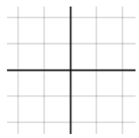
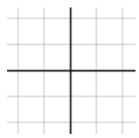
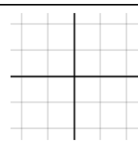
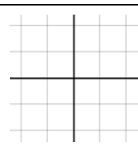
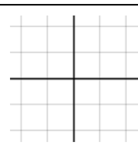
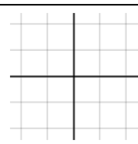
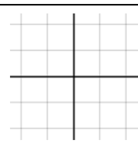
Pre-Calculus and Calculus Module 3 Vectors and Parametric Equations

Section 3.1 Introduction to Vectors

Practice Problems 3.1

For Problem 1-7, follow the steps to complete the chart using a drawing, vector notation, and angle and magnitude.

Step	**Vector Drawing**	**Vector Notation**	**Angle and Magnitude**
1. Down 3			
2. Right 1			
3. Left 4			
4. Up 1			
5. Up 2, Right 2			
6. Down 1, Right 1			
7. Down 1, Left 1			

For Problem 8-10, use the information from Steps 1-7 to solve the problem.

8. The length of the vector is the magnitude of the vector. Name pairs of vectors that have the same magnitude.

9. Name pairs of vectors that have the same direction.

10. Vectors are equivalent if they have the same magnitude and direction. Are any of the vectors from the above steps equivalent? If the answer is yes, name them.

For Problem 11-13, draw the vector for the given situation.

11. 640 km/h due east for 1 hour

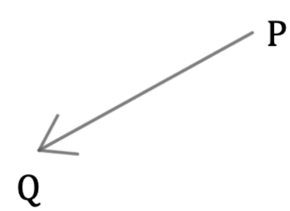
12. 70 mph due west for 2 hours

13. 58 ft./sec. due south for $\frac{1}{2}$-minute.

For Problem 14 and 15, draw a vector equivalent to the given vector.

14.

15.

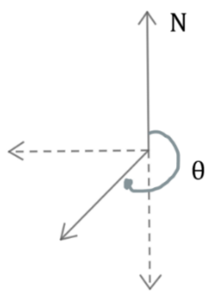


For Problem 16 and 17, use the given information to solve the problem.

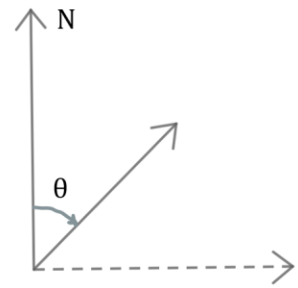
In navigation, the course of travel is the clockwise angle, θ, from due north. Due north is the initial side and the angle of the course of the object represents the path, or terminal side; this is called a bearing.

Estimate the bearing for the given course of travel.

16.



17.



For Problem 18 and 19, use the given information to solve the problem.

Let due north be the home port of a ship. Draw the ship's path given the bearing.

18. $330°$

19. $30°$

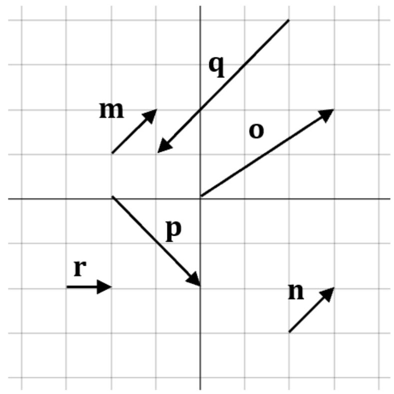
For Problem 20, follow the instructions given to solve the problem.

20. Draw the path of a ship that travels due east for 2 hours and then at a bearing of $45°$ due northeast for 3 hours.

Section 3.2 Vector Notation

Practice Problems 3.2

For Problem 1-5, use the graph below to answer the question.



1. Name a pair of equivalent vectors.

2. Draw a vector that has the same magnitude as **q** but opposite direction. Call is **s**.

3. Which vector has a direction of due east?

4. Which vector has a larger magnitude: **m** or **r**?

5. What is the magnitude and angle of vector **p**?

For Problem 6-10, use the given information to solve the problem.
The component form of vector **m** is $\langle 1, 1\rangle$. Write the component forms of vectors **n**, **o**, **p**, **q**, and **r**.

6. **n** = ⟨_______________⟩

7. **o** = ⟨_______________⟩

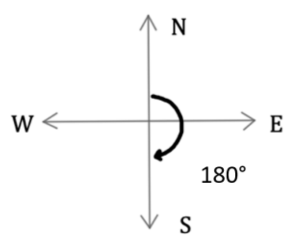
8. **p** = ⟨_______________⟩

9. **q** = ⟨_______________⟩

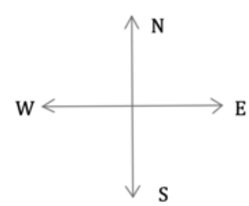
10. **r** = ⟨_______________⟩

For Problem 11-13, use the given information to name the bearing and then use colored pencils to draw a diagram of the bearing.

As stated in the last section, bearings can also be used to describe directions. These are angles measured clockwise from due north. For example, due south has a bearing of 180°.



11. A bearing of due west



12. A bearing of due east

13. A bearing of due northwest

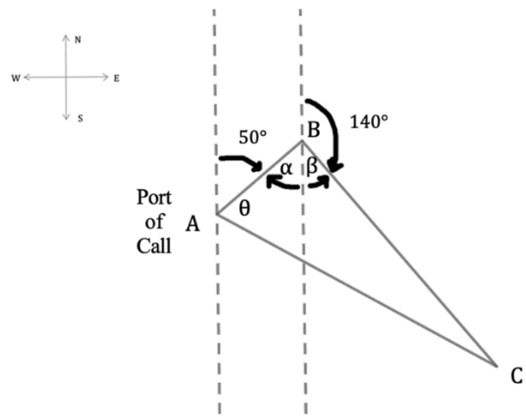
For Problem 14 and 15, use the information given to find the bearing and then draw a diagram of the bearing.

14. If you travel $15°$ east of due south, what is your bearing? Draw the diagram.

15. If you travel $45°$ west of due north, what is your bearing? Draw the diagram.

For Problem 16-20, use the given information to solve the problem.

A boat travels for 1 hour on a course $50°$ east of due north at 32 mph, then changes course to $140°$ east of due north for 2 hours at the same speed.



16. Find the measure of angle α and angle β. (Hint: Use due north as parallel lines cut by a transversal.)

17. After 3 hours, how many total miles has the boat traveled since it left the Port of Call?

18. What is the measure of angle θ?

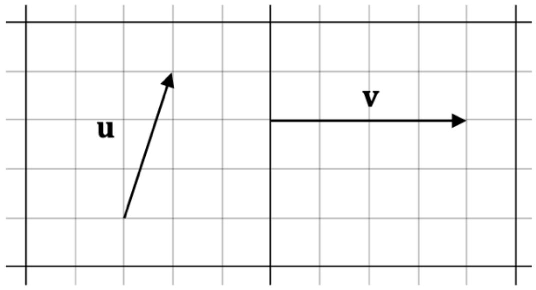
19. What is the bearing of the boat from the port of call after 3 hours?

20. What is the distance from the boat back to the port of call?

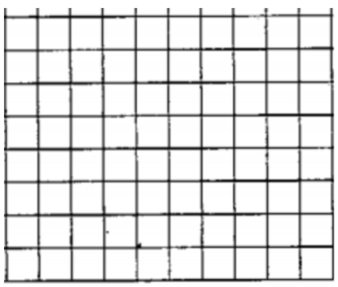
Section 3.3 Resultant Vectors

Practice Problems 3.3

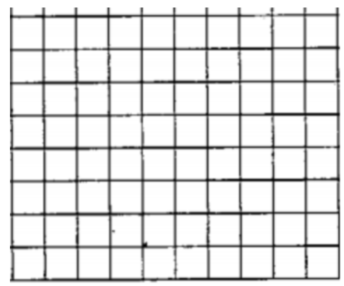
For Problem 1-5, use the given **u** and **v** below to solve the problem.



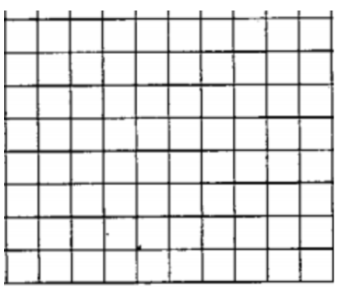
1. Draw the vector $\mathbf{u} + \mathbf{v}$.



