## Geometry and Trigonometry Module 2 Data Analysis

Section 2.1 Median-Median Line
Practice Problems 2.1
For Problem 1-6, use the information below to solve the problem.

The table below shows the data for total fat and total calories in sandwiches at fast-food restaurants.

| Total Fat <br> (oz.) | Total <br> Calories |
| :---: | :---: |
| 14 | 360 |
| 35 | 630 |
| 3.5 | 310 |
| 43 | 700 |
| 18 | 340 |
| 34 | 520 |
| 29 | 550 |
| 100 | 1,354 |
| 20 | 780 |

1. Order the $x$-values from least to greatest and divide the data into thirds. (Make sure the $y$-value moves with the $x$-value.)
2. The $x$-median will be the middle point of each group. Why? The $y$-median will need to be determined in each group. Why?
3. Find $M_{1}, M_{2}$, and $M_{3}$.
4. Find the slope, $y$-intercept, and equation for $M_{1}$ and $M_{3}$.
5. Find the slope-intercept equation and $y$-intercept for $M_{2}$. (Use the slope from Problem 4 to do this.)
6. What is the mean of the three $y$-intercepts? What is the equation of the median-median line?

The standard form of a linear equation is $a x+b y=c$. The slope-intercept form is $b y=-a x+c$, which can be simplified to $y=-\frac{a}{b} x+\frac{c}{b}$. This equation is the same as $y=m x+b$ when $-\frac{a}{b}$ is the slope ( $m$ ) and $\frac{c}{b}$ is the slopeintercept ( $b$ ).
For Problem 7-11, convert the standard form equation to the slope-intercept form.
7. $2 x-3 y=5$
8. $2 x+y=10$
9. $5 x+10 y=15$
10. $8 y=-16$
11. $3 x+3 y=-3$

For Problem 12-14, write the slope-intercept equation in standard from.
For example, $y=\frac{2}{3} x+4 \rightarrow-\frac{2}{3} x+y=4$. Multiply each side by -3 to eliminate the negative sign in front and clear the denominator: $(-3)-\frac{2}{3} x+(-3) y=4(-3) \rightarrow 2 x-3 y=-12$.
12. $y=6 x-2$
13. $y=-\frac{1}{2} x+5$
14. $y=-x+15$

For Problem 15-20, identify the slope and $y$-intercept in the equation.
15. $y=-\frac{1}{3} x+2$
16. $y=5 x$
17. $y=-2 x+4.2$
18. $10 y=20 x+20$ (Hint: divide each term by 10)
19. $2 x+4 y=8$ (Hint: firstly, convert to slope-intercept form)
20. $2 x+y=14$

## Section 2.2 Average-Mean Line

Practice Problems 2.2
For Problem 1 and 2, fill in the table.

1. Using the graph from Example 2, make a table showing the actual distance of the vertical lines (the deviations). This can be found by subtracting the data point $y$ from the average-mean line, where $y=616$.

| Total Fat $(\boldsymbol{x})$ | Total Calories $(\boldsymbol{y})$ | Deviation $(\boldsymbol{y}-\overline{\boldsymbol{y}})$ |
| :---: | :---: | :---: |
| 3.5 | 310 |  |
| 14 | 360 |  |
| 18 | 340 |  |
| 20 | 780 |  |
| 29 | 550 |  |
| 34 | 520 |  |
| 35 | 630 |  |
| 43 | 700 |  |
| 100 | 1,354 |  |

2. Some of the deviations are negative. Complete the table below to find the absolute value of the deviations.

Why are the acceptable answers all positive values?

| Total Fat (x) | Total Calories $(\boldsymbol{y})$ | Deviation $\mid(\boldsymbol{y}-\overline{\boldsymbol{y}} \mid$ |
| :---: | :---: | :---: |
| 3.5 | 310 |  |
| 14 | 360 |  |
| 18 | 340 |  |
| 20 | 780 |  |
| 29 | 550 |  |
| 34 | 520 |  |
| 35 | 630 |  |
| 43 | 700 |  |
| 100 | 1,354 |  |

For Problem 3 and 4, use the table below to solve the problem.

| Saturated <br> Fat $(\boldsymbol{x})$ | Total Fat <br> $(\boldsymbol{y})$ |
| :---: | :---: |
| 5 | 14 |
| 10 | 35 |
| 1 | 3.5 |
| 19.5 | 43 |
| 5 | 18 |
| 13 | 34 |
| 10 | 29 |
| 20 | 54 |

3. Use the table to find $\bar{y}$.
4. Make a graph showing the deviations (the distance of each data point from the average-mean line).


