Geometry and Trigonometry Module 4 Polygons and Quadrilaterals

Section 4.1 Classifying Polygons

Practice Problems 4.1

For Problem 1-6, answer the word problem.

1. How many people sing in a trio?

2. How many people sing in a quartet?

3. How many people sing in a quintet?

4. If bilateral sides of a figure are equal, how many sides are equal?

5. A city is celebrating its bicentennial anniversary. How many hundreds of years is that?

6. A nonvenal celebration would be after how many years?

For Problem 7-10, tell whether the statement is true or false.

7. If two polygons have the same perimeter, then they are congruent.

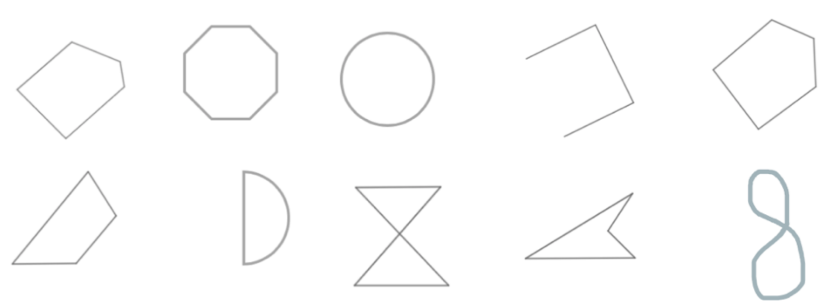
8. If two polygons have the same area, then they are congruent.

9. If two polygons are congruent, then the corresponding angles and the corresponding sides are congruent.

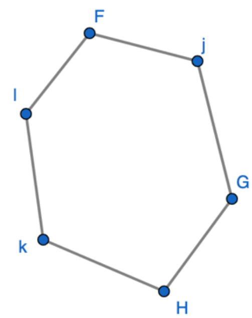
10. If the corresponding sides and the corresponding angles of two polygons are congruent, then the two polygons are congruent.

For Problem 11 and 12, follow the instructions given to solve the problem.

11. A polygon is a closed plane figure made up of segments. Circle the polygons below. Put an 'X' through the figures that are not polygons.



For Problem 12-20, use figure FjGHkl below to solve the problem.



12. What angles are consecutive to angle H?

13. What angles are opposite angle H?

14. Name the polygon by its sides.

15. Is the polygon concave or convex?

16. Draw all diagonals from angle F.

17. Name the pair of sides consecutive to vertex l.

18. What is the total number of diagonals in the hexagon?

19. If n is the number of sides in the figure, what is the number of diagonals from each vertex? What is a formula for the total number of diagonals in any polygon (an n-gon)?

20. Consider the number of triangles formed inside a polygon when the diagonals are drawn from one vertex. If each triangle formed by the diagonals is $180°$ degrees, what is the sum of the interior angles in the figure? What is a formula for the sum of the interior angles of an n-gon?

Section 4.2 Classifying Quadrilaterals

Practice Problems 4.2

For Problem 1-10, tell whether the statement is true or false.

1. It is possible to have three obtuse angles in a quadrilateral.

2. A regular polygon has all its sides congruent and all its angles congruent.

3. Another name for a regular triangle is isosceles.

4. Another name for a regular quadrilateral is a rhombus.

5. In a parallelogram, two of the angles may be $80°$ and $100°$ degrees.

6. All the angles in a quadrilateral add up to $180°$ degrees.

7. A quadrilateral is a rhombus if and only if it has four congruent sides.

8. A rhombus is a quadrilateral.

9. A square is an equiangular rhombus.

10. A square is an equiangular rectangle.

For Problem 11 and 12, follow the directions to answer the questions.

11. Cut two strips of paper the width of a ruler. Lay one strip on top of the other to form an ‘X’ and cut off the ends of the top piece that extend over the bottom piece. The shape you will have is a rhombus. Draw diagonals from each corner to the opposite corner.

a) Are the diagonals perpendicular?

b) Do the diagonals bisect each other?

c) Do the diagonals bisect the angles?

d) Are the diagonals axes of symmetry?

12. Cut one strip of paper the width of a ruler. Cut another strip double the width of a ruler. Lay one on top of the other to form an "X" and cut off the ends of the top piece that extend over the bottom piece. The shape you will have is a parallelogram. Draw diagonals from each corner to the opposite corner.

a) Are the diagonals perpendicular?

b) Are the diagonals congruent?

c) Do the diagonals bisect the angles?

d) Are the diagonals axes of symmetry?

For Problem 13-16, tell whether the statement is: always true; never true; sometimes true.

13. The diagonals of a square are perpendicular bisectors of each other.

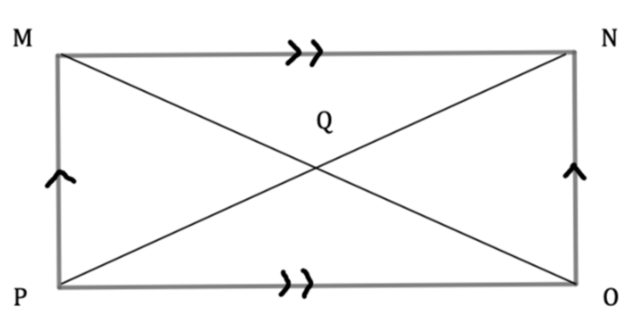
14. The diagonals of a parallelogram are congruent.

15. The consecutive angles of a square are congruent and supplementary.

16. The diagonals of a rhombus are congruent.

For Problem 17 and 18, use the given theorem(s) to solve the problem.

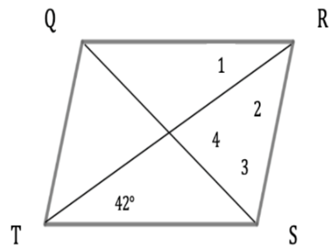
17. The Rectangle Diagonals Theorem says a parallelogram is a rectangle if and only if the diagonals are congruent. Below, MNOP is a rectangle. The arrows indicate parallel lines. One of the properties of a rectangle is that the diagonals of a parallelogram bisect each other.



a) If MQ = 5 cm., how long is PN?

b) If $m\ \angle$ONQ is 64°, how many degrees is $\angle$QPO?

18. The Rhombus Diagonals Theorem says a parallelogram is a rhombus if and only if its diagonals are perpendicular. The Rhombus Opposite Angles Theorem says a parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles. Below, QRST is a rhombus.



Find the angles measures:

$m\angle 1 =$

$m\angle 2 =$

$m\angle 3 =$

$m\angle 4 =$

For Problem 19 and 20, use the information below to write the Corollaries. A Corollary can be easily proven given a theorem.

The Rhombus Corollary says that a quadrilateral is a rhombus if and only if it has four congruent sides.

19. Write the biconditional statement for the Rectangle Corollary.
Hypothesis: A quadrilateral is a rectangle.
Conclusion: A quadrilateral has four right angles.

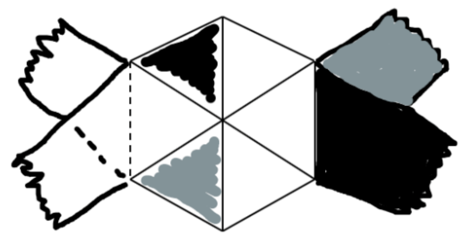
20. Write the biconditional statement for the Square Corollary.
Hypothesis: A quadrilateral is a square.
Conclusion: A quadrilateral is a rhombus and a rectangle.

Section 4.3 Angle Sums of Polygons

Practice Problems 4.3

1. Tie a paper hexagon.

1) Use two long strips of paper that are equal in width.
2) Tie a square knot as shown in the diagram below.
3) Tuck the ends of each strip into the loop of the other strip.
4) Pull tight and flatten the crease.
5) Cut off the lengths hanging out from the sides of the hexagon.



For Problem 2, complete the chart.

2.