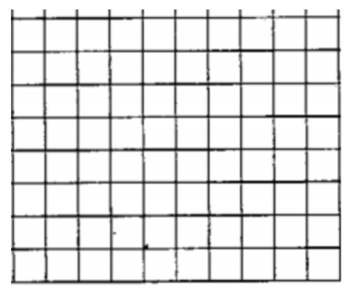
2. Using vectors **u** and **v**, draw the vector $\mathbf{u} - \mathbf{v}$.



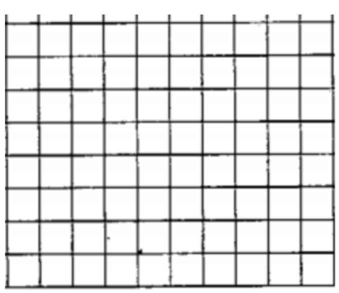
3. Draw the vector $2\mathbf{u}$.



4. Draw the vector $-2\mathbf{v}$.

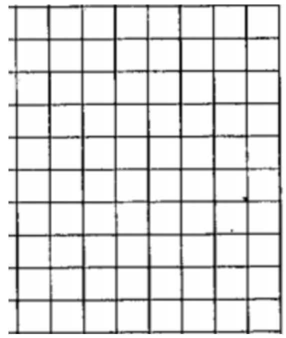


5. Draw the vector $2\mathbf{u} + 3\mathbf{v}$.

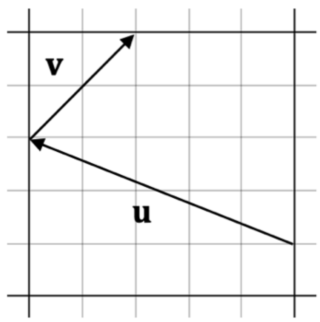


For Problem 6-15, follow the instructions to solve the problem. For **u** and **v** refer to Problem 7.

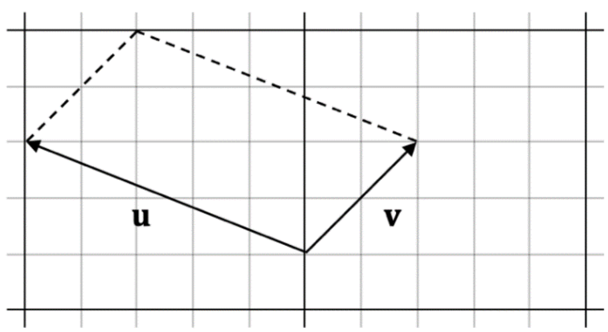
6. A boat travels due east for 5 km. and then due south for 10 km. Draw the vector quantities and their sum if each square of the grid is 1 km^2.



7. The triangle method places the terminal point of **u** at the initial point of **v** for **u** + **v**. Then **u** + **v** extends from the tail of **u** to the head of **v**. Draw the vector **u** + **v** using the triangle method.

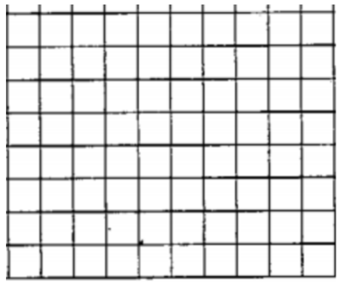


8. The parallelogram method displays **u** and **v** as adjacent sides that start at a common point. Then **u** + **v** extends from that common point to the opposite vertex in the parallelogram. Draw the vector **u** + **v** using the parallelogram method.



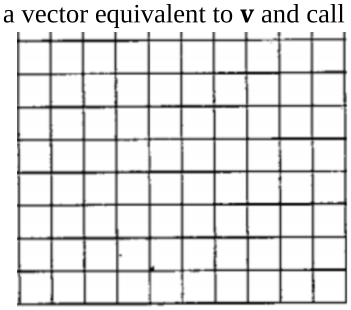
9. Are the resultant vectors in Problem 7 and Problem 8 equivalent? Why or why not?

10. Draw the resultant of $\mathbf{v} - \mathbf{u}$ using the vectors.

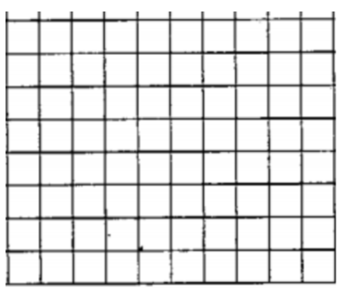


11. What is the length of the resultant vector when a boat travels due east for 5 km. and then due south for 10 km.?

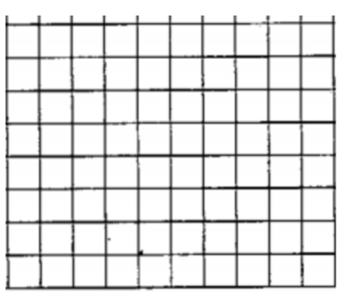
12. a) Draw a vector equivalent to **v** and call it **m**.



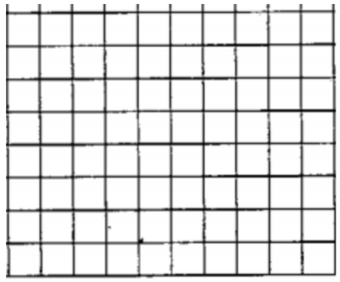
b) Draw 2**m** and call it **n**.



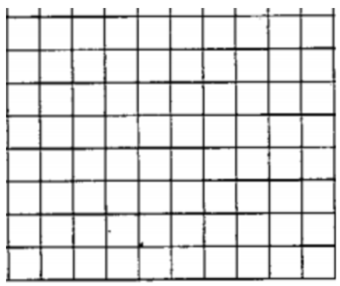
c) Draw the resultant of $\mathbf{m} + \mathbf{n}$.



13. Is $\mathbf{u} - \mathbf{v}$ equal to $\mathbf{v} - \mathbf{u}$? Draw a diagram and demonstrate why or why not.



14. Let each square on the grid represent 10 km.^{2}. Draw a resultant vector for a plane traveling due south for 70 km. and due east for 100 km.



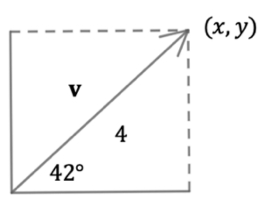
15. What is the magnitude of the resultant vector in Problem 14?

For Problem 16-18, fill in the blank(s).

16. The sum $\mathbf{m} + \mathbf{n}$ is the resultant vector from the tail of **m** to the head of **n** if the _______________ of **m** is placed at

the _______________ of **n**.

17. Given the diagram below, point (x, y) is located at the ____________________ of **v**.



18. The magnitude of **v** from Problem 17 is ____________________.

19. What is the horizontal component of **v**?

20. What is the vertical component of **v**

Section 3.4 Scalar Multiplication

Practice Problems 3.4

For Problem 1, follow the instructions to solve the problem.

1. If $\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$, find $\mathbf{u} + \mathbf{0}$.

For Problem 2-5, use the given vectors to solve the problem.

$$\mathbf{u} = \begin{pmatrix} 2 \\ 4 \\ -1 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 5 \\ 1 \\ 3 \\ -2 \end{pmatrix}$$

2. $\mathbf{u} - \mathbf{v}$

3. $3\mathbf{u}$

4. $\frac{1}{2}\mathbf{u}$

5. $\mathbf{u} + 2\mathbf{v}$

Problem 6 – 20 are Euclidean vectors and the inner dot product is commutative. Therefore, the rows and columns may be added. Linear Algebra differentiates rows and columns as mutually dual spaces which cannot be added. When you are working in linear algebra, add rows to rows and columns to columns.

6. If $\mathbf{u} - 2\mathbf{v} = \mathbf{0}$, how are the components of $\mathbf{u}$ and $\mathbf{v}$ related? Solve for $\mathbf{v}$ in terms of $\mathbf{u}$.

7. Find $(-2, 2) + 2\begin{pmatrix}4\\8\end{pmatrix}$.

8. If $\begin{pmatrix}6\\3\\-5\end{pmatrix} + \begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix} = \begin{pmatrix}10\\-9\\-5\end{pmatrix}$, find the components $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$.

9. If $\begin{pmatrix}0\\0\end{pmatrix} + \begin{pmatrix}v_1\\v_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$, what do you know about the components $\mathbf{v}_1$ and $\mathbf{v}_2$?

10. If $0\begin{pmatrix}u_1\\u_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$, what do you know about the components $\mathbf{u}_1$ and $\mathbf{u}_2$?

For Problem 11-13, let $\mathbf{u}$ be $(4, -2)$ ($\mathbf{u} = (4, -2)$) and $\mathbf{v}$ be $\begin{pmatrix}3\\0\end{pmatrix}$ ($\mathbf{v} = \begin{pmatrix}3\\0\end{pmatrix}$).

