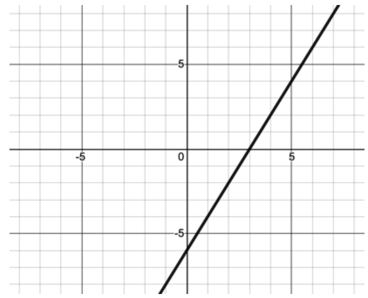

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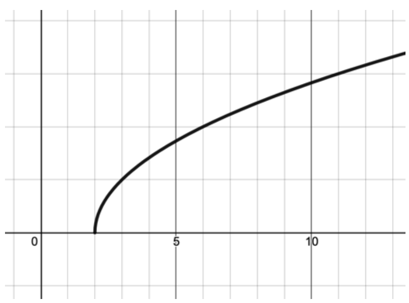

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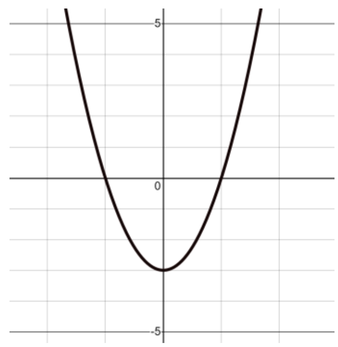

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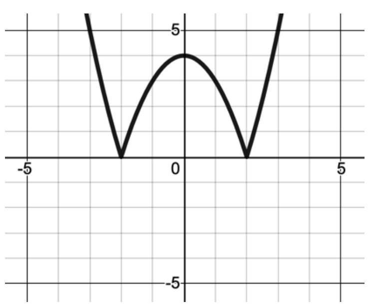

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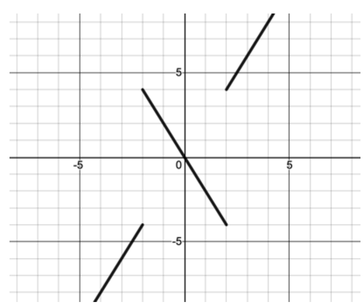

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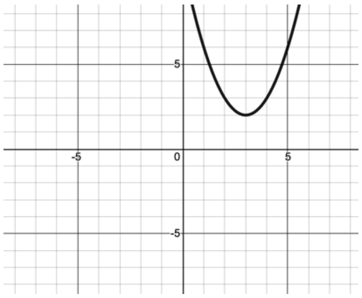

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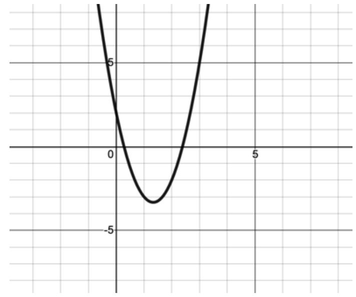

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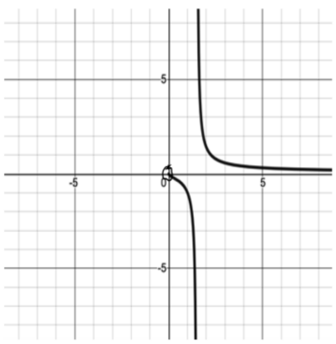

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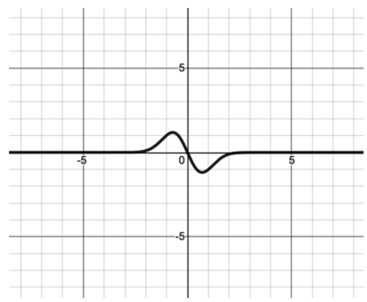

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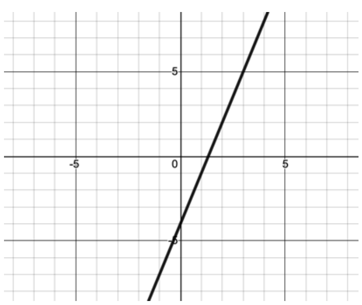

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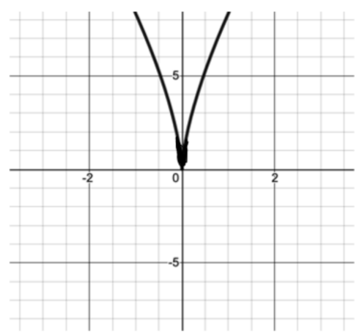

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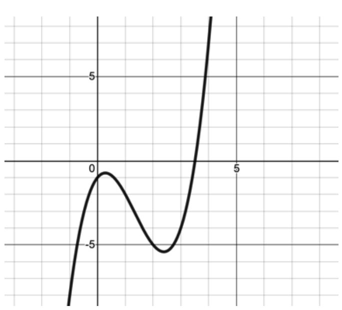

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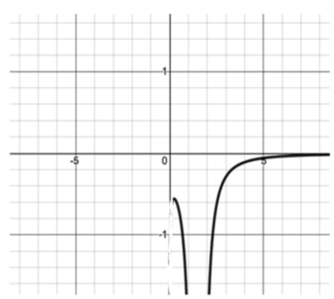

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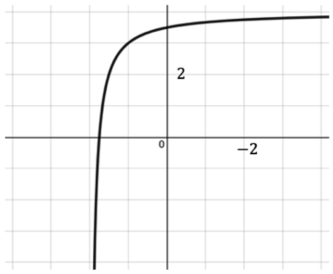