Number of Sides of a Polygon	Number of Triangles in a Polygon	Sum of Interior Angles	Degree of Each Interior Angle
5	3	540°	108°
6			
7			
8			
9			
10			
11			
12			
n			

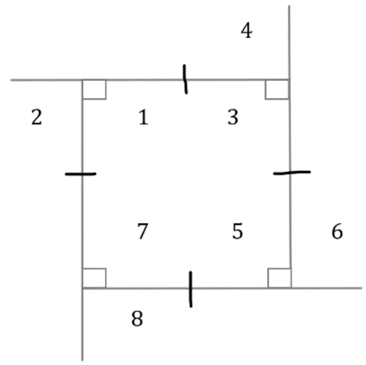
For Problem 3-5, use the chart above to complete the problem.

3. Add another column to the chart for sums of the angles of a polygon and call it **Degree of Each Exterior Angle**. Then complete the chart with the new column.

4. How many sides does a polygon have whose angles have a sum of 1,980° degrees?

5. Does the polygon in Problem 4 have to be equilateral? Does it have to be equiangular?

For Problem 6-12, use the figure below to solve the problem.



6. Which property states $m\angle 1 + m\angle 2 = 180°$ degrees?

7. For how many pairs of angles does the property from Problem 6 hold true for?

8. How do you know the figure is a square?

9. What is the sum of the 4 pairs of angles (numbered 1-8)?

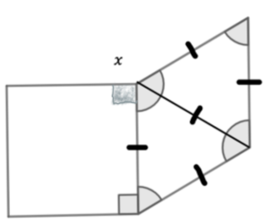
10. What is the sum of $m\angle 1 + m\angle 3 + m\angle 5 + m\angle 7$?

11. What is the sum of the 4 pairs of angles minus the 4 in Problem 10?

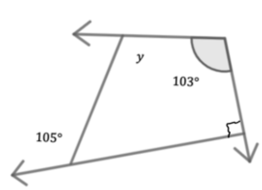
12. What property allows $m\angle 2 + m\angle 4 + m\angle 6 + m\angle 8$ to equal $360°$ degrees?

For Problem 13 and 14, find the measure of the given angle.

13. Find the measure of x.



14. Find the measure of y.



For Problem 15-20, solve the word problem.

15. What is the sum of the measures of the exterior angles of a 96-gon?

16. What is the measure of each interior angle of an equiangular triangle?

17. Does a triangle have to be equilateral for the sum of the exterior angles to be $360°$ degrees?

18. Does a triangle have to be equiangular for each exterior angle to measure $120°$ degrees?

19. In the third century, clay tablets could have the shape of a regular 16-gon. If a piece of a tablet was discovered in an archaeological dig such that each interior angle of the 3 angles on the piece was $160°$ degrees, was this a third century tablet?

20. What is the measure of each angle of a tablet that is a regular 16-gon?

Section 4.4 The Geometry of Tangrams

Practice Problems 4.4

For Problem 1-4, use the shapes from the lesson notes to solve the problem.

1. Trace the triangle labeled “3” onto the center of a piece of paper. Put the right angle in the lower left corner and mark it with a small square. Call the left side of the triangle a and the base b. Let the hypotenuse on the right be c.

Which three pieces can be traced onto the piece of paper to make a square with side a being the right side of the square? Can the same three pieces fit to make a square with side b as the top side of the square? Which two pieces fit to make a square with side c as the left side of the tilted square? How do these two pieces relate to the three pieces on the other two sides of the square? How do you think the Chinese used this to discover the Pythagorean Theorem?

a) There are five right triangles in total. Let the length of the original side of the Tangram square be n. What are the lengths of the sides of triangles “1” and “2” in terms of n?

b) What is the hypotenuse of triangle “3” in terms of n?

c) What is the area of the square in terms of n?

d) What are the side lengths of the small triangles “4” and “5” in terms of n?

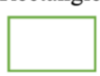
e) What is the area of triangles “4” and “5” in terms of n? What is the area of triangle “3” in terms of n?

f) What is the area of the two squares on the sides of the triangle, a and b, in terms of n?

g) Since triangles “3” and “4” and “5” Fit into triangles “1” or “2”, what is the area of the square formed on side c of the triangle in terms of n?

2. Complete the chart below. You have already completed some of these. For example, you created a square from two triangles. You can draw that in the first box under the square and to the right of “2.” It looks like the shape in step 2 (from the lesson notes when constructing the Tangram set). The solutions are given numerically, and some have geometric representations.

You may want to work on completing the chart over the course of the next week or so. Just come back to it throughout each day and draw in the solutions. You could also copy it to a poster board if you need more space to draw. Some may not have solutions; for example, you cannot make a rectangle with only two of the Tangram pieces.

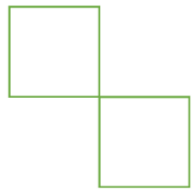
Make These Shapes							
Number of Pieces to Use	Square 	Rectangle 	Triangle 	Trapezoid 	Trapezoid 	Parallelogram 	Hexagram 
2							
3							
4							
5							
6							
7							

3. See if you can make squares, rectangles, parallelograms, and trapezoids using 5, 6, or 7 pieces.

Now, we will be playing a game called Polyominoes. A polyomino is a shape made of a connected set of congruent squares. So, in the game *Polyominoes*, we begin with a given number of squares and two restrictions: 1) Each square must have an edge in common with at least one of the other squares in the set; 2) Squares that touch must have either an edge or vertex in common.

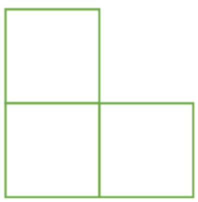
Domino

Not a Domino

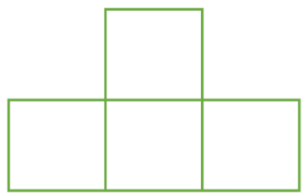


There is only one way to combine two squares to make a domino. Any other domino is a duplication through rotation or reflection.

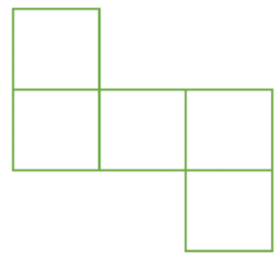
Triomino



Tetromino



Pentomino



a) Given 3 squares, how many triominoes can you draw?

b) Given 4 squares, how many tetrominoes can you draw?

c) Given 5 squares, how many pentominoes can you draw?

d) Can you make rectangles using each of the different pentomino configurations?

Section 4.5 The Geometry of Star Polygons

Practice Problems 4.5

For Problem 1-6, solve the word problem.

1. Find all the star polygons when $n = 8$. How many are there?

2. When numbers 1-8 are used for m, which form star polygons, and which do not? Why do you think this is the case?

3. Two numbers are called relatively prime when their only shared factor is 1. For numbers 1-8, which numbers are relatively prime to 8? Do these form star-polygons? Which numbers share a common factor other than 1? Do these form star polygons?

4. Now that you know that m must be relatively prime to n to form a star polygon, how many unique star polygons can be made when $n = 12$?

5. Can a star polygon ever be formed when $n = 2$? Why or why not? When will star polygons begin repeating in terms of n?

6. If we change the rules so that we are not starting at the same point each time, other star polygons can be created. For example, we can get a 6-star polygon from two inverted triangles.

To create a 6-star polygon, begin at one point and connect every second point around the circumference to form a triangle. Move one point right or left of the initial point and begin connecting every second point around the circumference from this point to form another triangle. Now, we have a 6-pointed star.

Our new rule allows us to start at one point and connect to a second, third, or fourth point before moving to the next point and repeating the pattern at each point. What is the sum of the angle measures of the 6-star polygon using this new rule? What does the 6-star polygon look like when $m = 3$ using the new rule?

In a true star, you can make a complete circuit from one point back to the start without lifting your pencil. In a 6-star polygon, you must lift your pencil to complete the figure, so we are calling this a truncated star polygon.

For Problem 7-10, fill in the table and use it to solve the problems.

7. If the points of an n-gon are connected every mth point, the formula for finding the angle measure sums is $180°|n - 2m|$ (using the new rule to create star polygons). Let us investigate angle measure sums by using the formula to complete the table below.