For Problem 5 and 6, fill in the table.
5. Fill in the deviations for the table below (calculate the distance of the vertical lines by taking $y-\bar{y}$.)

| Saturated Fat $(\boldsymbol{x})$ | Total Fat $(\boldsymbol{y})$ | Deviation $(\boldsymbol{y}-\overline{\boldsymbol{y}})$ |
| :---: | :---: | :---: |
| 5 | 14 |  |
| 10 | 35 |  |
| 1 | 3.5 |  |
| 19.5 | 43 |  |
| 5 | 18 |  |
| 13 | 34 |  |
| 10 | 29 |  |
| 20 | 54 |  |

6. Find the absolute value of the deviations to complete the table below. Which two data points may be outliers?

| Saturated Fat $(\boldsymbol{x})$ | Unsaturated Fat $(\boldsymbol{y})$ | Absolute Value of the Deviation <br> $(\|\boldsymbol{y}-\overline{\boldsymbol{y}}\|)$ |
| :---: | :---: | :---: |
| 5 | 14 |  |
| 10 | 35 |  |
| 1 | 3.5 |  |
| 19.5 | 43 |  |
| 5 | 18 |  |
| 13 | 34 |  |
| 10 | 29 |  |
| 20 | 54 |  |

The slope is the rate of change or $\frac{\Delta y}{\Delta x}$. This is found by using the formula $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$. Two points that lie on the same line are $(3,6)$ and $(-1,-4)$. The slope of the line is $m=\frac{-4-6}{-1-3}=\frac{-10}{-4}=\frac{5}{2}$.

For Problem 7-11, find the slope given the two points on a line.
7. $(4,3)$ and $(8,7)$
8. $(1,10)$ and $(2,6)$
9. $(0,0)$ and $(8,-2)$
10. $(-3,-6)$ and $(5,9)$
11. $(-4,10)$ and $(-2,7)$

For Problem 12-15, use the information below to solve the word problem.
The slope formula leads to the point-slope form of an equation: $y-y_{1}=m\left(x-x_{1}\right)$. This equation can be found using one point and the slope.
When the points are $(3,6)$ and $(-1,4)$ (like in the example for Problem $7-11)$ the slope $(m)$ is equal to $\frac{5}{2}$. We can use the first point, $(3,6)$, and the slope to find the equation as shown below:

$$
\begin{gathered}
y-6=\frac{5}{2}(x-3) \\
y-6=\frac{5}{2} x-\frac{15}{2} \\
y=\frac{5}{2} x-\frac{15}{2}+6 \\
y=\frac{5}{2} x-\frac{15}{2}+\frac{12}{2} \\
y=\frac{5}{2} x-\frac{3}{2} \\
-\frac{5}{2} x+y=-\frac{3}{2} \\
(-2)-\frac{5}{2} x+(-2) y=\left(-\frac{3}{2}\right)(-2) \\
5 x-2 y=3
\end{gathered}
$$

12. Write the equation for Problem 11 in point-slope form using the first point from Problem 11. Convert to standard form.
13. Write the equation for Problem 11in point-slope form using the second point from Problem 11. Convert to standard form.
14. The two equations for Problem 12 and 13 are the same. What is the $y$-intercept of the equation?