11. Is $\mathbf{u} + \mathbf{v}$ equal to $\mathbf{v} + \mathbf{u}$?

12. Is $\mathbf{u} - \mathbf{v}$ equal to $\mathbf{v} - \mathbf{u}$?

13. Is $2(\mathbf{u} + \mathbf{v})$ equal to $2\mathbf{u} + 2\mathbf{v}$?

For Problem 14-20, follow the instructions given to solve the problem.

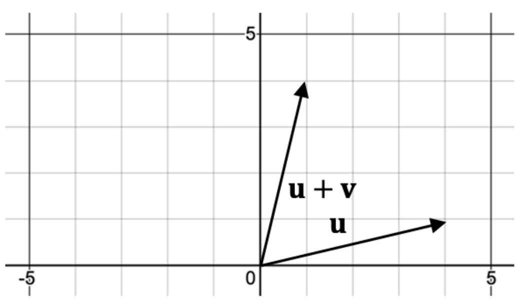
14. a) What property holds true for vector addition?

b) Does the distributive property hold true for vector addition?

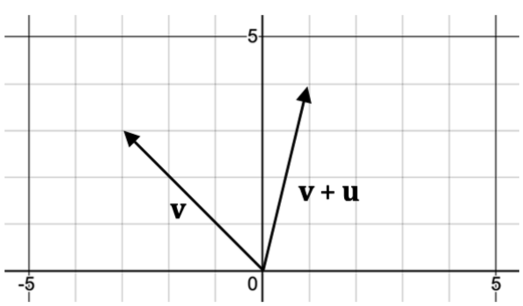
15. a) Is $\mathbf{u} - \mathbf{v}$ equal to $\mathbf{u} + -\mathbf{v}$?

b) Is $\mathbf{u} - \mathbf{v}$ equal to $-\mathbf{v} + \mathbf{u}$?

16. Draw vector $\mathbf{v}$ from the head of $\mathbf{u}$ to the head of $\mathbf{u} + \mathbf{v}$. What is the resultant vector?



17. Draw vector $\mathbf{u}$ from the head of $\mathbf{v}$ to the head of $\mathbf{v} + \mathbf{u}$. What is the resultant vector?



18. What property is demonstrated in Problem 16 and Problem 17? How do you know?

19. a) Draw $\mathbf{u} + -\mathbf{u}$ on the graph in Problem 16.

b) Draw $\mathbf{v} + -\mathbf{v}$ on the graph in Problem 17.

c) What is the magnitude of these resultant vectors?

20. Do the resultant vectors from Problem 19 have any meaningful direction? What is the name of the resultant vectors?

Note that in Euclid's definition a point has no parts and that means no dimensions. However, many points next to each other form a line which has one dimension. Therefore, one may say the zero vector has no dimension and lacks any meaningful direction or may be the direction of ones choosing.

Section 3.5 Unit Vectors and Magnitude

Practice Problems 3.5

For Problem 1-3, write the vector in component form.

1. $3\boldsymbol{i} - 2\boldsymbol{j}$

2. $-5\boldsymbol{i} + 10\boldsymbol{j}$

3. $-2.6\boldsymbol{i} - 4.1\boldsymbol{j}$

For Problem 4-6, write the vector as linear combinations of $\boldsymbol{i}$ and $\boldsymbol{j}$ in the form a$\boldsymbol{i} + b\boldsymbol{j}$.

4. $\langle -7, 8 \rangle$

5. $\langle -5, 0 \rangle$

6. $\langle 0, 3 \rangle$

For Problem 7, find the magnitude of each vector in Problem 1-3.

7. a) $3\boldsymbol{i} - 2\boldsymbol{j}$ b) $-5\boldsymbol{i} + 10\boldsymbol{j}$

   c) $-2.6\boldsymbol{i} - 4.1\boldsymbol{j}$

For Problem 8, find the magnitude of each vector in Problem 4-6.

8. a) $\langle -7, 8 \rangle$ b) $\langle -5, 0 \rangle$

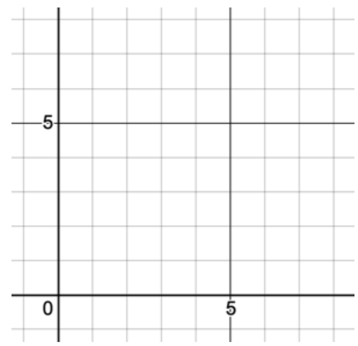
   c) $\langle 0, 3 \rangle$

For Problem 9 and 10, follow the instructions given to solve the problem.

9. Let $\mathbf{u}$ equal $\boldsymbol{i} + \boldsymbol{j}$, and $\mathbf{v}$ equal $2\boldsymbol{i} + 2\boldsymbol{j}$, and $\mathbf{w}$ equal $2\mathbf{u} + 2\mathbf{v}$.

a) Find $\mathbf{w}$ in component form.

b) Draw vectors $\mathbf{u}$ and $\mathbf{v}$ on the graph.



c) Find $\|\mathbf{w}\|$ (the magnitude of $\mathbf{w}$).

10. Fill in the blanks given $\mathbf{u} = \mathrm{a}\boldsymbol{i} + \mathrm{b}\boldsymbol{j}$ and $\mathbf{v} = \mathrm{c}\boldsymbol{i} + \mathrm{d}\boldsymbol{j}$.

If vector $\mathbf{u}$ is equivalent to vector $\mathbf{v}$, then $\mathrm{a} =$ ___________ and $\mathrm{d} =$ ___________.

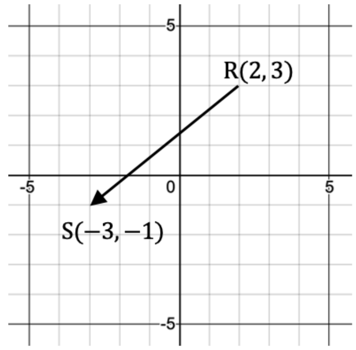
For Problem 11-13, given $\mathrm{L}(-1, 3)$ and $\mathrm{M}(2, 5)$ solve the problem.

11. Express $\overrightarrow{\mathrm{LM}}$ in component form.

12. Express $\overrightarrow{\mathrm{LM}}$ in the form $\mathrm{a}\boldsymbol{i} + \mathrm{b}\boldsymbol{j}$.

13. What is the magnitude of $\overrightarrow{\mathrm{LM}}$?

For Problem 14 and 15, use the graph below to solve the problem.



14. Express $\overrightarrow{RS}$ in…

a) Component form

b) a$\boldsymbol{i}$ + b$\boldsymbol{j}$ form

15. What is the magnitude of $\overrightarrow{RS}$?

For Problem 16-20, let $\mathbf{s}$ equal $6\boldsymbol{i} + 4\boldsymbol{j}$ and $\mathbf{t}$ equal $2\boldsymbol{i} - \boldsymbol{j}$.

16. a) Find the resultant vector of $\mathbf{s} + \mathbf{t}$. Call it $\mathbf{u}$.

b) What is the magnitude of the resultant vector $\mathbf{u}$?

17. a) Find the resultant vector of $\mathbf{t} + \mathbf{s}$. Call it $\mathbf{v}$.

b) What is the magnitude of the resultant vector $\mathbf{v}$?

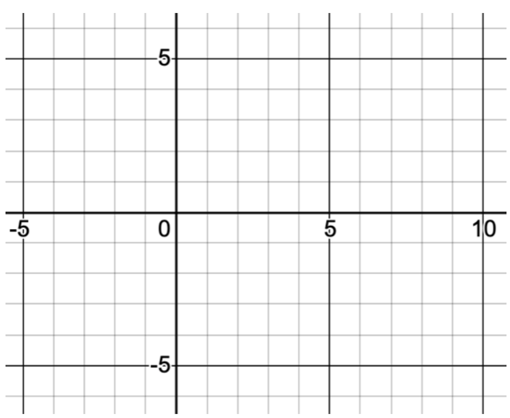
18. What do you notice about Problem 16 and 17? Explain why this is so.

19. a) What is the magnitude of $\mathbf{s}$?

b) What is the magnitude of $\mathbf{t}$?

c) Find $\|\mathbf{s}\| + \|\mathbf{t}\|$.

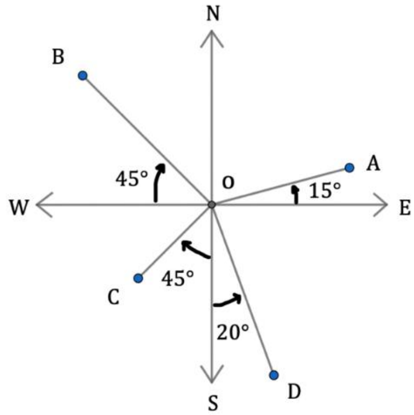
20. Use the given graph to draw the vectors and explain why $\|\mathbf{s} + \mathbf{t}\| < \|\mathbf{s}\| + \|\mathbf{t}\|$.