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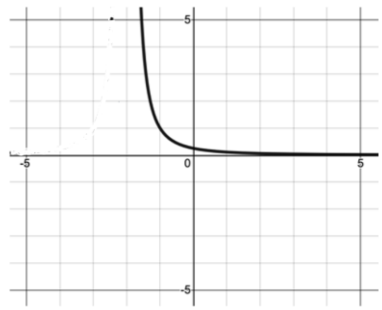

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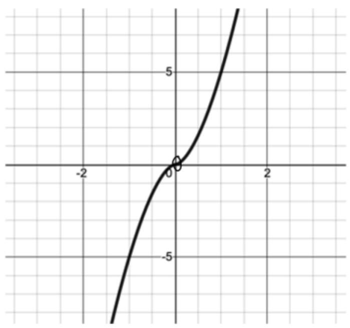

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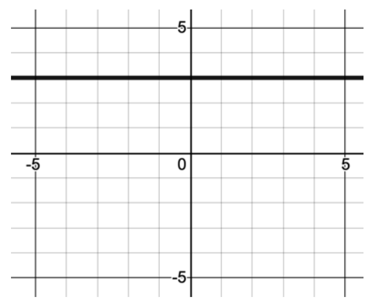

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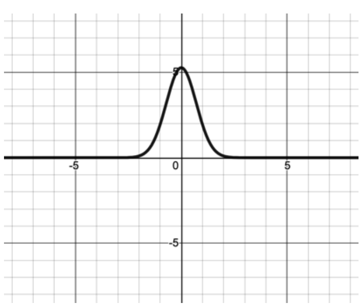

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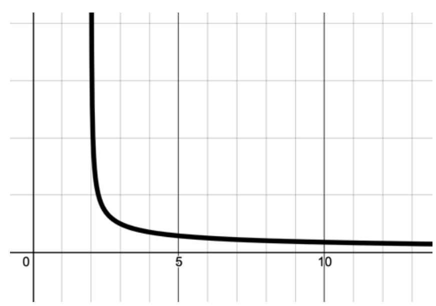
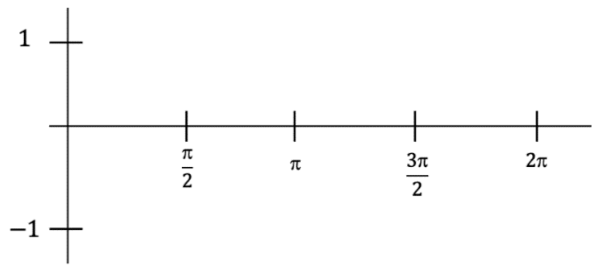


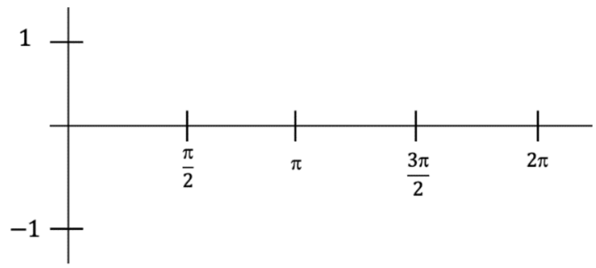
Section 7.6 The Derivatives of Sine and Cosine

Practice Problems 7.6

For Problem 1 and 2, draw the graphs given the information and use them to solve the problem.

1. Draw the graph of $g(x) = \cos x$ and $g'(x)$ just below it.





2. Explain why the derivative of $\cos x$ looks the way it does.

For Problem 3-6, find the derivative of y with respect to x ($\frac{dy}{dx}$).

3. $y = 4x$

4. $y = -3x^5$

5. $y = -10$

6. $y = -x^4$

For Problem 7-10, use the given information to choose the correct derivative with respect to x for the function.

The notation for the first derivative used by Leibniz was $\frac{dy}{dx}$ and the notation for the second derivative was $\frac{d^2y}{dx^2}$.

7. $y = -4x^2$

a) $\frac{d^2y}{dx^2} = -8$

b) $\frac{d^2y}{dx^2} = -4$

c) $\frac{d^2y}{dx^2} = -16x^2$

d) $\frac{d^2y}{dx^2} = -4x$

8. $y = 3x^2$

a) $\frac{d^2y}{dx^2} = 3$

b) $\frac{d^2y}{dx^2} = 6$

c) $\frac{d^2y}{dx^2} = 9x^4$

d) $\frac{d^2y}{dx^2} = 12x^3$

9. $y = x^4$

a) $\frac{d^2y}{dx^2} = x^8$

b) $\frac{d^2y}{dx^2} = 4x$

c) $\frac{d^2y}{dx^2} = 12x^2$

d) $\frac{d^2y}{dx^2} = 3x^5$

10. $y = -4x$

a) $\frac{d^2y}{dx^2} = 0$

b) $\frac{d^2y}{dx^2} = -8$

c) $\frac{d^2y}{dx^2} = 16x^2$

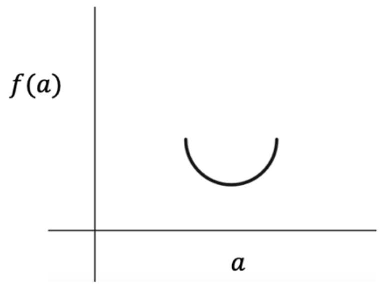
d) $\frac{d^2y}{dx^2} = -4x^2$

Section 7.7 Local and Global Extrema and Critical Points

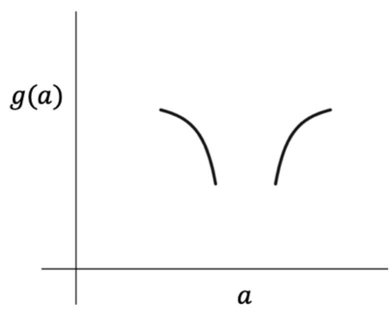
Practice Problems 7.7

For Problem 1 and 2, use the graph given to solve the problem.