$\boldsymbol{n}$	$\boldsymbol{m = 1}$	$\boldsymbol{m = 2}$	$\boldsymbol{m = 3}$	$\boldsymbol{m = 4}$	$\boldsymbol{m = 5}$
5	$540°$	$180°$			Not a star
6	$720°$				
7	$900°$		$180°$		$540°$
8	$1{,}800°$	$1{,}440°$	$720°$	$360°$	

8. Draw the diagrams for the shapes of $n = 5$ and $n = 6$.

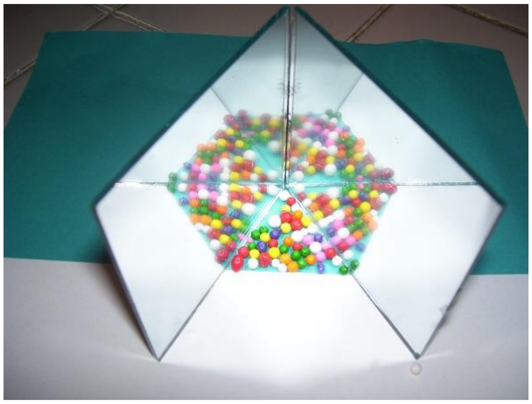
9. When do the angle sum measures tend to repeat the initial one when $m = 1$?

10. What are the patterns in the table when m is increasing for odd numbers of n? What are the patterns in the table when m is increasing for even numbers of n?

Section 4.6 Central Angles and Apothems

Practice Problems 4.6

For Problem 1, use the picture below to solve the problem.



1. What is the measure of the swing angle of the mirror for the regular hexagon in the mirror?

For Problem 2-9, fill in the chart and use it to help solve the problems.

2. Use the formula for the central angle to complete the chart below. Make the polygons in the chart using the mirror and measure the swing angle to confirm your answers.

Polygon	**Number of Sides**	**Measure of Swing Angle**
Triangle		
Quadrilateral		
Pentagon		
Hexagon		
Heptagon		$51.42°$
Octagon		
Nonagon		
Decagon		

3. What is the measure of a central angle of a square?

4. How many sides does a regular polygon have if the central angle is approximately $32.72°$ degrees?

5. What is the sum of the measure of the interior angles of a regular pentagon?

6. Find the measure of each interior angle of a regular pentagon.

7. As the number of sides of a polygon increases to 84, then to 201, etc. what happens to the polygon inscribed in the circle?

8. How could you use the apothem to determine the radius of the circle in which the polygon is inscribed if the length of the side of the polygon is given?

9. The apothem of each polygon and the length of the side are given in the chart below. Find the area of each polygon and complete the chart.

Polygon	**Number of Triangles**	**Apothem (a)**	**Length of Side (s)**	**Area of Polygon**
Pentagon	5	5	7.2	
Hexagon		4	3.8	
Septagon		6	3.9	
n-gon				

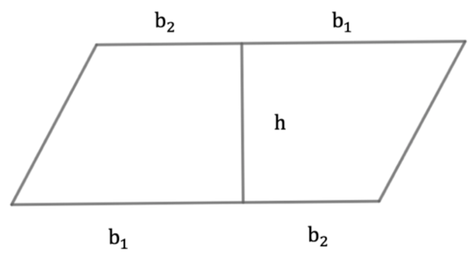
For Problem 10, solve the word problem.

10. A soccer ball has 20 regular hexagons and 12 regular pentagons on its surface. If the side length of each hexagon and each pentagon measures 4.3 cm., and the apothem of the hexagon is 3.9 cm. and the apothem of the pentagon is 3.1 cm., find the approximate surface area of the soccer ball.

Section 4.7 Areas of Polygons

Practice Problems 4.7

For Problem 1-4, use the given figure to answer the questions and/or solve the problem.



1. The height divides the parallelogram into two congruent trapezoids. By now, you know the formula for the area of a parallelogram, and you will use this information to derive the formula for the area of a trapezoid.

a) What is the base of the parallelogram?

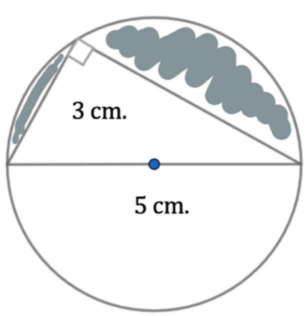
b) What is the height of the parallelogram?

c) What is the area of the parallelogram?

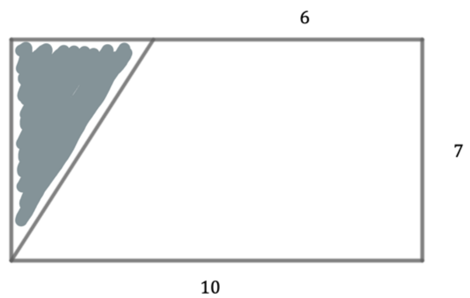
d) How does the area of the original trapezoid compare to c)?

e) What is the formula for the area of a trapezoid in terms of base and height?

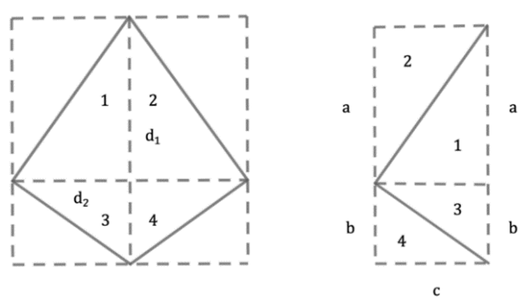
2. You know the formula for the area of a circle is: $A = \pi r^2$ (the formula for circles will be derived when we study circles). Use this formula and all you have previously learned to find the area of the shaded region in the figure below.



3. Find the area of the shaded region in the figure below.



4. The pieces of the kite in the figure below on the left have been rearranged to form the figure on the right. Answer the given questions to derive the formula for the area of a kite.



a) How does the rectangle on the right compare to the original rectangle in which the kite on the left is inscribed?

b) Which diagonal on the figure to the left represents the base $(a + b)$ on the figure to the right?

c) Which part of the diagonal on the figure to the left represents the height (c) on the figure to the right?

d) Using $(a + b)$ for base and c for height in the area formula for the triangle, find the formula for the area of a kite in terms of d_1 and d_2.

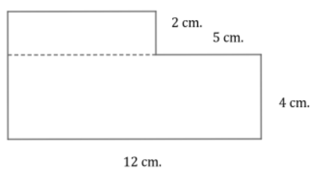
For Problem 5 and 6, use the information from Problem 4 to solve the problem.

5. If $d_1 = 24$ cm. and $d_2 = 12$ cm., find the amount of glass needed to cover the kite in the window (rectangle) shown in the diagram to the left.

6. Using the information from Problem 5, what is the amount of glass needed to cover the area of the window (rectangle) that is not covered by the kite?

For Problem 7-9, find the area of the given shape.

7.



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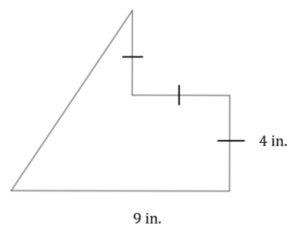

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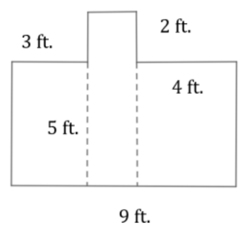
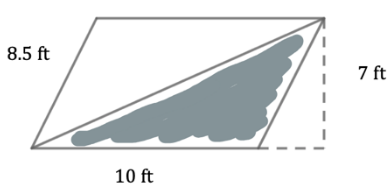


For Problem 10-14, find the area of the shaded region in the given figure.

10.



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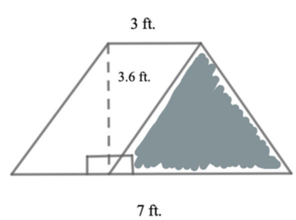

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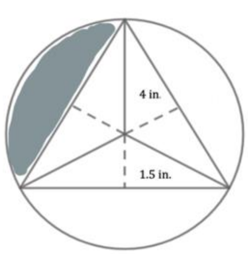
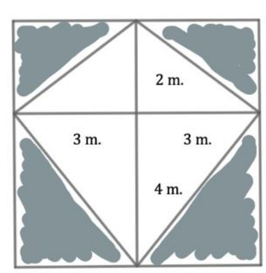


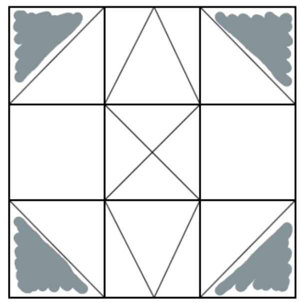
The center of the circle is the point where the segments of the equilateral triangles intersect, whose base is 6.35 inches.