Section 3.6 Directions and Bearings

Practice Problems 3.6

For Problem 1-4, use the graph below to give the direction.



1. o to A
2. o to B
3. o to C
4. o to D

For Problem 5-8, use the graph above to give the bearing.

5. $\overrightarrow{OA}$
6. $\overrightarrow{OB}$
7. $\overrightarrow{OC}$
8. $\overrightarrow{OD}$

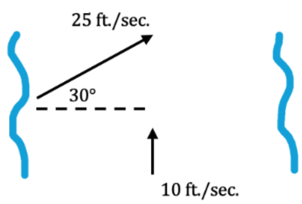
For Problem 9 and 10, use the information given to solve the problem.

9. A plane takes off and travels at a bearing of $45°$. After 1 hour, the plane has traveled 750 km.; how far east of takeoff is the plane?

10. A walker leaves home and travels 5 km. south and 2 km. east. What is the bearing from the start point of the walker's home?

For Problem 11-16, use the information below to solve the problem.

A boat is traveling at 25 feet per second at an angle of $30°$ up the river.



11. Find the x-component of the boat without allowing for the effects of the current.

12. Find the y-component of the boat without allowing for the effects of the current.

13. Use the component form to represent the vector. This is the velocity of the boat without allowing for the effects of the current.

14. The current of the river is 10 feet per second due north. The motion the boat actually follows is the boat motion allowing for the effects of the river.

a) What is the vector in component form that represents the current of the river?

b) Add the two vectors together in component form to find the actual motion of the boat.

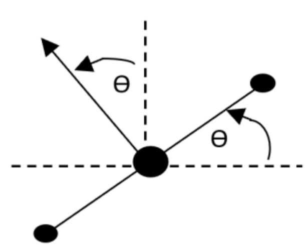
15. The river is 80 feet wide. Use the x-component of the velocity of the actual motion of the boat along with the distance formula $d = rt$ to find how long it takes the boat to get across the river.

16. How far up the river will the boat be when it gets to the other side of the river? Use the y-component of velocity of the actual motion of the boat along with the distance formula to solve this problem.

For Problem 17-20, use the information below to solve the problem.

Lift is a force vector perpendicular to the wings of an airplane. The lift vector resolves to horizontal and vertical components when an airplane banks for a turn. The vertical component has a magnitude equal to the weight of the plane in order for it to stay up in the air. Centripetal force is the horizontal component that makes a plane stay on a curved path.

Given a plane with a weight of 250,000 lbs. banking at angle θ, solve Problem 17-20.



17. Find the vertical component (magnitude) of the plane for the following angles:

a) $0°$

b) $10°$

c) $20°$

18. Find the horizontal component (centripetal force) of the plane for the following angles:

a) $0°$

b) $10°$

c) $20°$

19. How does the angle of the bank of the plane relate to its circular motion?

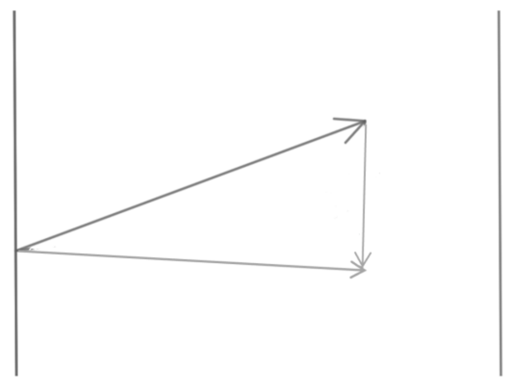
20. When the plane is not banking, what angle does it fly at? Does it fly straight or in a circular pattern?

Section 3.7 Vectors and the Law of Sines and Cosines

Practice Problems 3.7

For Problem 1-5, use the given information to label the parts of the diagram and answer the questions.

A boat is heading upstream with a bearing of 70° at a rate of 30 ft./sec. The current is heading downstream at 7 ft./sec.



1. Label the vectors on the diagram.

2. The actual motion of the boat is the boat motion allowing for the effects of the stream's current. Label the vector that represents the actual motion v.

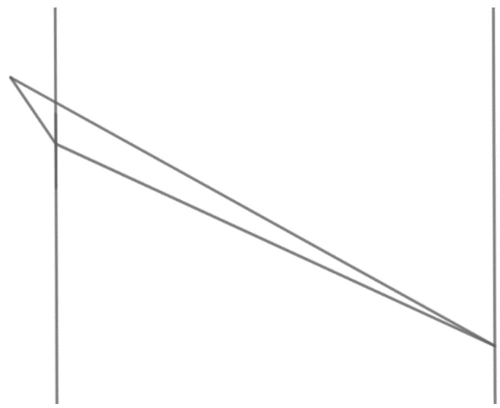
3. Label any known angles in the triangle. Label the true angle between the bearing and the true course of the boat θ.

4. Use the Law of Cosines to find v. What is the actual rate with the current?

5. Find θ to get the true bearing of the boat.

For Problem 6-10, use the given information to label the parts of the diagram and answer the questions.

A plane is flying at 600 mph on a course with a bearing of 295°. It is pushed along with a 100 mph wind that is blowing to the northwest and has a direction of N34°W.



6. Label the diagram with angles and vectors. Label the true vector v and the unknown angle θ.

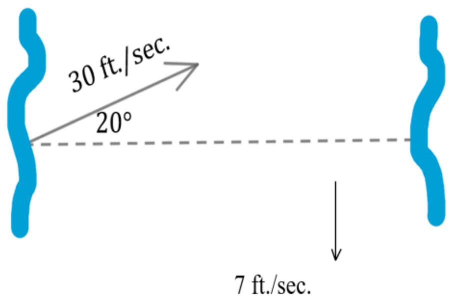
7. What is the resultant speed of the plane?

8. What is the bearing of the resultant direction?

9. If the plane flies for 2 hours, what will its true bearing be?

10. After 2 hours, how far will the plane be from its starting point?

For Problem 11-15, use the given diagram and component form to solve the problem.
A boat is traveling at a rate of 30 ft./sec. upstream at an angle of 20°.



11. Find the x-component of the vector that represents velocity without allowing for the stream's current.

12. Find the y-component of the vector that represents velocity without allowing for the stream's current.

13. a) Write the velocity of the boat, without allowing for the stream's current, in component form.

b) Write the component form that represents the stream's current. (Hint: Downstream is opposite of upstream.)

14. Add the two components together to represent velocity allowing for the stream's current.

15. Use right triangle trigonometry and the velocity vector allowing for the stream's current to find the true bearing of the boat.

For Problem 16-20, use the situation given to solve the problem.

16. If a boat travels at 70° west of south, what is its bearing?

17. If the same boat travels at 70° east of south, what is its bearing?

18. If a boat travels at a bearing of 0°, what direction is it traveling?

19. If a boat travels at a bearing of 180°, what direction is it traveling?

20. a) What is a bearing of due east?

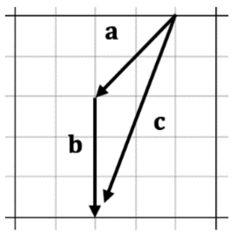
b) What is a bearing of due west?

Section 3.8 Converting Vectors to Component Form

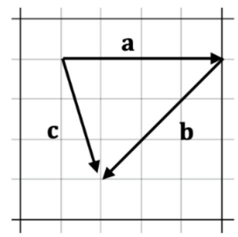
Practice Problems 3.8

For Problem 1-3, find the components for each vector. Write the components (**c**) of the resultant vector.

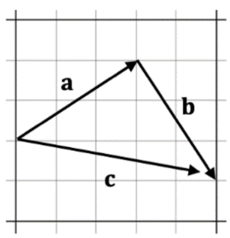
1.



2.



3.



For Problem 4-6, write the vector in component form.

4. $\boldsymbol{c} = -2\boldsymbol{i} + 5\boldsymbol{j}$

5. $\boldsymbol{d} = -10\boldsymbol{j}$

6. $\boldsymbol{f} = 3\boldsymbol{i}$

For Problem 7 and 8, write the components in vector form.

7. $m = \langle -4, -8 \rangle$

8. $n = \langle -5, 0 \rangle$

For Problem 9 and 10, use the information below to solve the problem.