1. Given the plot of $f(x)$, what can you say about the second derivative at the value $x < a$, $x = a$ and $x > a$.



2. Given the plot of $g(x)$, what can you say about the second derivative at the value $x < a$, $x = a$ and $x > a$.



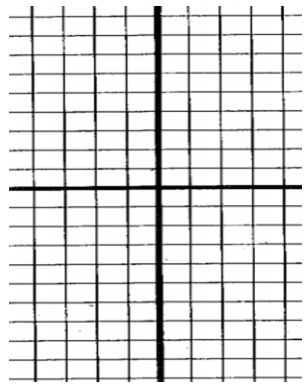
For Problem 3-8, use the given information to solve the problem.

In Example 3 from the Lesson Notes, $h(x)$ was equal to $\frac{1}{4}x^4 + \frac{5}{3}x^3 + 3x^2$ and $h'(x)$ was equal to $x^3 + 5x^2 + 6x$. This was factored to $h'(x) = x(x+3)(x+2)$ and resulted in x-intercepts at $x = 0$, $x = -3$, and $x = -2$.

3. Find $h''(x)$.

4. Find $h''(0)$, $h''(-3)$, and $h''(-2)$.

5. The critical points of the first derivative are 0, -3, and -2. Calculate and plot the second derivative at these critical points. What does the curve resemble?

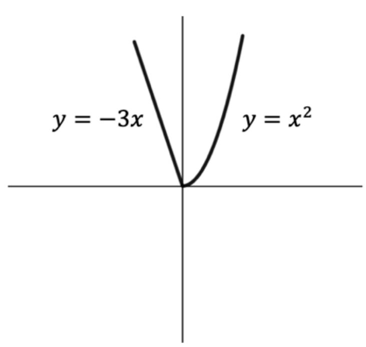


6. Recall that a function is concave up when the second derivative is positive. Is this a local minimum or local maximum?

7. If the second derivative is negative, the function is concave down. Is this a local minimum or a local maximum?

8. From Problem 4, does h have a local minimum or local maximum at $h''(0)$, $h''(-3)$, and $h''(-2)$?

For Problem 9, use the graph below to answer the questions that follow.



a) What is the slope of the tangent line on the left side of the graph when $x = 0$?

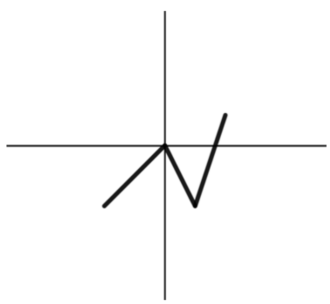
b) What is the slope of the tangent line on the right side of the graph when $x = 0$?

c) Is the graph continuous over the given interval?

d) Does the derivative exist at $x = 0$?

For Problem 10-12, use the graph given to answer the questions.

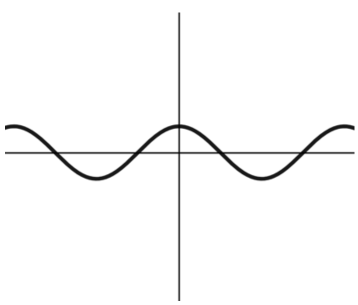
10.



a) Is the graph continuous at every point in its domain?

b) Is the graph differentiable for every point in its domain?

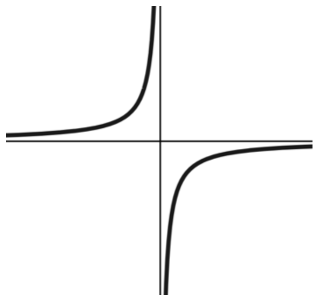
11.



a) Is the graph continuous at every point in its domain?

b) Is the graph differentiable for every point in its domain?

12.



a) Is the graph continuous at every point in its domain?

b) Is the graph differentiable for every point in its domain?

For Problem 13, solve the multiple-choice problem.

13. Circle the type of function that is not differentiable at every point in its domain.

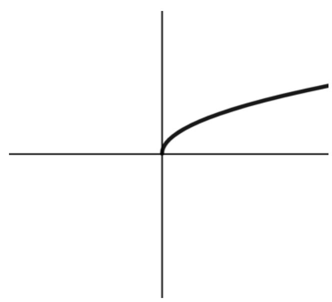
a) Polynomial Functions

b) Trigonometric Functions

c) Rational Functions

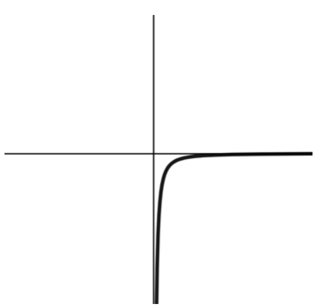
d) Step Functions

For Problem 14-17, use the graph of the function $f(x) = \sqrt[2]{x}$ to solve the problem.