For Problem 16-20, identify the point and the slope of the equation.
15. $y-3=2(x-1)$
16. $y+3=5(x+8)$
17. $y-7=4(x+9)$
18. $y+8.2=\frac{2}{3}(x-6)$
19. $y=\frac{1}{2}(x-3)$
20. $y-4=4 x$

Section 2.3 Standard Deviation
Practice Problems 2.3
For Problem 1-4, use the given data to solve the problem.

| Day (x) | Laps Run (y) |
| :---: | :---: |
| 2 | 7 |
| 4 | 10 |
| 6 | 18 |
| 8 | 17 |
| 10 | 22 |

1. Find $\bar{y}$.
2. Fill in the missing numbers below for the average-mean line and the scale of the $x$ and $y$ axis.

3. Complete the table below by calculating the mean deviations.

| Day $(\boldsymbol{x})$ | Laps Run $(\boldsymbol{y})$ | Mean Deviations $(\boldsymbol{y}-\overline{\boldsymbol{y}})$ |
| :---: | :---: | :---: |
| 2 | 7 |  |
| 4 | 10 |  |
| 6 | 18 |  |
| 8 | 17 |  |
| 10 | 22 |  |

4. Complete the table below for the mean deviations and the squares of the mean deviations.

| Days (x) | Laps Run $(\boldsymbol{y})$ | Mean Deviations $(\boldsymbol{y}-\overline{\boldsymbol{y}})$ | Mean Deviations Squared <br> $(\boldsymbol{y}-\overline{\boldsymbol{y}})^{2}$ |
| :---: | :---: | :---: | :---: |
| 2 | 7 |  |  |
| 4 | 10 |  |  |
| 6 | 18 |  |  |
| 8 | 17 |  |  |
| 10 | 22 |  |  |

For Problem 5, fill in the blanks.
5. Using the distance from the data point to the mean line as the side length of a square can help complete a square to the right of the data point.
a) Each square is a geometric representation of the square of the $\qquad$ .
b) If all the areas of the squares are added, that is a numerical representation of the sum of the
$\qquad$ of the $\qquad$ _.

6. Find the sum of the squares of the mean deviations and calculate the sample standard deviation using the following formula:

$$
s=\sqrt{\frac{\Sigma(\mathrm{y}-\bar{y})^{2}}{n-1}}
$$

7. Calculate the coefficient of variance from Problem 6 using the following formula:

$$
C=\frac{s}{\bar{y}}
$$

What does this tell you about the data? Is it uniform or varied?
8. What is the sum of the squares of the mean deviations in Problem 4? (This will be used again in Section 2.13.)

For Problem 9-13, find the $x$-intercepts for each equation. (Hint: This is when $y=0$.)
9. $x-y=5$
10. $2 y=5$
11. $x=-6$
12. $x+y=2 x+2 y$
13. $3 x+6 y=12$

For Problem 14-18, find the $y$-intercepts for each equation. (Hint: This is when $x=0$.)
14. $2 x+1=0$
15. $y=\frac{1}{3} x+4$
16. $5 x-10 y=-15$
17. $y=-24$
18. $2 x+4 y=-8$

For Problem 19 and 20, find the $x$-intercepts and $y$-intercepts for each equation.
19. $2 x+3 y-10=2$
20. $3 x-7 y=2$

## Section 2.4 Residual Deviation

Practice Problems 2.4
For Problem 1-3, use the table below to solve the problem.

| Saturated Fat $(\boldsymbol{x})$ | Total Fat $(\boldsymbol{y})$ | Predicted Values <br> $(\widehat{\boldsymbol{y}})$ | Residuals <br> $(\boldsymbol{y}-\widehat{\boldsymbol{y}})$ | Squares of the <br> Residuals $(\boldsymbol{y}-\widehat{\boldsymbol{y}})^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 14 | 16.5 |  |  |
| 10 | 35 | 29 |  |  |
| 1 | 3.5 | 6.5 |  |  |
| 19.5 | 43 | 52.75 |  |  |
| 5 | 18 | 16.5 |  |  |
| 13 | 34 | 36.5 |  |  |
| 10 | 29 | 29 |  |  |
| 20 | 54 | 54 |  |  |

1. Find the residuals and the square of the residuals in the table above.
2. Why are the four values in the right-hand corner of the table zero?
3. What is the sum of the squares of the residuals?