The formula for a displacement vector given an initial point (x_1, y_1) and terminal point (x_2, y_2) is $\langle x_2 - x_1, y_2 - y_1 \rangle$.

9. Find the component form of $\overrightarrow{RS}$ given point $R(4, -2)$ and point $S(-3, 0)$.

10. What is $2 \cdot \overrightarrow{RS}$ in component form?

For Problem 11-13, given the magnitude and standard angle, change the vector to component form.

11. $\|\mathbf{v}\| = 4$ $\theta = 45°$

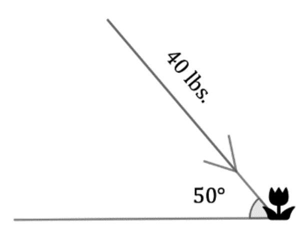
12. $\|\mathbf{v}\| = 10$ $\theta = 135°$

13. $\|\mathbf{v}\| = 6$ $\theta = 120°$

For Problem 14 and 15, use the information below to solve the problem.

Vectors are used in physics to find force, which has magnitude and direction.

The diagram below displays the following situation:
You are edging your yard and pushing with a force of 40 lbs. at an angle of 50° from the handle of the edger to the ground.

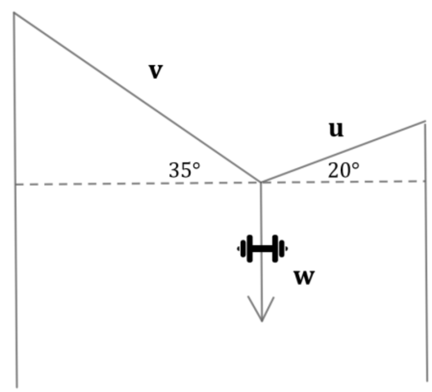


14. Find the horizontal and vertical components and write the force vector in component form.

15. The horizontal force is pushing the edger ahead. How much force is being applied parallel to the ground?

For Problem 16-20, use the information and diagram below to solve the problem.

A mathematical puzzle has a weight swinging along a wire between two small poles. It weighs 4 pounds and is represented by the vector **w**. It stopped swinging when **u** and **v** (the force on each side of the wire) were at the given angles.



16. What is the force vector **w** in component form $\langle a, b\rangle$?

17. The standard angle for force vector **u** is ___________; **u** can be written in component form as $\langle \|\mathbf{u}\| \cos 20°, \|\mathbf{u}\| \sin 20°\rangle$.

18. The standard angle for force vector **v** is 145°; **v** can be written in component form as $\langle \|\mathbf{v}\|$________, $\|\mathbf{v}\|$________$\rangle$.

19. When the sum of the force vectors is equal to zero, they are at an equilibrium point. These points can be written as follows:

$$\|\mathbf{u}\| \cos 20° + \|\mathbf{v}\| \cos 145° = 0$$

$$\|\mathbf{u}\| \sin 20° + \|\mathbf{v}\| \sin 145° = 4$$

Now solve for $\|\mathbf{u}\|$ in terms of $\|\mathbf{v}\|$. The equation is done for you below:

$$\|\mathbf{u}\| \cos 20° + \|\mathbf{v}\| \cos 145° = 0$$

$$\|\mathbf{u}\|(0.94) + \|\mathbf{v}\|(-0.82) = 0$$

$$\|\mathbf{v}\|(-0.82) = 0 - \|\mathbf{u}\|(0.94)$$

$$\|\mathbf{v}\| = \frac{-0.94}{-0.82}\|\mathbf{u}\|$$

$$\|\mathbf{v}\| = 1.146\|\mathbf{u}\|$$

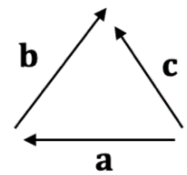
Substitute $\|\mathbf{v}\|$ in terms of $\|\mathbf{u}\|$ in the second equation for equilibrium.

20. Now that you have found the magnitude of $\mathbf{u}$, substitute that value in the first equation for equilibrium to find $\|\mathbf{v}\|$.

Section 3.9 The Dot Product

Practice Problems 3.9

For Problem 1-5, use the given diagram and information to solve the problem.



Vector **d** is $\langle \mathbf{d}_1, \mathbf{d}_2 \rangle$ in component form.

1. Write vector **a** and vector **b** in component form.

2. Write the resultant vector **c** in component form.

3. Use $\mathbf{a} = \mathbf{c} - \mathbf{b}$ to write the component form of vector **a**.

4. Let $\mathbf{b} = \mathbf{c} - \mathbf{a}$ to write the component form of vector **b**.

5. If **a** and **b** are two nonzero vectors, find the dot product where θ is the angle between them.

For Problem 6-10, let $\mathbf{a} = 4\boldsymbol{i} + \boldsymbol{j}$ and $\mathbf{b} = -\boldsymbol{i} - 3\boldsymbol{j}$.

6. Write vector **a** and vector **b** in component form if vector **d** is $\langle \mathbf{d}_1, \mathbf{d}_2 \rangle$ in component form.

7. Write the resultant vector in component form and call it **c**.

8. Find $\mathbf{a} \cdot \mathbf{b}$.

9. Find $\|\mathbf{a}\|\|\mathbf{b}\|$.

10. What is the measure of the angle between **a** and **b**?

For Problem 11-13, given the vector, find the dot product of $\mathbf{x} \cdot \mathbf{y}$.

11. $\mathbf{x} = \langle -1, -1 \rangle$ $\mathbf{y} = \langle 0, 4 \rangle$

12. $\mathbf{x} = \langle -2, 5 \rangle$ $\mathbf{y} = \langle 4, 7 \rangle$

13. $\mathbf{x} = \langle 0, 0 \rangle$ $\mathbf{y} = \langle 2, \frac{1}{2} \rangle$

For Problem 14-16, find the magnitude of the vectors **x** and **y** in Problem 11-13.

14. $\|\mathbf{x}\| =$ $\|\mathbf{y}\| =$

15. $\|\mathbf{x}\| =$ $\|\mathbf{y}\| =$

16. $\|\mathbf{x}\| =$ $\|\mathbf{y}\| =$

For Problem 17-19, use the dot product and magnitude to find the angle θ between the two vectors **x** and **y** in Problem 11-13.

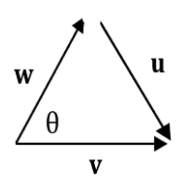
17. $\theta =$

18. $\theta =$

19. $\theta =$

For Problem 20, use the information given to solve the problem.

20. Let $\mathbf{v} = \mathrm{a}\boldsymbol{i} + \mathrm{b}\boldsymbol{j}$ and $\mathbf{w} = \mathrm{c}\boldsymbol{i} + \mathrm{d}\boldsymbol{j}$. Use the Law of Cosines with the diagram to prove that $\mathbf{v} \cdot \mathbf{w} = \mathrm{ac} + \mathrm{bd}$.



<u>Section 3.10 Unit and Orthogonal Vectors</u>

<u>Practice Problems 3.10</u>

For Problem 1-4, use the component form to solve the vector problems.

1. Find m and n if $m\boldsymbol{i} + n\boldsymbol{j} = 3\boldsymbol{i} + 5\boldsymbol{j}$.

2. Find r and s if $r\boldsymbol{i} + s\boldsymbol{j} = 3\boldsymbol{i}$.

3. Find q and p if $q\boldsymbol{i} + p\boldsymbol{j} = \frac{4}{9}\boldsymbol{j}$.

4. If $\overrightarrow{\mathrm{OP}} = 2\boldsymbol{i} - \boldsymbol{j}$ and $\mathrm{P} = (4, 5)$, find the coordinates of O.

For Problem 5-8, let $\mathbf{u} = 2\boldsymbol{i} - 3\boldsymbol{j}$ and $\mathbf{v} = -4\boldsymbol{i} + \boldsymbol{j}$.

5. Find $\mathbf{u} + \mathbf{v}$.

6. Find $\|\mathbf{v}\|$.

7. Find $3\mathbf{v}$.

8. Find $\mathbf{v} \cdot \mathbf{u}$.

9. Find $\frac{1}{2}\mathbf{v}$.

10. Find $\mathbf{u} - \mathbf{v}$.

For Problem 11 and 12, use the dot product to find the angle between **w** and **v**.

11. $\mathbf{w} = 3\boldsymbol{i}$ $\mathbf{v} = -4\boldsymbol{i} + 4\boldsymbol{j}$

12. $\mathbf{w} = -2\boldsymbol{i} - 2\boldsymbol{j}$ $\mathbf{v} = -2\boldsymbol{i} + 5\boldsymbol{j}$

For Problem 13-17, follow the instructions to solve the problem.