14. Which graph below is f' and which is f''?

a)



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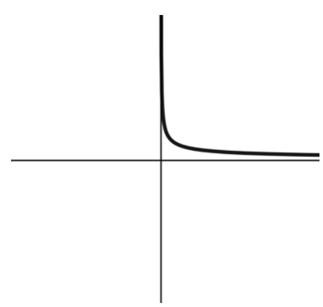


15. Tell whether the graph is increasing or decreasing over its domain.

a) f over the domain $[0, \infty)$

b) f' over the domain of f

c) $f"$ over the domain of f

16. Tell whether the graph is concave up or concave down.

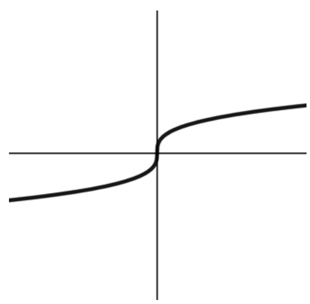
a) f

b) f'

c) $f"$

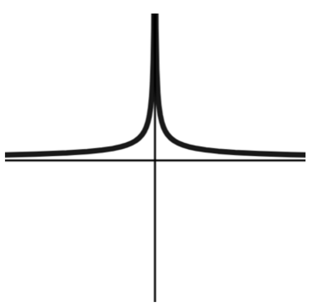
17. Are there any extreme values or points of inflection for f?

For Problem 18-20, use the graph of the function $g(x) = \sqrt[3]{x}$ to solve the problem.

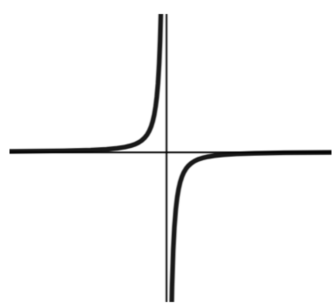


18. Which graph below is g' and which is $g"$?

a)



b)



19. Answer the following questions regarding the graphs above.

a) When is $g' > 0$?

b) Is g always increasing or decreasing?

c) Are there any extreme values for g?

d) What is $g'(x)$?

20. Answer the following questions regarding the graphs above.

a) When is $g" > 0$?

b) Is g concave up or concave down over $(-\infty, 0)$?

c) When is $g" < 0$?

d) Is g concave up or concave down over $(0, \infty)$?

e) Where is the point of inflection for g?

f) What is $g"(x)$?

Section 7.8 Finding the Equation of the Tangent Line

Practice Problems 7.8

For Problem 1-8, use the information given to solve the problem.

1. Find the equation to the tangent line for the graph of $f(x) = x^3 - 2x^2 + 3x - 5$ at the fixed point $x = 3$.

2. Use your calculator to draw $f(x)$ and $t(x)$ from Problem 1.

3. Complete the table for $f(x)$ and $t(x)$ as x gets closer and closer to a. In this case, the fixed point is $x = 3$ so a is equal to 3. How does the table demonstrate that $t(x)$ is the tangent of $f(x)$ at $x = 3$?

$\boldsymbol{x}$	$\boldsymbol{f(x)}$	$\boldsymbol{t(x)}$	**Error** $= \boldsymbol{f(x) - t(x)}$
2.90			
2.95			
2.99			
3			
3.01			
3.05			
3.10			

4. Let $g(x)$ be $0.2x^4$. Find $g'(x)$ and $g'(2)$. Write an equation for the tangent line at $x = 2$.

5. Sketch a graph of $g(x)$ and the tangent line from Problem 4 to verify that your answer is the tangent line to the function.

6. Let $h(x) = x^2 + 5x - 2$. Find $h'(1)$ and $h'(2)$. At the local extrema, the tangent line will be horizontal and have a slope of 0. Set $h'(x) = 0$ and solve for x algebraically. Graph the equation and use the trace button on the graphing calculator to find the maximum and/or minimum to see if these values agree.

7. Let y be $x^4 - 2x^2$ and follow the steps to find out if the equation has any horizontal tangents. First, calculate $\frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^2)$. Next, set the equation for $\frac{dy}{dx}$ equal to 0 and solve for x. Find the corresponding points on the curve of the equation and verify these using a calculator.

8. Given the function $y = x^2 - 2x + 3$, show that the tangent line is horizontal at the point $(1, 2)$.

For Problem 9-11, find the derivative of the function at the given value to solve the problem.

9. $y = \frac{x^2}{2} + 3x - 4$ at $x = 1$

10. $y = 5x^4 + 2x^2 - 5$ at $x = -2$

11. $y = -2x^3 - 2x^2 - 2x$ at $x = 0$

For Problem 12-14, find the equation of the line tangent to the given function at the given point.

12. $y = \frac{x^2}{2} + 2x - 2$ at $x = 2$

13. $y = x^2 + 5x + 4$ at $x = -1$

14. $y = \frac{1}{3}x^3 - 2x$ at $x = 3$

For Problem 15-20, use the given information and table to solve the problem.

Let us revisit the Tootsie Roll Pop® average rate of change of volume problem from Section 7.1. Previously, we calculated this value using what we knew about tangent lines and limits, average rate of change, and instantaneous rates of change. Now that we have learned about derivatives, let us use this method to calculate the value of the average rate of change of volume of a Tootsie Roll Pop®.

Below is the table from Section 7.1 of the Tootsie Roll Pop® experiment.

Time (s.)	**Radius r (cm.)**	**Volume V ($cm.^3$)**	$\frac{dv}{dt}$
0	1.5120	14.4784	
30	1.4642	13.1496	
60	1.4324	12.3105	
90	1.4001	11.5079	
120	1.3528	10.3706	
150	1.3201	9.6557	
180	1.2732	8.6461	
210	1.2255	7.7094	
240	1.1937	7.1242	
270	1.1459	6.3030	
300	1.1141	5.7922	
330	1.0663	5.079	
360	1.0027	4.2225	

15. Find the formula for $\frac{dv}{dt}$ (the rate of change of volume over time). Find the derivative of the formula for volume of a sphere.