For Problem 4-8, use the given information to solve the problem.
4. Use the first two points in the table to find a linear equation that models the data.
5. Complete the table below using the equation from Problem 4 as the line of best fit to find $\hat{y}$.

| Day $(\boldsymbol{x})$ | Laps Run (y) | Predicted Values <br> $(\hat{\boldsymbol{y}})$ | Residuals $(\boldsymbol{y}-\widehat{\boldsymbol{y}})^{\mathbf{2}}$ | Squares of the <br> Residuals $(\boldsymbol{y}-\widehat{\boldsymbol{y}})^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 7 |  |  |  |
| 4 | 10 |  |  |  |
| 6 | 18 |  |  |  |
| 8 | 17 |  |  |  |
| 10 | 22 |  |  |  |

6. What is the sum of the squares of the residuals for Problem 5?
7. The calculator linear regression of line of best fit is $y=1.85 x+3.7$. Use that to complete the table below.

| Day $(\boldsymbol{x})$ | Laps Run $(\boldsymbol{y})$ | Predicted Values <br> $(\widehat{\boldsymbol{y}})$ | ${\text { Residuals }(\boldsymbol{y}-\widehat{\boldsymbol{y}})^{\mathbf{2}}}^{\text {La }}$Squares of the <br> Residuals $(\boldsymbol{y}-\widehat{\boldsymbol{y}})^{2}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 7 |  |  |  |
| 4 | 10 |  |  |  |
| 6 | 18 |  |  |  |
| 8 | 17 |  |  |  |
| 10 | 22 |  |  |  |

8. Which is smaller: the sum of the squares of the residual in Problem 6 or the sum of the squares of the residuals from Problem $8\left(\mathrm{SS}_{\mathrm{res}}\right)$ ? What line is a better line of best fit and why?

For Problem 9-11, graph the lines using the slope and $y$-intercept.
For example, in $y=-3 x+4, m=-\frac{3}{1}$, and $b=4$. For the slope of this equation, go to $(0,4)$, draw a point, then go down 3 and right 1 and draw another point. Connect the two points to sketch the line.
9. $y=4 x+2$
10. $y=-x+3$
11. $y=-\frac{2}{3} x-1$

For Problem 12 and 13, use the previous problem given to solve the problem.
12. How can you tell from the graph of Problem 11 that the slope is negative?
13. How can you tell from the slope of Problem 9 that the line will go up from left to right?

For Problem 14 and 15, solve the word problem.
14. If two lines have the same slope, they are parallel. Write an equation for a line that has a $y$-intercept of 4 and is parallel to the line in Problem 10.
15. If two lines are perpendicular, they have a slope that is the opposite reciprocal of the other. Write an equation for a line that has a $y$-intercept of -6 and is perpendicular to the line in Problem 10.

For Problem 16-20, use the information below to solve the problem.
If you have an equation in point-slope form you can identify a point and the slope. Plot the point then follow the rise and the run of the slope to draw the next point on the graph. Connect the two points to sketch the line.

For example, in $y-3=\frac{1}{2}(x+4)$, the point on the line is $(-4,3)$ and the slope is $m=\frac{1}{2}$.
16. $y-2=4(x-1)$
17. $y+3=\frac{1}{3}(x-1)$
18. $y=5(x+7)$
19. In graphing form, Problem 16 is $y=4(x-1)+2$. What are $h$ and $k$ ?
20. Find $(h, k)$ for Problem 17. This is not the vertex because a linear equation has no vertex. What is it?

## Section 2.5 Coefficient of Correlation Practice Problems 2.5

The calculator was used to find the coefficient of correlation for the line of best fit from the experiment in the lesson comparing the percent of sucrose to the density of the beverage sample. For the practice problems, you will calculate it by hand using everything you have learned so far in this module. How close is it to the calculator result?