13.



14. Without calculating, what is the area of the non-shaded region in the parallelogram in Problem 10?

For Problem 15-20, use the quilted square below to answer the problem. (One quilted square is made up of 9 squares.)



15. If the quilted square has an area of 144 square inches, what is the length of the side of each square?

16. What is the area of the shaded triangles in the four corners?

17. What is the area of the interior of the figure in the middle column of the quilted square?

18. How many square feet of material is covered by the quilted square?

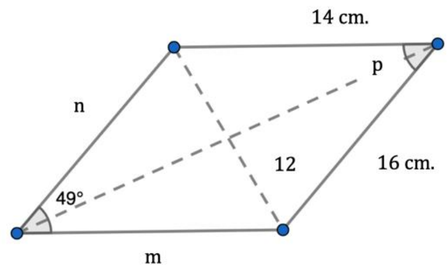
19. Sixteen quilted squares are needed to make a baby blanket. How many square yards of batting are needed between the front and back of the quilt?

20. If a 4-inch border surrounds the blanket in Problem 19, how many square feet of material are needed to make the blanket?

Section 4.8 Properties of Parallelograms

Practice Problems 4.8

For Problem 1-4, find the missing sides and angles in the parallelogram displayed.



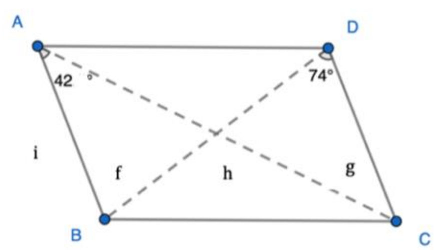
1. m =

2. n =

3. angle P =

4. Find the perimeter of the parallelogram. The perimeter of a polygon is the sum of the lengths of the sides.

For Problem 5-10, use the parallelogram ABCD to find the missing sides and angles and answer the questions.



5. angle g =

6. angle f =

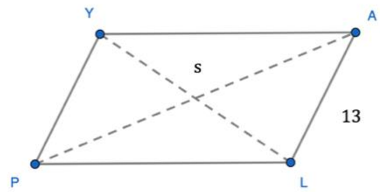
7. angle h =

8. If the perimeter of ABCD is 80 inches and side BC is 26 inches, find the length of the other three sides of the parallelogram.

9. If angle ABC is 110°, what is the measure of angle BDA?

10. What are the measures of angles DAC and BCA?

For Problem 11-15, use the parallelogram PLAY to solve the problem.



11. If PL = $3x + 8$ and the perimeter of PLAY is 66, find x.

12. If PL = $3x + 3$ and AY = $4x - 12$, find the distance of PL.

13. If the perimeter of PLAY is 46, what is the distance of AY?

14. If angle ALY has a measure of 63° and angle YPA has a measure of 25°, what is the measure of angle s?

15. Find all four sides of the parallelogram if PL = $3x + 8$ and LA = $x + 11$, and the perimeter is 118 and side LA is not 13.

For Problem 16-19, answer true or false.

16. All rectangles are parallelograms.

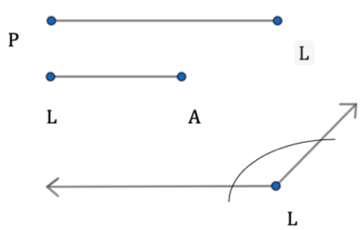
17. All quadrilaterals are parallelograms.

18. The Parallelogram Diagonals Conjecture states that diagonals bisect the angles of the parallelogram.

19. The opposite angles of a parallelogram are supplemental.

For Problem 20, use the given diagrams to solve the problem.

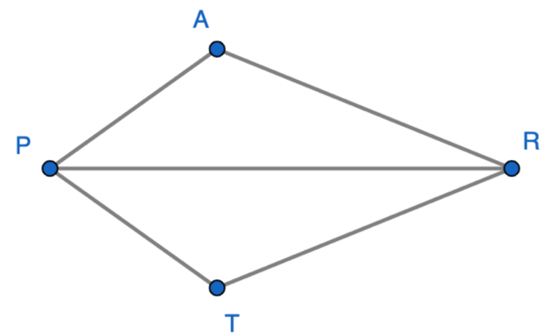
20. If you are given side $\overline{PL}$, side $\overline{LA}$, and $\angle L$, how would you construct a parallelogram PLAY?



Section 4.9 Properties of a Kite

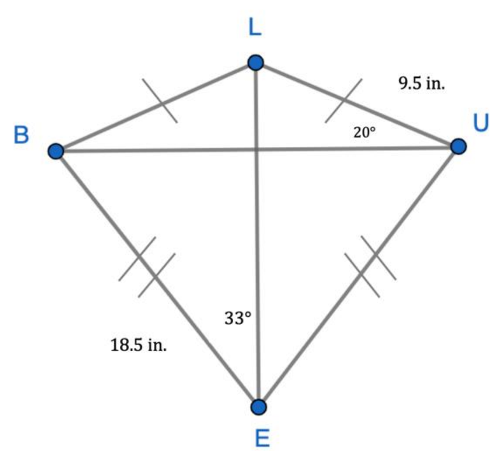
Practice Problems 4.9

For Problem 1-6, use the kite PART to solve the problem.



1. angle APR ≅ angle TPR since ___________________________________ bisects _____________________
2. angle ARP ≅ angle TRP since ___________________________________ bisects _____________________
3. side PR ≅ side PR by the _____________________ property
4. angle PAR ≅ angle PTR because _____________________ angles in a kite are congruent
5. Name the vertex angles in the kite PART.
6. If the two triangles PAR and PTR share a common side and all three angles are congruent, what conjecture can we make about triangles PAR and PTR?

For Problem 7-11, use the kite BLUE to solve the problem.



7. The measure of angle ELU = ____.
8. If the measure of angle LEU = __________, the measure of angle EUB is ____________________.

9. The measure of side BL = __________

10. The measure of side EU = __________

11. The perimeter of the kite BLUE = ____

For Problem 12 and 13, solve the word problem.

12. To find the area of a kite, multiply the lengths of the diagonals and then divide the product by 2. What is the area of a kite that has diagonals with lengths of 26.5 cm. and 38.2 cm.?

13. a) Balinda is going to build a kite. She has two dowel rods measuring 24” and 10” and lays them in a cross, so that the long rod is 9” above the shorter rod and 15” below it. She is going to cover the kite in mylar material. How much mylar does she need to cover the kite?

b) Balinda adds panels to each edge. The panels are 1” wide at each edge to fold down so that ½” is in the front and ½” is in the back. The panels will be looped front to back to hold the string along each edge. How much mylar does she need for the panels? How much mylar material does she need to build the kite? Round to the nearest tenth.

For Problem 14 and 15, fill in the blank.

14. When all four sides of a kite are congruent, the kite can be named a ______________________.

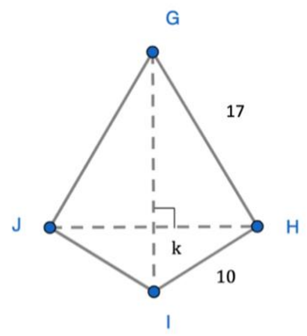
15. When all four angles are right angles as well, then the kite can be named a _____________________.

For Problem 16-20, solve the word problem.

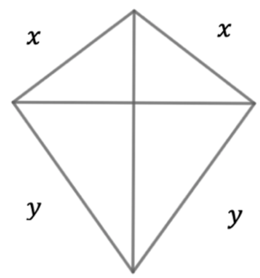
16. The kite pictured is called a DART. Is it a concave polygon or convex polygon?