13. Find the vector **v** that is orthogonal to $\mathbf{u} = 2\boldsymbol{i} + \boldsymbol{j}$.

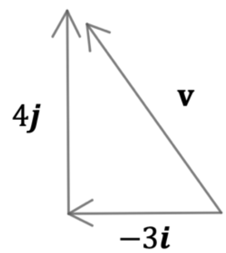
14. Find another vector that is orthogonal to $\mathbf{u} = 2\boldsymbol{i} + \boldsymbol{j}$.

15. Find a unit vector orthogonal to $\mathbf{u} = -4\boldsymbol{i} + \boldsymbol{j}$.

16. Find the magnitude of the vector from Problem 15.

17. Find a unit vector orthogonal to $\mathbf{u} = \frac{2}{3}\boldsymbol{i} - \boldsymbol{j}$. Find its norm.

For Problem 18-20, use the diagram below to solve the problem.



18. Find the magnitude of **v**.

19. Find the horizontal and vertical components of the unit vector.

20. Find a unit vector orthogonal to **v**.

Section 3.11 Vector Equations

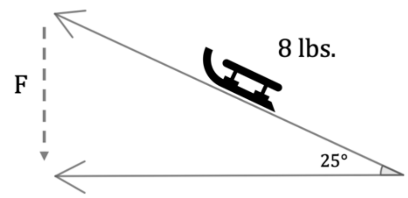
Practice Problems 3.11

For Problem 1-10, use the information below to solve the problem.

Work that is done when force is applied over a given distance is another use for the dot product: $w = F \cdot d$, where w is work, F is force, and d is displacement.

For Problem 1, use the information and diagram given to solve the problem.

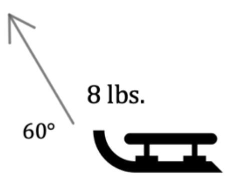
Suppose a sled is pulled up a hill. The sled weighs 8 lbs. and the incline of the hill is at an angle of 25°.



1. How much force is needed to keep the sled from sliding back down the hill?

For Problem 2-10, use the information and diagram given to solve the problem.

At the top of the hill the ground is flat. A young man pulls a sled with 8 lbs. of force along the ground with the rope at an angle of 60°.



2. What is the horizontal component of the force vector for the sled?

3. What is the vertical component of the force vector for the sled?

4. What is the force vector for the sled? ($F = \langle ?\,, ?\, \rangle$)

5. If the sled was pulled for 20 feet of displacement, what is the displacement vector for the sled? ($d = \langle ?\,, ?\, \rangle$) (Remember, there is only horizontal displacement; there is no vertical displacement.)

6. How much work was done to pull the sled 20 feet? (Remember, $w = F \cdot d$ and $F \cdot d$ is the dot product, not multiplication.)

7. Suppose this sled is pushed up the base of the hill for 20 feet. Use the base of the hill with an incline of $25°$. What is the displacement vector for the sled? (Find the horizontal and vertical components; $d = \langle ?\,, ?\, \rangle$.)

8. How much work was done to push the sled up the hill using the incline from Problem 7? (The answer for work is measured in ft.· lbs.)

9. Find a vector equation of the line through the points $(-1, 3)$ and $(4, 5)$.

10. Write a vector equation for the line through the points using the unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$.

For Problem 11 and 12, write a vector equation for the line through the points using the unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$.

11. $\mathbf{v} = \langle -2, -4 \rangle + t\langle 3, 5 \rangle$

12. $\mathbf{v} = \langle -\frac{1}{4}, \frac{2}{3} \rangle - t\langle 1, 1 \rangle$

For Problem 13-16, use the information from Problem 11 and/or Problem 12 to solve the problem.

13. If the vector **v** in Problem 11 is the velocity of a remote-controlled car at time t, what are the horizontal and vertical components for velocity after 3 seconds in meters per second?

14. What is its speed of the remote-control car after 3 seconds in units of meters per second?

15. If the vector **v** in Problem 12 is the velocity of a marble at time t, what are the horizontal and vertical components for velocity after 9 seconds in meters per second?

16. What is its speed of the marble after 9 seconds in units of meters per second?

For Problem 17-20, use the two points $(-1, 5)$ and $(3, 7)$ on a line to solve the problem.

17. a) What is the slope of the line?

b) Write the equation of the line in point-slope form and convert to standard form.

18. Find the vector with initial point $\langle -1, 5\rangle$ and terminal point $\langle 3, 7\rangle$. How does it relate to the slope of the line?

19. Use the initial points to write the vector equation for the line with the parameters t for time and **v** for the velocity vector. What is the endpoint when $t = 2$? Does this point work in the equation of the line?

20. Write another vector equation for the line using the terminal point. Use the parameter t (time) and **v** (velocity) in meters per second. Show that this point also lies on the line.

Section 3.12 Parametric Equations and Motion

Practice Problems 3.12

For Problem 1-4, use the situation given to solve the problem.

1. A small boat travels past a lighthouse. Due west is the x-coordinate and due south is the y-coordinate. The parametric travel equations are $x = 23t$ and $y = 127$. What will the coordinates of the boat be after half an hour if t is the time in minutes?

2. A large boat travels past the same lighthouse as the small boat in Problem 1 at the origin. The travel equations are $x = 54t$ and $y = 221$. What will the coordinates of the boat be after half an hour?

3. A snail begins at point $(0, 4)$ and travels along the line $y = 4$ at $+2.5$ units per second for 6 seconds. At what point will the snail end?

4. A snail begins at point $(4, 4)$ and travels along the line $x = 4$ at -3 units per second for 1.5 seconds. At what point will the snail end?

For Problem 5-8, use the given information to solve the problem.

The takeoff of a model drone is given by the parametric functions:

$$x(t) = 3^t$$
$$y(t) = 3^{t+1}$$

5. Complete the table from 0-5 seconds for the takeoff of the model drone. The first row is done for you.

t	x	y
0	$3^0 = 1$	$3^1 = 3$
1		
2		
3		
4		
5		

6. What are the coordinates for takeoff at 4 seconds?

7. If x and y are measured in feet, how far has the drone moved vertically after 3 seconds?

8. If x and y are measured in feet, how far has the drone moved horizontally after 2 seconds?

For Problem 9-11, use the given parametric functions to solve the problem.

$x(t) = 2t$ Let $x(t)$ be x

$y(t) = 6 - t$ Let $y(t)$ be y

9. Solve for t in terms of x in the first equation.

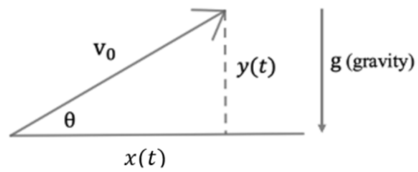
10. Substitute that value for t in the second equation. What is the $y = f(x)$ equation for the following parametric equations?

$$x(t) = 2t$$
$$y(t) = 6 - t$$

11. Solve the parametric equation for $t = 4$ seconds. Substitute $x(t)$ for x and $y(t)$ for y in the equation $y = f(x)$, which you found in Problem 8. Do these coordinates satisfy the equation? If so, what does that mean? If not, what does that mean?

For Problem 12-15, use the given information and diagram to solve the problem.
Let us derive the equation for the parametric equation for the projectile motion that was used in this section of Pre-Calculus and Calculus.

The diagram below shows an object with an initial launch with a velocity of v_0 at an angle of θ°. The horizontal motion is $x(t)$ and the vertical motion is $y(t)$.



12. Find $\cos\theta$ and $\sin\theta$ in terms of $x(t)$ and $y(t)$, then solve for $x(t)$ and $y(t)$.

13. Add x_0 to the horizontal equation to account for any horizontal shift from the origin at launch. Add y_0 to the vertical equation to account for any vertical shift from the origin at launch. Write the new equations.

14. These equations do not account for the effects of gravity on the vertical movement since gravity pulls to the center of the earth at a rate of $-16t^2$ feet per second. Add the influence of gravity to the $y(t)$ equation from Problem 13 to complete the equation.

15. Write the equation in terms of $f(x)$ for the two following parametric equations:

$$x(t) = -5 + 3t$$
$$y(t) = 10 - 4t$$

For Problem 16-20, use the given information to solve the problem.
Pendulum motion is a real-world application of parametric equations. At rest, the pendulum is at the origin on the x-y coordinate plane above the horizontal floor.

A pendulum is pulled to a displacement of 37 cm. It is released at time $t = 0$ seconds and swings back and forth between 37 cm. and -37 cm. The time to complete one period on the stopwatch is 4.6 seconds. The displacement varies sinusoidally with time.