## Linear Regression and Statistical Data

1. Complete the table below with the values for $x$ (volume) and $y$ (mass) from the beverage sample.
2. Use the linear regression equation you found for the beverage sample and let it be $y$. This is the predicted value of $y$ based on the equation for the line of best fit. Fill in this column in the table.
3. Find the difference between the predicted value, $\hat{y}$, and the actual value of $y$ from the experiment. This is called the residual. Fill in this column in the table.
4. Find the squares of the residuals. Fill in this column in the table.
5. Add up all the squares of the residuals. This is called the sum of squares of the residuals, or $\mathrm{SS}_{\text {res }}=$ $\qquad$ _.
6. Add up all the values of the $y$-coordinates and divide the sum by the total number of $y$-values. This is called the mean $(\bar{y})$. The mean is $\qquad$
7. Find the difference between the $y$-values and the mean. This is called the deviation or mean deviation. Fill in this column in the table.
8. Find the squares of the deviation and fill in this column in the table.
9. Add up all the squares of the deviations. This is called the sum of the squares of the deviations, or $\mathrm{SS}_{\mathrm{dev}}=$
$\qquad$ -.
10. The $\mathrm{SS}_{\text {res }}$ is a minimum of all possible values. The mean deviation is from the horizontal or constant function, the average-mean line. The residual is from the line of regression, or line of best fit, and is smaller. If you remove the fractional difference, $\mathrm{SS}_{\mathrm{dev}}$, of one from the other you get an even better fit. This is called the coefficient of determination.

$$
r^{2}=\frac{\left|S S_{\mathrm{res}}-\mathrm{SS}_{\mathrm{dev}}\right|}{S S_{\mathrm{dev}}}
$$

$$
r^{2}=
$$

11. The $r$ is called the correlation coefficient and it measures the fit of the data to the function. If it is 1 or -1 , it is a perfect fit. The correlation between the function and data can be strong, weak, or not at all if $r=0$.

$$
\sqrt{r^{2}}=\quad r=
$$

12. The function that identifies a pattern in Chemistry that is a theoretical law would give an exact fit, but when you collect data from an experiment, it is not an exact fit. Is your correlation a good fit or not?

| Volume (x) | Actual Value <br> $(\boldsymbol{y})$ | Predicted <br> Value $(\hat{\boldsymbol{y}})$ | Residual <br> $(\boldsymbol{y}-\widehat{\boldsymbol{y}})$ | Residual <br> Squared <br> $(\boldsymbol{y}-\widehat{\boldsymbol{y}})^{2}$ | Deviation <br> $(\boldsymbol{y}-\overline{\boldsymbol{y}})$ | Deviation <br> Squared <br> $(\boldsymbol{y}-\overline{\boldsymbol{y}})^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 10.19 |  |  |  |  |  |
| 20 | 20.53 |  |  |  |  |  |
| 30 | 31.47 |  |  |  |  |  |
| 40 | 41.01 |  |  |  |  |  |
| 50 | 52.06 |  |  |  |  |  |
| 60 | 62.71 |  |  |  |  |  |
| 70 | 72.86 |  |  |  |  |  |
| 80 | 83.33 |  |  |  |  |  |
| 90 | 94.31 |  |  |  |  |  |

Section 2.6 Permutations, Combinations, and Binomial Probability Distributions Practice Problems 2.6
For Problem 1-6, state whether or not the situation represents a binomial experiment. If it does not, explain why.

1. Four cards are selected from a deck of 52 one at a time and are not replaced after each pick. The number of threes taken from the deck is recorded.
2. Four cards are selected from a deck of 52 one at a time and are then replaced after each pick. The number of threes taken from the deck is recorded.
3. A die is rolled ten times. Whenever it lands on two it is recorded.
4. Red, green, and blue marbles are in a bag. Five trials that consist of picking one marble and recording its color are performed. The marble is replaced each time.
5. A coin is flipped eight times. Whenever it lands on tails it is recorded.
6. Of college students surveyed in $2017,68 \%$ said they did not get more than eight hours of sleep a night. Fifty college students are selected at random and asked if they slept more than eight hours the previous night.

For Problem 7-12, consider flipping a coin five times (one side is tails and the other is heads).
7. Complete a probability distribution table for the experiment. If the coin lands on tails it is a success.

| 0 Successes | 1 Success | 2 Successes | 3 Successes | 4 Successes | 5 Successes |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

8. Complete the binomial probability distribution table for $k$ and $p(k)$.

| $\boldsymbol{k}$ | $\mathrm{n}_{\mathbf{k}}$ | $\boldsymbol{p}^{\boldsymbol{k}}$ | $\boldsymbol{q}^{\boldsymbol{n - \boldsymbol { k }}}$ | $\boldsymbol{p}(\boldsymbol{k})$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 |  |  |  |  |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |

9. Make a histogram to represent the binomial probability distribution.

10. Find the probability of getting two tails.
11. Find the probability of getting two or three tails.
12. Find the probability of getting at least two tails.