17. What is the possible number of 90° angles a kite could have if it were not a rhombus or a square?

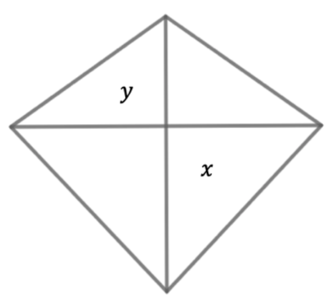
18. The length of the diagonal JH in the shape below is 16 cm. What is the length of side k? Find the length of diagonal GI.



19. What is the formula for the perimeter of the kite in terms of x and y? Find the perimeter if $x = 13.2$ cm. and $y = 18.4$ cm.



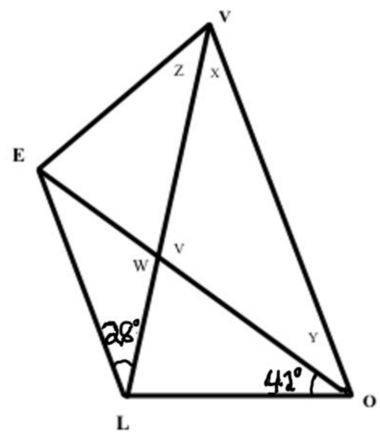
20. What is the formula for the area of the kite in terms of x and y if d_1 is y and d_2 is x? Find the area of the kite if $x = 19$ cm. and $y = 12.5$ cm.



Section 4.10 Properties of Trapezoids

Practice Problems 4.10

For Problem 1-5, use the figure below to find the missing angles.



LOVE is an isosceles trapezoid with parallel bases LE and OV and two congruent sides: LO and EV. Find the angle measures for Problem 1-6 given that the measure of angle EVO is 70° and angle OLE is 110°.

1. angle X = __________

2. angle Y = __________

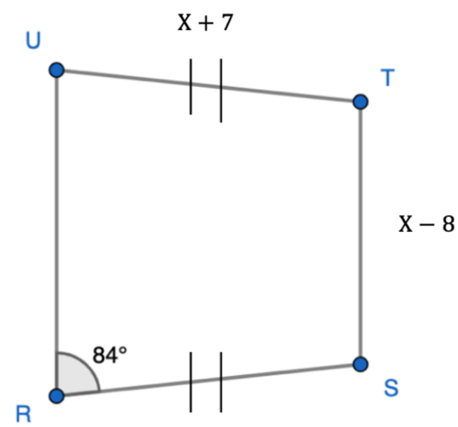
3. angle Z = __________

4. angle V = __________

5. angle W = __________

6. What conjecture can you make about the diagonals of an isosceles triangle?

For Problem 7-10, use the figure below to find the missing measurements for isosceles trapezoid RSTU.



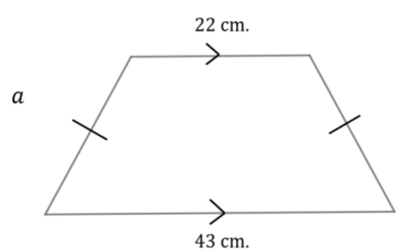
7. angle T = __________ 8. angle U = __________

9. If the perimeter of RSTU is 58 cm. and the length of side UR is 10 cm., find the length of each of the congruent sides.

10. What is the length of the shorter base in RSTU given the information in Problem 8?

For Problem 11-16, find the missing segments or angles given the trapezoid.

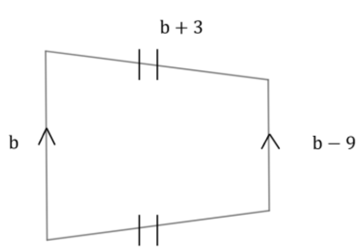
11.



Perimeter = 79 cm.

$a =$ __________

12.



Perimeter = 65 cm.

b = __________

13. What is the length of the midsegment of the trapezoid in Problem 11?

14. What is the length of the midsegment of the trapezoid in Problem 12?

The median length is the average of the two base lengths. The equation for the median length is: $m = \frac{b_1 + b_2}{2}$ or $m = \frac{1}{2}(b_1 + b_2)$. If we substitute m in the formula for the area of a trapezoid, we get $A = mh$.

15. What is the area of a trapezoid if the length of the midline is 18.4 mm. and the height is 12.1 mm.?

16. What is the area of a trapezoid if the length of the midline is 14.6 mm. and the height is 6.9 mm.?

For Problem 17-20, answer the word problem.

17. If a quadrilateral has angle measures of 60° degrees, 60° degrees, 120° degrees, and 120° degrees, is the quadrilateral a kite or a trapezoid? Explain your reasoning.

18. If the midsegment of a trapezoid has a length of 0, what is the name of the shape created?

19. How many lines of symmetry does a trapezoid have that is not isosceles?

20. What is the maximum number of right angles a trapezoid can have that is not isosceles?

Section 4.11 Dilations, Vectors, and Translations

Practice Problems 4.11

For Problem 1-4, tell whether the dilations of figures are enlargements or reductions given the scale factor.

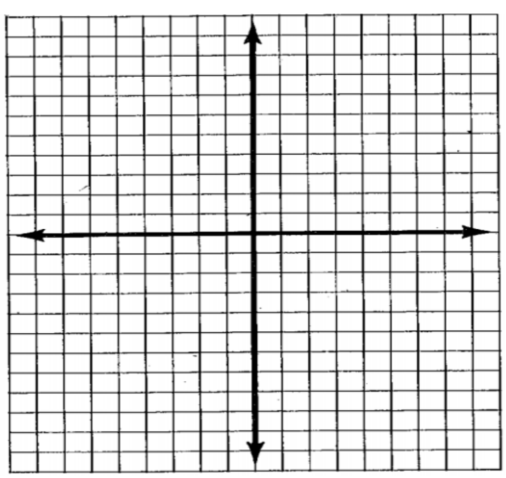
1. 2
2. $\frac{1}{8}$
3. $\frac{3}{10}$
4. 22

For Problem 5-8, write the rule for the coordinates of a dilation given the scale factor.

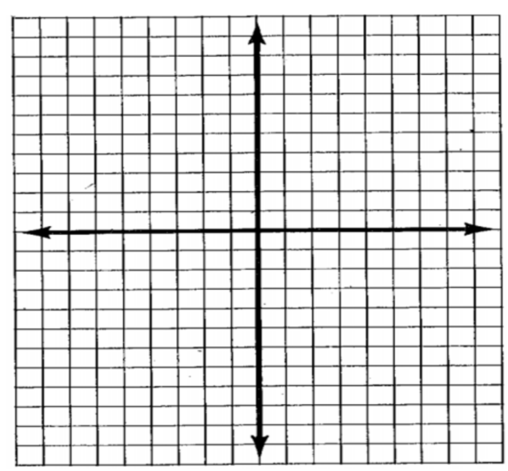
5. $\frac{2}{3}$
6. $\frac{5}{7}$
7. 11
8. 4.2

For Problem 9 and 10, given the coordinates of the pre-image and the scale factor, find the coordinates of the image. Tell whether it is an enlargement or reduction and write the coordinate rule for the dilation. Graph both the pre-image and image and label the points of the coordinates.

9.
Quadrilateral
$G\ (-3, 0)$
$L\ (-2, 2)$
$A\ (1, 4)$
$D\ (1, -3)$
Scale factor: 3

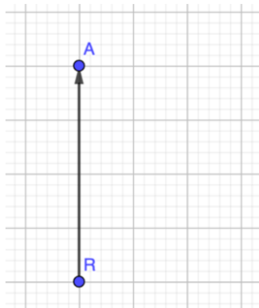


10.
Triangle
$K\ (1, -8)$
$A\ (4, 4)$
$M\ (12, 0)$
Scale factor: $\frac{1}{4}$

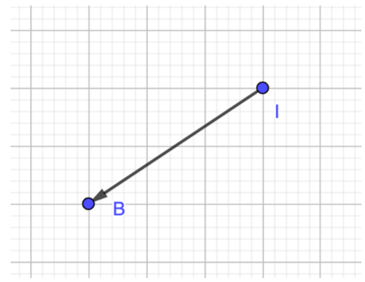


For Problem 11-14, write the vector in component form and find the horizontal and vertical components. Use the large gray squares as one unit. Try it with the small gray squares as one unit. What do you notice?