16. Sketch a graph of the periodic function and write an equation for x as a function of time.

17. Sketch a graph of the periodic function and write an equation for y as a function of time.

18. What is the horizontal and vertical position of the pendulum at 2.3 seconds?

19. If the swing was between 25 cm. and -25 cm., how does that change the parametric equations?

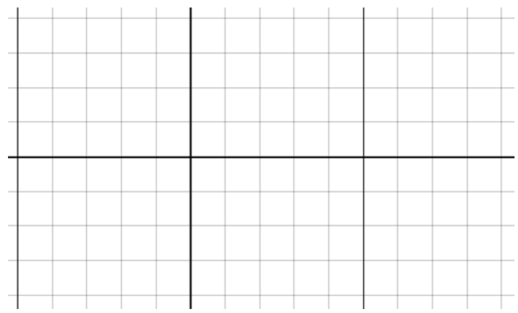
20. If the time for one period was 2.7 seconds on a stopwatch, how does that change the parametric equations?

Section 3.13 Graphing Parametric Equations

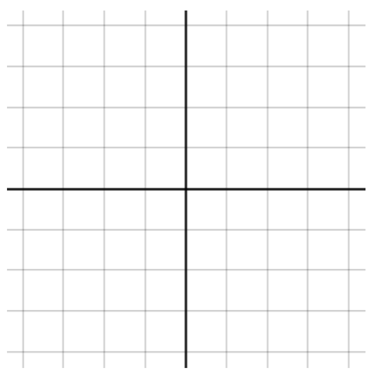
Practice Problems 3.13

For Problem 1-9, follow the instructions given to solve the problem.

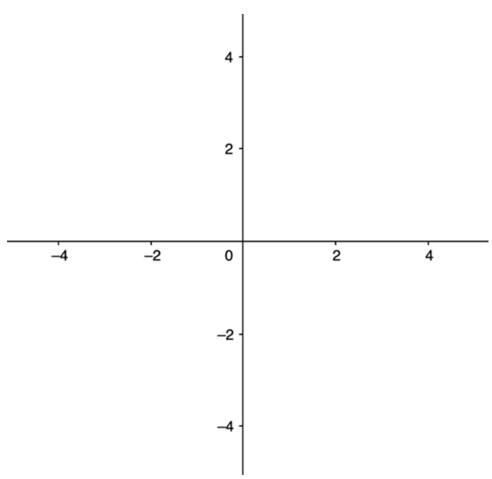
1. Draw the graph of the motion of the snail who begins at point $(0, 4)$ and travels along the line $y = 4$ at $+2.5$ units per second for 6 seconds.



2. Draw the graph of the motion of the snail who begins at point $(4, 4)$ and travels along the line $x = 4$ at -3 units per second for 1.5 seconds.



3. The equation for a circle is $x^2 + y^2 = r^2$. The equation $(x - 2)^2 + y^2 = 3.2^2$ is the graphing form of a circle with center at $(2, 0)$ and a radius of $r = 3.2$. Sketch the circle on the graph.



4. The parametric equations for the circle from Problem 3 are shown as follows:

$$x(t) = 2 + 3.2\ \sin t$$
$$y(t) = 3.2\ \cos t$$

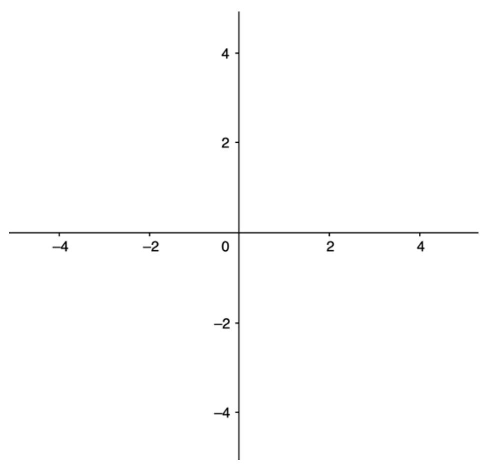
(The equation $y(t) = 3.2\ \cos t$ is the same as $y(t) = 0 + 3.2\ \cos t$. Let t be radians.)

Complete the table for the parametric equations:

t	$x(t)$	$y(t)$
-1	-0.692	________
0	________	________
1	4.692	1.728
2	4.909	________
3	________	-3.167

Locate these points on the sketch in Problem 3.

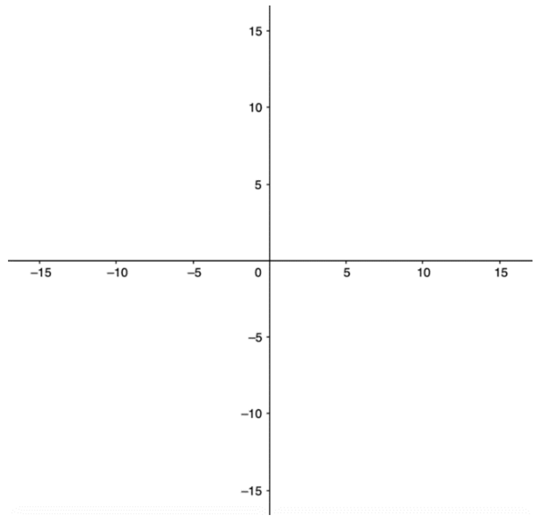
5. Draw the parametric equations from Problem 4 on the graphing calculator and copy it onto the graph.



6. Write the graphing form for the equation for a circle with a diameter of 10 and center at $(-7, 2)$.

7. Write the parametric equation for a circle with a diameter of 10 and center at $(-7, 2)$.

8. Use the parametric mode on the graphing calculator to graph the circle in Problem 7 and then sketch it below.



9. Use the function mode on the same page of the graphing calculator to draw the circle for Problem 6. Is it the same circle?

For Problem 10-15, use the information below to solve the problem.
The parametric equations below are for a small jet and a large jet as they pass a radar tower located at the origin.

Small Plane:	Large Plane:
$x(t) = 0 + 50t$	$x(t) = 0 + 80t$
$y(t) = 1{,}000 + 0t$	$y(t) = 1{,}200 + 0t$

10. What will the coordinates for the small airplane be after $t = 6$ seconds?

11. What will the coordinates for the large airplane be after $t = 6$ seconds?

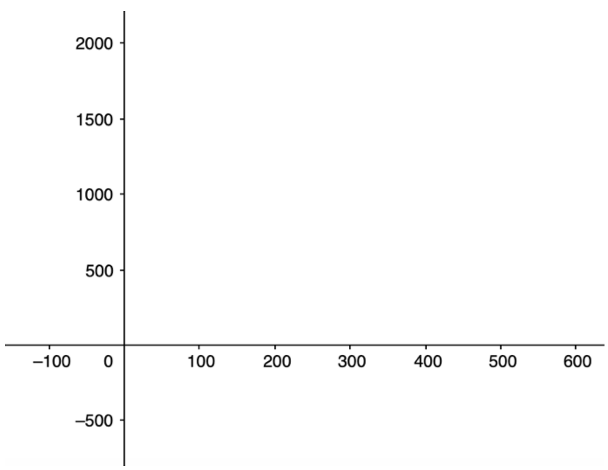
12. If x represents kilometers east of the tower and y represents kilometers north of the tower, what does each ordered pair represent?

13. Complete the tables below for the horizontal and vertical motion of the airplanes from $t = 0$ to $t = 6$ seconds.

Small Plane		
t	x	y
0		
1		
2		
3		
4		
5		
6		

Large Plane		
t	x	y
0		
1		
2		
3		
4		
5		
6		

14. Complete the graph below for the motion of the airplanes in relation to the radar tower at $(0, 0)$.



15. Write the equation for y in terms of x for the small plane and large plane.

For Problem 16-20, follow the instructions given to solve the problem.

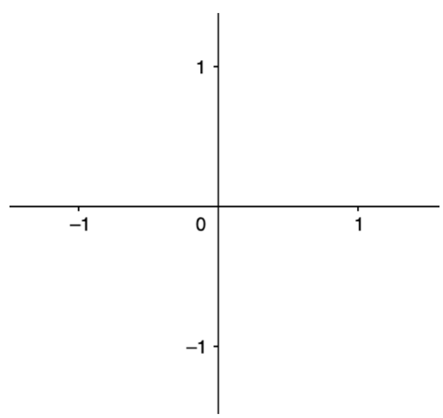
16. Complete the table for $x = \cos t$

t	$\cos t$
-6	
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	
6	

17. Complete the table for $y = \sin t$.

t	$\sin t$
-6	
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	
6	

18. Graph the ordered pairs for $(\cos t, \sin t)$ for $t = -6$ to $t = 6$. What do you think the graph will look like?



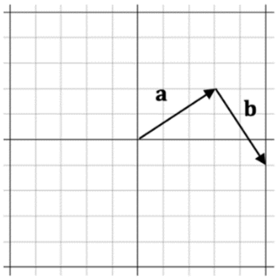
19. What is the parametric equation for a unit circle?