For Problem 13-15, evaluate the combination.
13. ${ }_{9} \mathrm{C}_{5}$
15. $\quad{ }_{2} \mathrm{C}_{1}$

For Problem 16-20, answer the word problem.
16. What other value of $k$ will also work for the combination ${ }_{6} \mathrm{C}_{2}={ }_{6} \mathrm{C}_{\mathrm{k}}$ ?
17. What other value of $k$ will also work for the combination ${ }_{5} \mathrm{C}_{3}={ }_{5} \mathrm{C}_{\mathrm{k}}$ ?
18. What pattern do you observe? Using the pattern, complete the following formula.

$$
{ }_{n} C_{r}={ }_{n} C
$$

For Problem 19 and 20, use the information below to solve the problem.

A deck of cards has 52 cards: 13 hearts, 13 diamonds, 13 spades, and 13 clubs.
19. How many different ways can a card of hearts be chosen from a deck of 52 cards? (Use a calculator to solve the problem.)
20. How many different ways can four cards be chosen that are all the same suit? (Use a calculator to solve the problem.)

Section 2.7 Normal Distributions
Practice Problems 2.7
For Problem 1 and 2, find the numbers in the given row of Pascal's Triangle.

1. The Seventh Row
2. The Eighth Row

For Problem 3 and 4, write the binomial expansion given the Binomial Theorem $\binom{n}{r}=\frac{n!}{(n-r)!r!}$ to find the binomial coefficients, or use the numbers from the rows of Pascal's Triangle from Problem 1 and 2.
3. $(a+b)^{7}$
4. $(x+y)^{8}$

For Problem 5 and 6, find the binomial coefficient. Use the Binomial Theorem or Pascal's Triangle.
5. $\quad\binom{7}{3}$
6. $\quad\binom{7}{4}$

For Problem 7-16, solve the word problem.
7. If the probability of success for a binomial experiment is $p=0.73$, what is the probability of failure?
8. If the probability of failure for a binomial experiment is $\mathrm{p}=0.21$, what is the probability of success?
9. If the probability of an event is $\mathrm{p}=0.05$ and $\mathrm{n}=4$, what is the probability of the event occurring twice? Let the event be making a free-throw shot.
10. What is the probability of rolling a 3 with one toss of a die?
11. What is the probability of rolling a 3 on a die twice in a row with two tosses of a die?
13. What is the probability of rolling a 3 with 6 tosses of a die?
15. What is the probability that you will flip 5 heads with 10 tosses of a coin?
16. Of people who buy life insurance, $65 \%$ are men. If 11 insurance owners are randomly contracted, what is the probability that 8 of the 11 will be men?

For Problem 17-20, fill in the blanks with a word or variable for the four conditions of a binomial distribution.
17. The number of observations $\qquad$ is fixed.
18. Each observation is $\qquad$ of the other observations.
19. There are only two outcomes, success or $\qquad$ _.
20. The probability of each success $\qquad$ stays the same.

## Section 2.8 Mean or Expected Value of a Binomial Experiment Practice Problems 2.8

For Problem 1-5, use the information below to solve the problem.
Ethan shoots free-throws with a $70 \%$ average.

| Shots Made ( $\boldsymbol{x})$ | Probability of Shots Made $(\boldsymbol{p}(\boldsymbol{x})$ ) |
| :---: | :---: |
| 5 |  |
| 4 |  |
| 3 |  |
| 2 |  |
| 1 |  |
| 0 |  |

1. Complete the table for $p(x)$.
2. Draw a graph of the histogram for the table.
3. Does the histogram resemble a normal distribution?
4. Find the mean of Ethan's next five free-throw attempts.
5. To find the expected value of makes when Ethan attempts five free-throws, we would take $5 \cdot 0.70$. What is the value of $5 \cdot 0.70$ and why would you expect it?