16. What is $\frac{dr}{dt}$? Use the slope of the line of best fit (slope of the tangent line) for the time versus radius graph.

17. Complete the $\frac{dv}{dt}$ column of the table. Substitute the radius at each 30-second interval and the $\frac{dr}{dt}$ from Problem 16 into the derivative from Problem 15.

18. Graph the rate of change in volume with respect to time. Explain why the data looks smoother than the same graph in Problem 8 of Section 7.1.

19. Find the value of the average of the change in volume over time by adding up all the rates of change of volume and dividing by the number of rates of change of volume, which is 13.

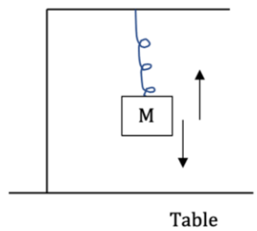
20. What is the rate of change of the volume of a Tootsie Roll Pop®? Why is this the most accurate method of finding this rate of change?

Section 7.9 The Chain Rule

Practice Problems 7.9

For Problem 1-4, use the given information to solve the problem.

A mass (M) is bouncing up and down on a spring. Its displacement is measured as the height above the table on which the spring and mass are stationed. As the mass is traveling upward at a decreasing rate, its velocity is positive and its acceleration is negative. Assuming the mass has been released and is springing up and down above the table, answer the questions that follow.



1. As the mass travels downward at a decreasing speed but increasing rate, is the velocity positive or negative? Is the acceleration positive or negative?

2. If the mass is moving toward the table while slowing down, is the velocity positive or negative? Is the acceleration positive or negative?

3. If there is no acceleration when the mass is springing upward, what can be said about the rate of change? What can be said about the velocity?

4. When would the mass have zero velocity and negative acceleration?

For Problem 5-8, use the Chain Rule to find the derivative of the expression/equation.

5. Differentiate $g(x) = (3x^5 + 10)^{1.2}$.

6. Find y' given $y = \sqrt{5x^2 + 2x}$.

7. $\frac{d}{dx} 3(\sin x)^2$

8. $h(x) = (-2x^4 + x^3 - 5x^2 + x)^4$

For Problem 9-12, find the differential $[dy]$. The function must be smooth and not vertical at any point.

9. $y = -x^3 - 1$

10. $y = x^2 - 2x + 5$

11. $y = \sqrt[5]{x}$

12. $y = 2x^3 - 9x + 7$

For Problem 13-16, given the information, find the velocity function, $v(t)$, and acceleration function, $a(t)$, of the function given.

A particle moves along a horizontal line whose position is given as the function $s(t)$ in which $t \geq 0$.

13. $s(t) = -t^3 + 10t^2$

14. $s(t) = -t^3 + 9t^2 - 22t$

15. $s(t) = t^2 - 14t + 18$

16. $s(t) = t^2 - 15t - 130$

For Problem 17-20, use the Chain Rule to differentiate the function given with respect to x.

17. $y = (-3x^5 - 10)^4$

18. $y = (x^7 + 4)^3$

19. $y = (x^3 - 4x)^3$

20. $y = (-5x^2 - 5)^4$

Section 7.10 The Product Rule

Practice Problems 7.10

For Problem 1-3, differentiate the function with respect to x.

1. $y = x^3 + 5x - 11$

2. $y = \cos x \sin x$

3. $y = (2x^3)(-x^4)$

For Problem 4-10, use the information given to solve the problem.

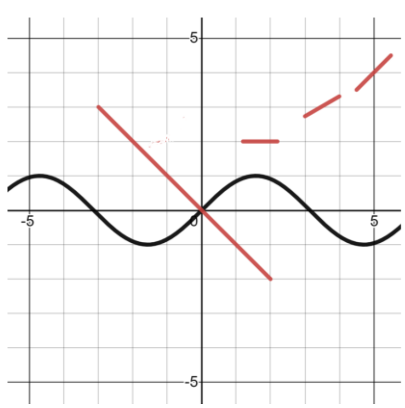
4. Find $\frac{dy}{dx}$ if $y = 3 \sin x$

5. Find $p'(x)$ if $p(x) = 3^{-4}$

6. Find $\lim_{x \to 3} \frac{(x+8)(x-3)}{x-3}$

7. If $y' = u'vw + uv'w + uvw'$, what does y equal?

8. Below is the graph of $y = \sin x$. There are four lines with the slope values of $-1,\ 0, \frac{\sqrt{3}}{3}$, and 1. Trace the x and y-axis and slope lines on tracing paper. Move the slope lines over the sine curve; keep the x-axis parallel and the y-axis parallel. See where each slope line fits along the curve. Use the graph to answer the questions that follow.



a) Which slope line is tangent to the sine curve at $x = -\pi$ or $x = -3.14$ and $x = \pi$ or $x = 3.14$?

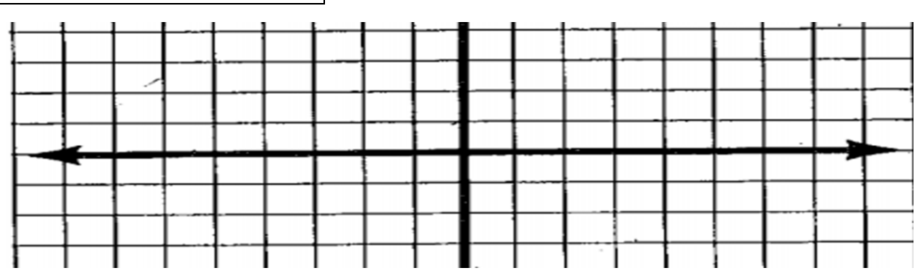
b) Which slope line is tangent to the sine curve at $x = -\frac{\pi}{2}$ or $x = -1.6$, $x = \frac{\pi}{2}$ or $x = 1.6$, and $x = \frac{3\pi}{2}$ or $x = 4.7$, and $x = -\frac{3\pi}{2}$ or $x = -4.7$?

c) One of the segments has a slope of $\frac{\sqrt{3}}{3}$ along the sine curve. At what values of x is it most tangent to the curve?

d) At what value of x is a line with the slope of $-\frac{\sqrt{3}}{3}$ most tangent to the curve?