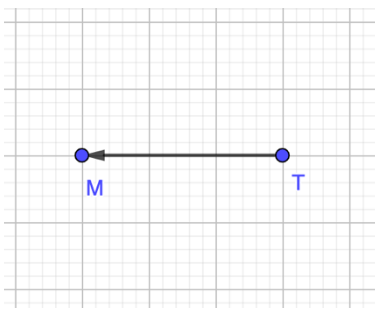
11.



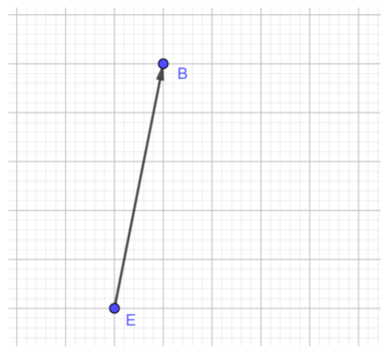
12.



13.



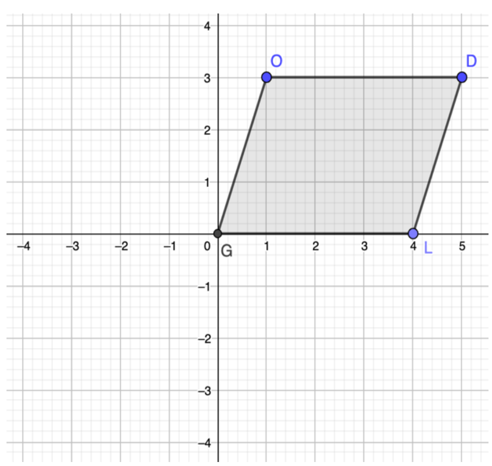
14.

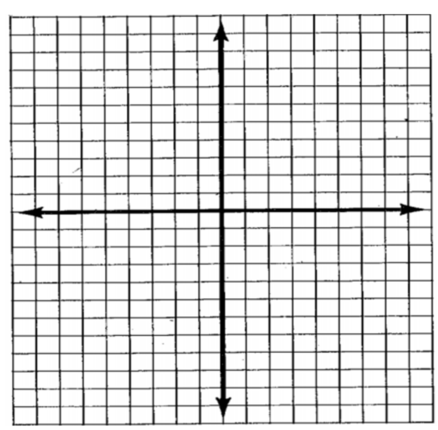


For Problem 15 and 16, translate the polygons using the given translation vector. Then graph the image and write the rule using the coordinates.

15.

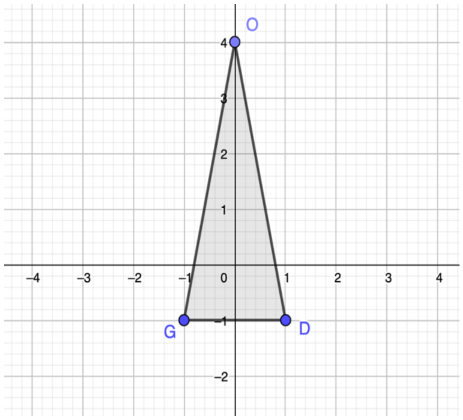
Vector: $\langle 2, 1 \rangle$
***G* (0, 0)**
***O* (1, 3)**
***D* (5, 3)**
***L* (4, 0)**

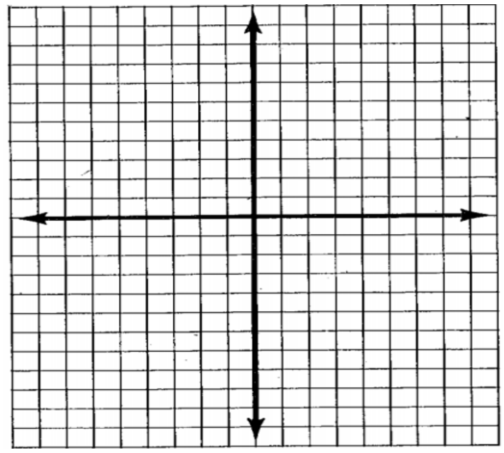




16.

Vector: $\langle \mathbf{0}, \mathbf{4} \rangle$
G **(-1, -1)**
O **(0, 4)**
D **(1, -1)**

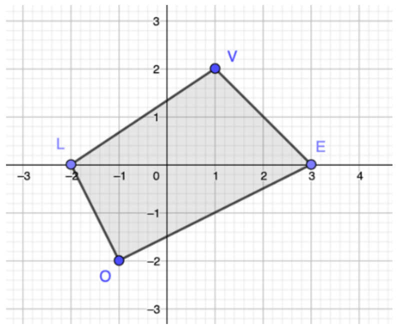




For Problem 17 and 18, given the coordinate rule and the pre-image, write the translation vector and graph the image after the transformation.

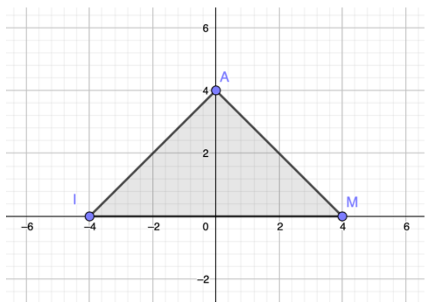
17.

L **(-2, 0)**
O **(-1, -2)**
E **(3, 0)**
V **(1, 2)**
$(\boldsymbol{x}, \boldsymbol{y}) \to (\boldsymbol{x} - \mathbf{1}, \boldsymbol{y} - \mathbf{1})$



18.

$I\ (-4, 0)$
$A\ (0, 4)$
$M\ (4, 0)$
$(x, y) \to (x + 3, y - 3)$



For Problem 19 and 20, use the given information to solve the problem.

19. Graph the triangle IAM from Problem 18 using a scale factor of $-\frac{1}{2}$. What happens to the image? Is it an enlargement or a reduction? Write the rule using the coordinates.

20. Emmalie and Brooklyn had photos made in a photo booth and want to enlarge the picture to make thank you cards. The photo booth picture has a width of 1.2 inches and a height of 1.2 inches. The thank you cards have a width 3 inches and a height of 3 inches. What is the scale factor for the dilation?

Section 4.12 Reflections and Rotations

Practice Problems 4.12

For Problem 1-8, given the point $(5, -3)$ as the pre-image, tell the coordinates of the point of the image after the given transformation.

1. $(x, y) \to (x + 4, y - 2)$

2. Reflection over the line $y = -x$

3. Reflection over the line $y = x$

4. Reflection over the y-axis

5. Reflection over the x-axis

6. Rotation about the origin of $270°$

7. Rotation about the origin of $180°$

8. Rotation about the origin of $90°$

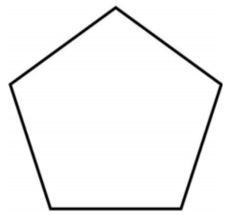
For Problem 9-12, use the following information to solve the problem:

If a figure has rotational symmetry, then it maps onto itself.

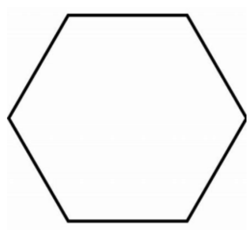
9. Does an isosceles trapezoid have rotational symmetry?

10. Does a circle have rotational symmetry?

11. How many lines of rotational symmetry does a regular pentagon have?



12. How many lines of rotational symmetry does a regular hexagon have?

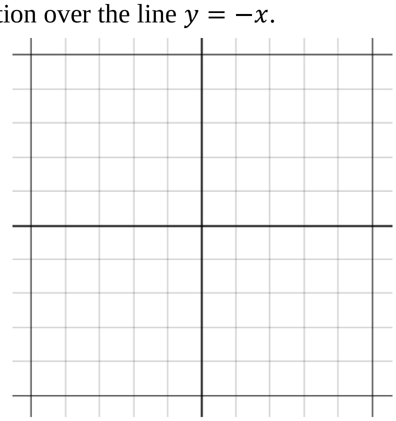


For Problem 13-16, use ΔTAP to solve the problem: ΔTAP has vertices T$(2, 1)$, A$(2, 4)$, and P$(5, 1)$.