20. What is the standard form equation for a circle centered at $(0, 0)$ with a radius of 1?

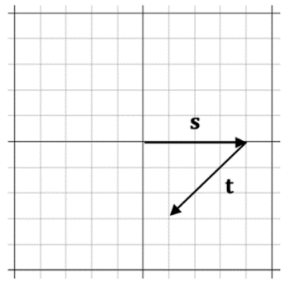
Section 3.14 Module Review

For Problem 1 and 2, given the vectors, draw the resultant vector and write it in component form.

1.

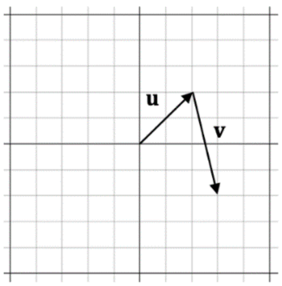


2.

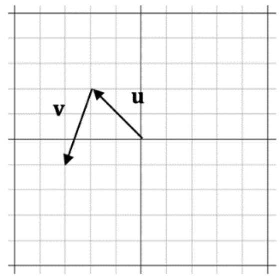


For Problem 3 and 4, given the vectors, follow the instructions to solve the problem.

3. Given the vectors, draw $\mathbf{u} + \mathbf{v}$.



4. Given the vectors, draw $\mathbf{u} - \mathbf{v}$.



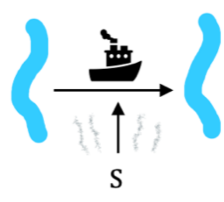
For Problem 5 and 6, sketch the vector given.

5. Bearing of 300°

6. Heading of 100°

For Problem 7-9, follow the instructions given to solve the problem.

7. If a boat is heading due east across a river with currents flowing from south to north, and is affected by the wind in the same direction (south to north), what will the bearing of the true course be between?



a) 0° to 90°

b) 90° to 180°

c) 180° to 270°

d) 270° to 360°

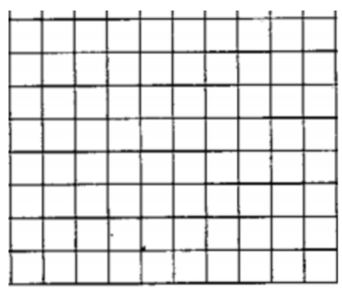
8. Multiply the column vector **u** by the scalar 8.

$$\mathbf{u} = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$$

9. Given the vector, which is the velocity of an airplane in meters per second, find the speed of the airplane, which is $\|\mathbf{v}\|$.

$$\mathbf{v} = 400\boldsymbol{i} + 300\boldsymbol{j}$$

Draw a graph of the velocity vector.



For Problem 10-15, use the given information to solve the problem.

Jazlin kicks a football to Silas at an angle of $45°$ to the horizontal ground with a velocity of $22\ \mathrm{m/s}$.

10. Find the horizontal and vertical velocity components of the football immediately after being kicked.

11. How far will the ball travel horizontally after t seconds? (This is the horizontal component of the parametric equations.)

12. How far will the ball travel vertically after t seconds? Gravity causes an object to accelerate downward at a rate of 9.8 m/s^2. Write an equation for the vertical travel of the ball including the effects of gravity using the equation $y(t) = -\frac{1}{2}gt^2 + v_i t$ where v_i is the initial velocity and g is gravity due to acceleration and t is time in seconds.

13. Assuming the kick occurred at $x = 0$ and $y = 0$ write the parametric equations for the projectile motion of the football.

14. Find the total time of the flight of the football.

15. What is the maximum height of the football in motion?

For Problem 16-20, tell whether the statement is true or false.

16. $\mathbf{v} + -\mathbf{v} = 0$

17. $\mathbf{u} - \mathbf{v} = -\mathbf{u} + \mathbf{v}$

18. $7\boldsymbol{i} - 3\boldsymbol{j} = \mathbf{u}$ is written in component form

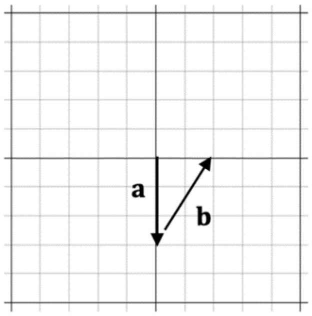
19. The norm of a vector is the vector parallel to it.

20. A vector has magnitude and direction.

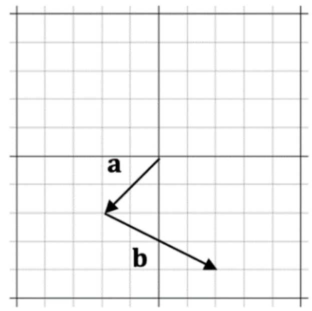
<u>Section 3.15 Module Test</u>

For Problem 1 and 2, given the vectors, draw the resultant vector and write it in component form.

1.

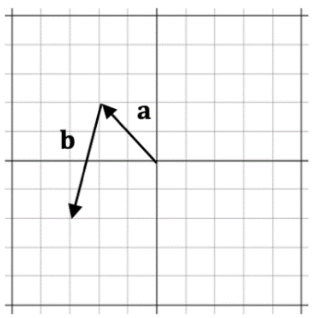


2.

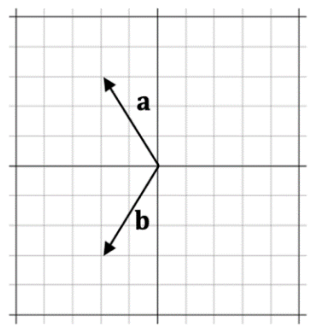


For Problem 3 and 4, given the vectors, follow the instructions to solve the problem.

3. Given the vectors, draw **a** + **b**.



4. Given the vectors, draw **a** − **b**.



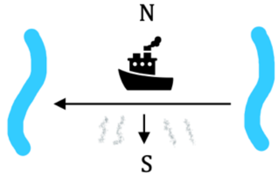
For Problem 5 and 6, sketch the given vector.

5. Bearing of 225°

6. Heading of 45°

For Problem 7-9, follow the instructions given to solve the problem.

7. If a boat is heading due west across a river with currents flowing from north to south, and is affected by the winds in the same direction (north to south), what will the bearing of the true course be between?



a) 0° to 90°

b) 90° to 180°

c) 180° to 270° d) 270° to 360°

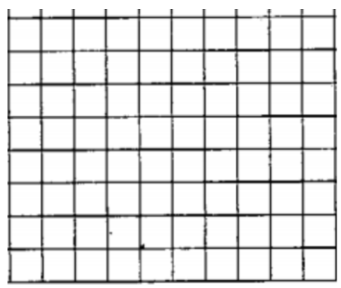
8. Multiply the row vector **v** by the scalar -3.

$$\mathbf{v} = \begin{bmatrix} \frac{1}{3} & 4 & -10 \end{bmatrix}$$

9. Given the vector, which is the velocity of an airplane in meters per second, find the norm of **u**, which is the speed of the airplane.

$$-225\boldsymbol{i} + 110\boldsymbol{j} = \mathbf{u}$$

Draw a graph of the velocity vector.



For Problem 10-15, use the given information to solve the problem.

Lindsey Vonn is a famous American skier. Suppose she leaves a ski jump with an initial velocity of 34.3 m/s at an angle of 45°.

10. Find the components of x and y. Do not take the effects of gravity into account yet.

11. How far will Lindsey Vonn travel horizontally after t seconds? (This is the horizontal component of the parametric equations.)

12. How far will Lindsey travel vertically after t seconds? Gravity causes an object to accelerate downward at a rate of 9.8 m/s^2. Write an equation for the vertical travel of Lindsey's jump adding the effects of gravity using the equation $y(t) = -\frac{1}{2}gt^2 + v_i t$ where v_i is the initial velocity, and g is gravity due to acceleration and t is time in seconds.

13. Assuming that the initial position of the jump is $x = 0$ and $y = 0$, write the parametric equations for the projectile motion of Lindsay's ski jump.

14. Find the total time of Lindsey's ski jump.

15. What is the maximum height of Lindsey's ski jump?

For Problem 16-20, tell whether the statement is true or false.

16. A scalar is a quantity that has magnitude but no direction.

17. $a\boldsymbol{i} + b\boldsymbol{j}$ represents a unit vector

18. Weight is a measure of force.

19. The terminal point of a vector is the tail.

20. Vectors may be added or subtracted.