9. Below are the slope values for several x values along the sine curve. Plot these points on a graph. What curve does this make? It is the derivative of the sine curve.

Approximate x Values	**Slope of the Tangent Line(s)**
$-\pi = -3.14$	-1
-2.2	-0.577
$-\frac{\pi}{2} \approx -1.6$	0
-1	0.577
0	1
1	0.577
$\pi/2 \approx 1.6$	0
2.2	-0.577
$\pi \approx 3.1$	-1
4.1	-0.577
$\frac{3\pi}{2} = 4.7$	0
5.3	0.577
$2\pi \approx 6.28$	1



For Problem 10-12, find $\frac{d^2y}{dx^2}$ given the function.

10. $y = 4x^3$

11. $y = 18x^2$

12. $y = -7x$

For Problem 13-16, find the instantaneous rate of change for the function at the value given.

13. $y = x^3 - 3$ at $x = -2$

14. $y = 2x^2 - x + 5$ at $x = -1$

15. $y = 5x^2 - 8x$ at $x = 0$

16. $y = 4x^3 - 2x^2 + x$ at $x = 3$

For Problem 17-20, find the equation of the tangent line to the function at the point given.

17. $y = 2x^2 - 3x$ at $x = -1$

18. $y = -5x^3 + 6$ at $x = 2$

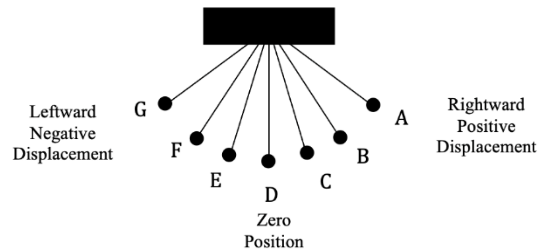
19. $y = -\frac{1}{x}$ at $x = 2$

20. $y = 3x^2 - 14x$ at $x = 5$

Section 7.11 The Quotient Rule

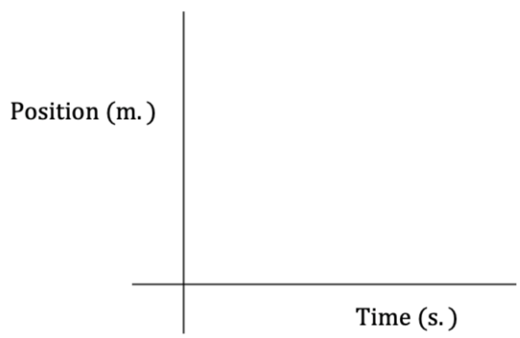
Practice Problems 7.11

For Problem 1-8, use the given diagram and information to solve the problem.



When the bob of a pendulum is completely vertical, the position is zero. Displacement to the right of equilibrium is positive position and displacement to the left of equilibrium is negative position. As the bob moves to the right, the displacement increases positively and then decreases positively back to zero. As the bob swings left of the vertical position, negative displacement increases and then decreases negatively as it swings back to zero.

1. Sketch the displacement graph of this periodic motion.



2. The position of the bob measured along the arc relative to its rest or equilibrium position at 0 is a function

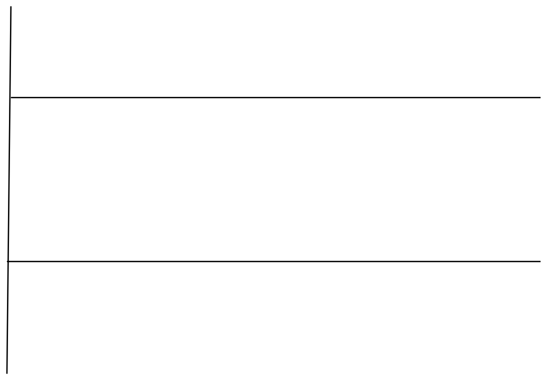
of the ___________________ wave of the time.

3. As the pendulum swings, a motion detector could be used to investigate its velocity with respect to time.

The velocity is least when the ____________________ is greatest.

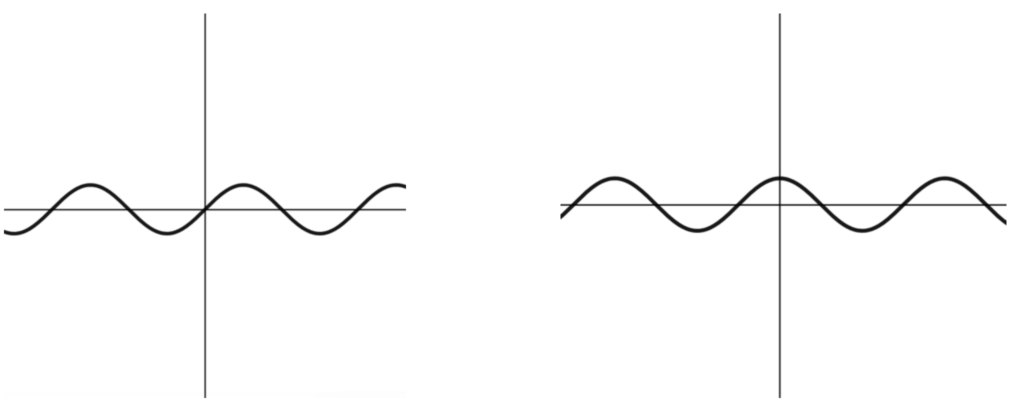
4. Where is the displacement when the velocity of the pendulum is zero?