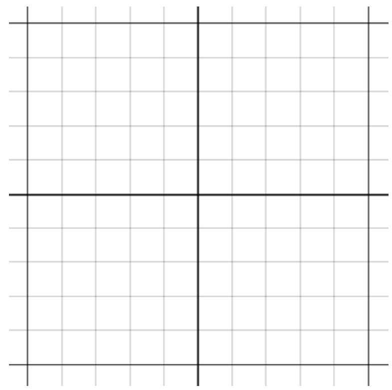
13. What are the coordinates of the vertices after a reflection over the x-axis?

14. What are the coordinates of the vertices after a reflection over the y-axis?

15. Draw the image after a reflection over the line $y = -x$.



16. Draw the image after a rotation of $270°$ about the origin.



For Problem 17-18, find the endpoints for line segment AB with endpoints A$(-4, -5)$ and B$(1, 2)$ after the given composition transformation.

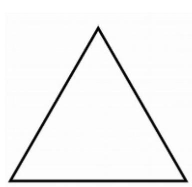
17. Translation: $(x, y) \rightarrow (x - 3, y)$ Rotation: $180°$ about the origin

18. Rotation: $90°$ about the origin Reflection: over the line $y = x$

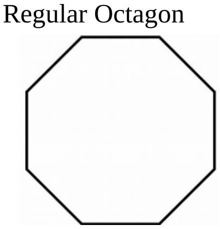
For Problem 19 and 20, given the information, solve the problem.

19. What angles of rotational symmetry apply to the given polygons?

a) Regular Triangle



b) Regular Octagon



20. Is rotating a polygon by $180°$ about the origin the same as the composition made up of a reflection over the y-axis followed by a reflection over the x-axis?

Section 4.13 Pick's Theorem and Euler's Formula

Practice Problems 4.13

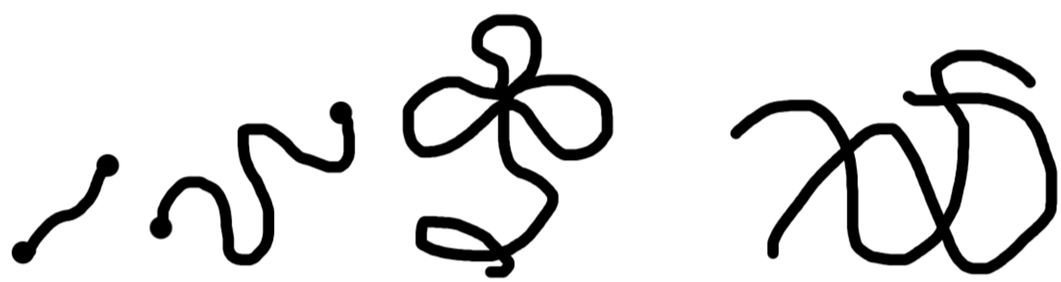
This section will include mathematical doodling and its relationship to geometry, graph theory, and recursion. You have already learned about recursion in Algebra 1 and 2, so you may return to the those to review recursion at any time. In Module 1, you were introduced to graph theory and Leonard Euler's circuits with the Konigsberg Bridge Problem when you studied discrete mathematics.

Leonard Euler was a Swiss mathematician and physicist. His father and grandfather (on his mother's side) were pastors. His father was also a good friend to Johann Bernoulli, who gave Euler educational lessons every Saturday after he recognized his brilliance. Every year, Euler would participate in the Paris Academy Prize Competition, which he won twelve times in his lifetime.

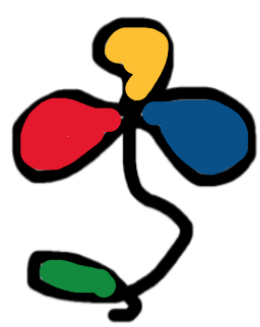
If they were all printed, Euler's works would amount to 60-80 volumes. Even at the end of his life, when he was going blind, he produced an article a week. In his 25 years of teaching at the Russian Academy of Science in St. Petersburg, Euler wrote over 380 articles. When requested to teach a German princess, he wrote over 200 letters to her concerning mathematics and philosophy. These letters ended up helping us understand his personality and religious beliefs, and when published as "Letters of Euler on Different Subjects in Natural Philosophy Addressed to a German princess," became his best-selling work in Europe and the United States.

Moreover, Euler became incredibly famous for his polyhedral formula of 3-dimensional solids. For now, we will explore Euler's formula using polygons and mathematical doodling.

All of us doodle at some point, maybe while sitting and waiting for someone or while talking on the phone. To understand doodling, each doodle is defined by points or vertices at the beginning and end of lines, curves, or loops, which may or may not be seen.



The straight line and curved line to the left are doodles with two vertices. The flower and squiggly lines to the right do not have clearly shown vertices where the lines cross or meet. The lines to the right do not contain links or loops, which have two distinct endpoints. The flower does contain loops and these enclosed spaces are called faces. The flower has four of these, which are shown below in different colors.



A doodle is connected if you can travel from one vertex to the other without lifting your pencil, as seen with the flower. In the woodland house below, the pencil must be lifted to complete the doodle.

For Problem 1-3 use the given figures/charts to solve the problem.

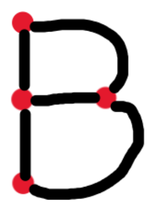
1. The letters A and B are shown with the vertices in red (where the edges meet or cross). The letter A has: 5 vertices, 1 face, 5 edges (between vertices). The letter B has: 4 vertices, 2 faces, 5 edges.

a) Which letters of the alphabet have the most vertices and which letters have the fewest vertices?

b) Which letters have the most edges? Which letters have the fewest edges?

c) Which letters have the same number of vertices and edges?

d) Can you state a relationship between the number of faces, edges, and vertices in a doodle?

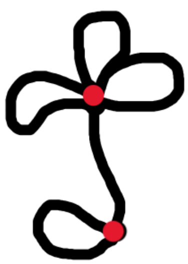


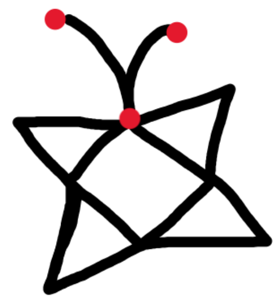
Vertices: __________ Vertices: __________

Faces: __________ Faces: __________

Edges: __________ Edges: __________

2. Complete the chart for the given doodles. Does it follow the same rule you established using the letters of the alphabet?





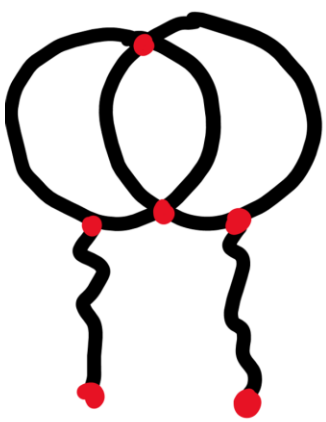
Vertices: __________ Vertices: __________ Vertices: __________

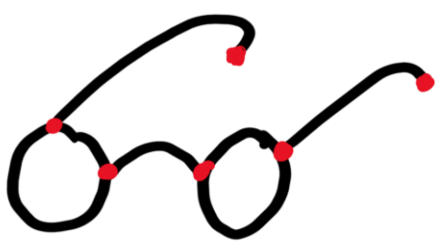
Faces: __________ Faces: __________ Faces: __________

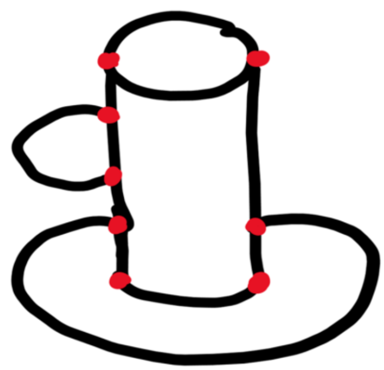
Edges: __________ Edges: __________ Edges: __________

What is the relationship between the vertices, edges, and faces? (This is called Euler's formula.)