5. Sketch the graph of velocity versus time on another graph below the position versus time graph. Mark the zero coordinate for velocity when there is zero that corresponds with the displacement from Problem 4.



6. At the equilibrium (resting position), the pendulum's velocity is increasing as the bob approaches it and decreasing as the bob moves away from it. How does the displacement graph relate to the velocity graph for the motion of the pendulum?

7. Using the two graphs below, fill in the blanks. The displacement is represented by the ____________ function and the velocity is represented by the _____________ function.



8. Given your knowledge about displacement, velocity, and derivatives, how does the velocity function relate to the displacement function for the motion of the pendulum?

For Problem 9 and 10, use the information given to solve the problem.

9. Differentiate $\frac{3x^4}{\sin x}$ with respect to x.

10. Use the Quotient Rule to find $h'(x)$ given $h(x) = \frac{(2x)^2}{\cos(x)}$.

For Problem 11-14, use the Sum and Difference Rule to differentiate the function with respect to x.

11. $y = 3x^3 + 5x^2$

12. $y = -2x^4 + 8x$

13. $y = 10x^4 - 2x^2$

14. $y = -x^6 - 4x^5$

For Problem 15-17, use the Product Rule to differentiate the function with respect to x.

15. $y = (4x^3 - 5) \cdot -x^2$

16. $y = 3x^4(2x + 6)$

17. $y = (-4x^2 + 1)(x^2 + 3)$

For Problem 18-20, use the Quotient Rule to differentiate the function with respect to x.

18. $y = \frac{3x^5}{2x^4+1}$

19. $y = \frac{3}{5x^3-2x^2}$

20. $y = \frac{x^2+1}{4x^2+3}$

Section 7.12 Derivatives of Trigonometric Functions

Practice Problems 7.12

For Problem 1-6, use the information given to solve the problem.

1. Given $y = \tan x$, find y'.

2. Differentiate $y = \cot x$.

3. Given $\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}}$, what do you think the derivative of the inverse secant function is? Do not do any differentiation, just look for patterns.

4. Find the derivative of the secant of x: $\frac{d}{dx}(\sec x)$.

5. If the derivative of the inverse tangent function is given, what do you think the derivative of the inverse cotangent function is? Like in Problem 3, do not do any calculations, just look for patterns.

Given $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$, find $\frac{d}{dx}(\cot^{-1}x)$.

6. A bell is bouncing on a spring that is hanging from a window frame. Its distance, y (in centimeters), varies sinusoidally with time, t (in seconds). A complete cycle takes 1.3 seconds. At $t = 0.2$, y reaches a maximum of 6 centimeters. The smallest the distance gets is 2 centimeters at $t = 0.85$.

a) Write an equation for y in terms of t using the general equation for a sinusoidal function:

$y = \mathrm{A}\cos(\mathrm{B}t - \mathrm{C}) + \mathrm{D}$. The amplitude is $|\mathrm{A}|$ (vertical dilation factor) and B is the angular frequency, C is the phase shift and D is the vertical shift.

b) How fast is the bell moving at $t = 1$ and $t = 3$?

For Problem 7-10, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

7. $4x = -3y^3 + 2$

8. $7 = 2x^4 - y^2$

9. $5y^2 + 9 = 5x$

10. $3x^2 - 2 = 9y^3$

Section 7.13 Derivatives of Exponentials and Logarithms

Practice Problems 7.13

For Problem 1-12, use the information given to solve the problem.

1. Use the Sum Rule with the rules you learned for exponents to differentiate $g(x) = 3^x + 2e^x$.

2. Given $g(x) = 3^x + 2e^x - 5$, find $g'(x)$.

3. Find $\frac{d}{dx}(5^x - 1.2e^x + x^3)$

4. If $f(x) = 4^x + e^x$, then $f'(x) = 4^x(1) \cdot \ln 4 + e^x \cdot 1$; where does the first 1 come from?

5. If $f(x) = 4^{2x} + e^x$, find $f'(x)$ using the chain rule and power rules?

6. Given $y = e^x \cdot x^4$, find y'.

7. What rule did you use to find y' in Problem 7?

8. Given $g(x) = e^{[x \cdot \cos 4x]}$, find $g'(x)$.

9. Find $\frac{d}{dt}(2^{\pi t})$.

10. Given $h(t) = \sin(2^{\pi t})$, find $\frac{d}{dt}h(t)$?

11. Is e^{2x} its own derivative?

12. Differentiate 4^x with respect to x.

For Problem 13-16, differentiate the function with respect to x.

13. $y = e^{3x^5}$

14. $y = \ln x^4$

15. $y = \ln 5x^3$

16. $y = e^{2x^2}$

For Problem 17-19, apply the Chain Rule twice to differentiate the function with respect to x.

17. $y = \ln(2 + e^{x^3})$

18. $y = e^{e^{4x^2}}$

19. $y = e^{3x}\cos(2x)$

For Problem 20, apply the rules of logarithms before differentiating the function with respect to x.

20. $y = \ln\left(\frac{x^4}{3x^2+2}\right)$

Section 7.14 Module Review

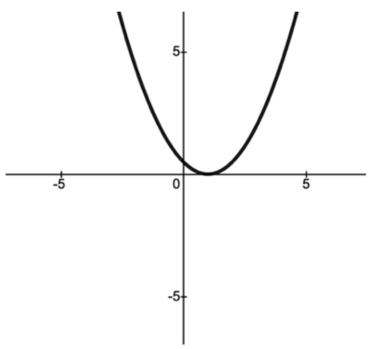
For Problem 1 and 2, find the average rate of change for the function over the interval given.

1. $y = 2x^2 + 2x - 1$ given the interval $[0, 1]$

2. $y = -x^2 + x - 2$ given the interval $[-1, 1]$

For Problem 3, use the given graph of the function to answer the questions that follow.

3.



$y = \frac{x^2}{2} - x + \frac{1}{2}$

a) What is the x-intercept?

b) What is the y-intercept?

c) What is the inflection point?

d) What is the interval when the graph is concave up?

e) What is the interval when the graph is concave down?

f) What is the relative minimum?

g) What is the relative maximum?

For Problem 4-8, find the first derivative for the function.

4. $f(x) = 5x^4 - 2x^3 - 3$

5. $f(x) = -\sqrt[3]{x} - 7x + 8$

6. $f(x) = -10x^3 - x^2 + 4x - 2$

7. $f(x) = \frac{1}{4}x^2 - 6x$

8. $f(x) = \frac{1}{\sqrt[4]{x}}$

For Problem 9 and 10, find the second derivative for the function.

9. $f(x) = \frac{2}{x^3}$

10. $f(x) = 6x^5 + 13x^2$

For Problem 11-14, find the velocity function $v(t)$ and acceleration function $a(t)$ for the position of a particle as it moves along a horizontal line, which is represented by the function $s(t)$.

11. $s(t) = t^3 - 10t^2 + 3t$

12. $s(t) = -t^3 + 15t^2$

13. $s(t) = t^2 + 20t - 8$

14. $s(t) = -t^2 - 17t - 4$

For Problem 15-20, use the information given to solve the problem.

15. What is the displacement of a particle moving along a horizontal line whose position is $s(t) = t^2 + 3t - 1$ over the interval $5 \leq x \leq 10$?

16. Find the equation of the line tangent to the function at the given point.

$$y = -2x^2 + 4x \text{ at } x = -1$$

17. Find the derivative of the given function.

$$y = -x^3 + 2x + 1 \text{ at } x = 2$$

18. Use the Chain Rule to find the derivative of $g(x) = (x^2 + 3x)^{\frac{1}{2}}$

19. Use Implicit Differentiation to find the derivative of $y = 2x^2 \cdot y^6$.

20. Use the Quotient Rule to find the derivative of $y = \frac{\cos x}{\sin x}$.

Section 7.15 Module Test

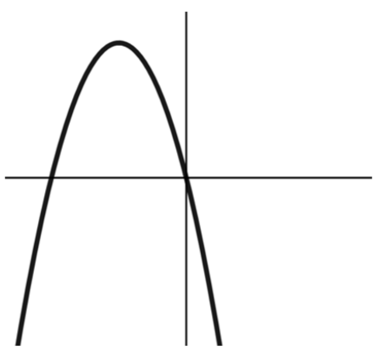
For Problem 1 and 2, find the average rate of change for the function over the interval given.

1. $y = x^2 + x$ given the interval $[-1, 2]$

2. $y = -2x^2 - 2x + 1$ given the interval $[1, 3]$

For Problem 3, use the given graph of the function to answer the questions that follow.

3.



$y = -\frac{x^2}{2} - 4x$

a) What are the x-intercepts?

b) What is the y-intercept?

c) What is the inflection point?

d) What is the interval when the graph is concave up?

e) What is the interval when the graph is concave down?

f) What is the relative minimum?

g) What is the relative maximum?

For Problem 4-8, find the first derivative of the function.

4. $f(x) = x^3 - 3x + 2$

5. $f(x) = -\sqrt{x} + 2x$

6. $f(x) = -3x^4 - x^3 + 2x^2 - 5$

7. $f(x) = \frac{3}{2}x^2 - 7$

8. $f(x) = \frac{1}{\sqrt[3]{x}}$

For Problem 9 and 10, find the second derivative for the function.

9. $f(x) = \frac{1}{x^2}$

10. $f(x) = 4x^4 - 8x^3$

For Problem 11-14, find the velocity function $v(t)$ and acceleration function $a(t)$ for the position of a particle as it moves along a horizontal line, which is represented by the function $s(t)$.

11. $s(t) = t^3 - 12t^2 + 14$

12. $s(t) = -t^3 + 5t^2 - 3t$

13. $s(t) = -t^2 + 18t - 14$

14. $s(t) = t^2 - 5t + 4$

For Problem 15-20, use the information given to solve the problem.

15. What is the displacement of a particle moving along a horizontal line whose position is $s(t) = -t^2 + 17t - 6$ over the interval $6 \leq x \leq 13$?

16. Find the equation of the line tangent to the function at the given point $y = 3x^3 + 2$ at $(2, 1)$.

17. Find the derivative of the function $y = -2x^3 + 5x - 3$ at $x = 3$.

18. Use the Chain Rule to find the derivative of the function $g(x) = (-2x^3 - 5x)^{\frac{3}{2}}$.

19. Use the Product Rule to find the derivative of $y = -3x^4 \cdot y^5$.

20. Use the Quotient Rule to find the derivative of $y = \frac{-5x^2+4x}{-2x^2-3}$.