3. Complete the chart for the given doodles. Does it follow the same rule you checked in Problem 2?







Vertices: __________ Vertices: __________ Vertices: __________

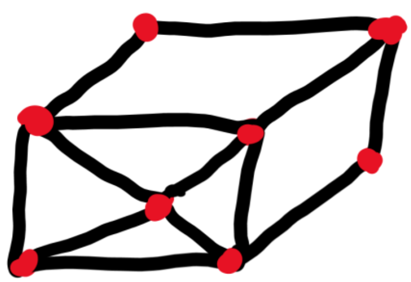
Faces: __________ Faces: __________ Faces: __________

Edges: __________ Edges: __________ Edges: __________

4. Circle doodles are made by drawing concentric circles one on top of another. Complete the chart for the given circle doodles. Let c represent the number of circles. Does Euler's formula work for all of these?

Circle Doodles	C	**Vertices**	**Faces**	**Edges**
	2	2	3	4
	3			
	4			
	c			

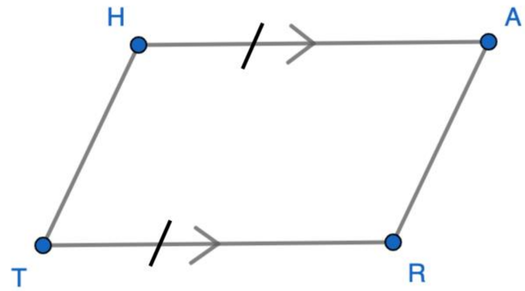
5. Here is one more doodle to use to check Euler's Formula for vertices, edges, and faces. Does it still work with a 3-D effect?



6. Have fun making your own doodles and checking Euler's Formula.

Section 4.14 Module Review

For Problem 1-7, given parallelogram HART solve the problem.



1. Name the one pair of congruent and parallel sides. How do you know?

2. True or False: The definition of a parallelogram is a quadrilateral with exactly one pair of parallel and congruent sides. (Explain why.)

3. If $\angle R$ is obtuse, is the quadrilateral a rectangle or a square?

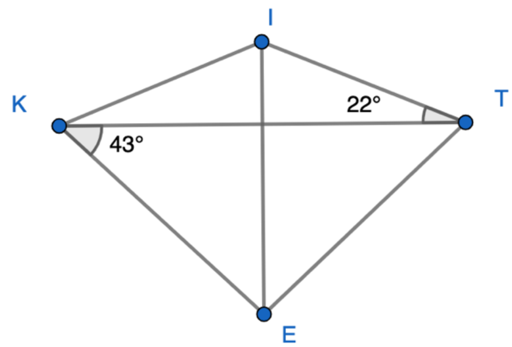
4. Is the quadrilateral a kite or a rhombus?

5. Is segment HT congruent and parallel to segment AR?

6. Do segments HR and TA bisect each other? Are they congruent?

7. Do the diagonals bisect the angles?

For Problem 8-18, use kite KITE to solve the problem. Angles KIT and KET are vertex angles.



8. What is the measure of angle ITE?

9. What is the measure of angle KIT?

10. What is the measure of angle KET?

11. If the length of ET is 13 cm., what other line segment of the kite is that same length?

12. If the length of KI is 11 cm., what other line segment of the kite is that same length?

13. If the length from I to where the diagonals meet is 9 cm., what is the length of KT?

14. If the length from E to where the diagonals meet is 10 cm., what is the length of the longest diagonal?

15. What is the area of the kite?

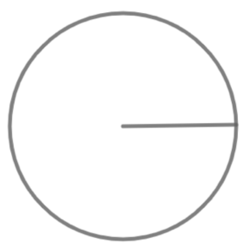
16. Is the kite a rhombus?

17. Is the kite a parallelogram?

18. Is the kite a trapezoid?

For Problem 19 and 20, use the given figure to solve the problem.

19. For the circle below, one radius is drawn from the center of the circle to the circumference. When there is one radius, the center of the circle is divided into one region or face. Draw a second radius and count the number of faces or regions of the circle. Do this four times to complete the chart and establish a relationship between the number of radii in a circle and the regions/faces of the circle.

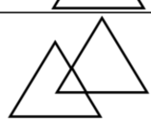


Radii	**Regions**	**Radii and Regions Relationship**
1		
2		
3		
4		
r		

a) Graph the radii on the x-axis and the regions on the y-axis. Graphically, what type of relationship is this?

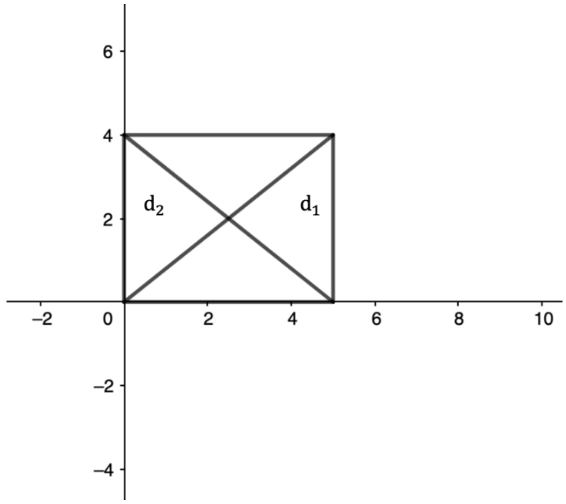
b) What is the equation that models the relationship between the radii and the regions?

20. Complete the chart for the triangle doodles. Let **t** be the number of triangles.

Triangle Doodles	**t**	**Vertices**	**Faces**	**Edges**
	1	3	1	3
	2			
	3			
	t			

Section 4.15 Module Test

For Problem 1-14, use the given figure to solve the problem. Let d_1 and d_2 be the diagonals of the figure.



1. How long is the base of the figure along the x-axis?

2. What other side has the same length as the base along the x-axis?

3. How long is the left side of the figure?

4. What other side has the same length as the left side of the figure?

5. Use the Pythagorean Theorem to find the length of d_1.

6. Use the distance formula to find the length of d_1.

7. Use the Pythagorean Theorem to find the length of d_2.

8. Use the distance formula to find the length of d_2.

9. What is the slope of the base of the figure?

10. What is the slope of the top of the figure?

11. What does this tell you about the top and bottom sides of the figure?

12. What is the slope of d_1?

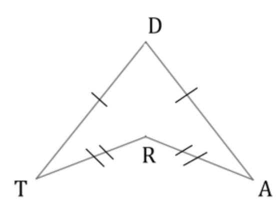
13. What is the slope of d_2?

14. What do the slopes of d_1 and d_2 tell you about the diagonals of the rectangle?

15. Is the figure a parallelogram?

16. Is the figure a rectangle or a square?

Use diagram DART to answer Problem 17-18. Side DA has a measure of 13 cm.

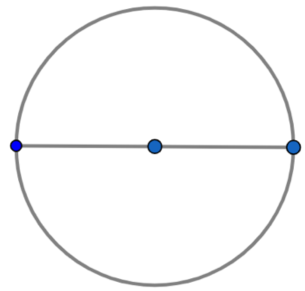


17. What is the perimeter of DART if side RA has a measure of 7 cm.?

18. Do the angles of DART have a sum of 360°?

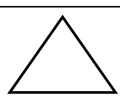
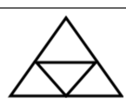
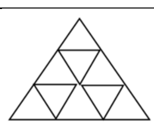
For Problem 19 and 20, use the given figure to solve the problem.

19. There is one diameter drawn in the circle and it divides the circle into two regions. A diameter is the line that runs from one side of a circle to the other side through the center point of the circle. Complete the chart to find the relationship between the diameters of a circle and the regions. Graph this and find the equation that models the data.



Diameters	**Regions**	**Diameter to Regions Relationship**
1		
2		
3		
4		
d		

20. Below is a doodle of Sierpinski triangles. Let **t** be the number of triangles on the side of the triangle. Complete the chart to find the formulas for the vertices, faces, and edges when there are **t** triangles on a side.

Sierpinski Doodles	**t**	**Vertices**	**Faces**	**Edges**
	1	3	1	3
	2			
	3			
	t			