For Problem 6-10, solve each problem for a family that has three children.
6. A family has three children. The sample space consists of eight outcomes. List the outcomes in the sample space. Let $B$ represent boy and $G$ represent girl.
7. Find the probability for each value of the random variable $x$. Let $x_{i}$ be the number of boys. Use the list of outcomes from Problem 6 to solve the problem and fill in the table.

| $\boldsymbol{x}_{\boldsymbol{i}}$ | $\boldsymbol{p}\left(\boldsymbol{x}_{\boldsymbol{i}}\right)$ |
| :---: | :---: |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |

8. Draw a graph of the histogram for the probability distribution in Problem 7.
9. Find the mean (or expected value) of the distribution.
10. Are the births of boys and girls equally likely?

For Problem 11-14, tell whether the statement is true or false.
11. A random variable is determined at the outset of an experiment.
12. A probability distribution is a function that maps the value of a random variable onto its probability.
13. There are twelve possible outcomes for a family of four children (when the order of birth is important).
14. The mean of a probability distribution is sometimes a value of the random variable of an experiment.

For Problem 15-20, use the experiment below to solve the problem.
Roll a pair of dice 20 times and add the sums together.
15. Complete the table below for the experiment.

| $\boldsymbol{x}$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ |  |  |  |  |  |  |  |  |  |  |  |

16. What is the sum of all the probabilities?
17. Graph the probability distribution.
18. Does this distribution look like the probability distribution for the sum of two dice?
19. Find the mean of the frequencies for the experiment.
20. Is the mean near 7? Why would you expect this?

## Section 2.9 Variance and Standard Deviation of a Binomial Distribution <br> Practice Problems 2.9

For Problem 1 and 2, answer the multiple-choice problem. There may be more than one answer.

1. Which of the following are measures of center?
a) range
b) median
c) mean
d) variance
e) standard deviation
f) interquartile range
2. Which of the following are measures of spread?
a) range
b) median
c) mean
d) variance
e) standard deviation
f) interquartile range

For Problem 3-7, answer the word problem.
3. If you know how far someone drives to work, would variance be in km . or square km .?
4. If you know how far someone drives to work, would standard deviation be in km . or square km .?
5. If two samples have the same mean, $\bar{x}_{1}=\bar{x}_{2}$, but different standard deviations, $s_{1}>s_{2}$, which sample will show more variability: $s_{1}$ or $s_{2}$ ?
6. If the heights of a group of students have a variance of 5.6 square inches, what is the standard deviation?
7. If the heights of a group of students have a standard deviation of 2.8 inches, what is the variance?

For Problem 8-11, use the information below to solve the problem.
Carlos computed the sum of the deviations for a set of data:

$$
\sum_{i=1}^{12}\left(x_{i}-\bar{x}\right)^{2}=720
$$

8. How many elements are in the set of data?
9. What is the sample variance of the set of data?
10. What is the standard deviation for the set of data?
11. Is it possible to find the mean of the set of data with the given information?

For Problem 12, solve the multiple-choice problem.
12. Which of the following is equal to $\frac{\sum_{i=1}^{n} x_{i}}{n}$ ?
a) $\frac{\bar{x}}{n}$
b) $\frac{n}{\bar{x}}$
c) $\bar{x}$
d) $\quad\left(x_{i}-\bar{x}\right)^{2}$

For Problem 13-15, answer the word problem.
13. Find the mean $\mu$ for a binomial distribution representing an experiment where $n=12$ and $p=0.35$.
14. Find the variance of $\sigma^{2}$ for a binomial distribution representing an experiment where $n=10$ and $p=0.4$.
15. Find the standard deviation $\sigma$ for a binomial distribution representing an experiment where $n=13$ and $p=$ 0.75 .

For Problem 16 and 17, use the information below to solve the problem.
A church compares the amount of time its members spend volunteering outside of the church each week to the amount of time its staff spends volunteering outside of the church each week.
16. Is the population of the church its members or its staff?
17. What is the sample: the members or the staff?

For Problem 18-20, solve the world problem.
18. Given the probability $p$, is the mean of a binomial distribution directly proportional to the number of trials $n$ ?
19. Given the probability $p$, is the variance of the binomial distribution directly proportional to the number of trials $n$ ?
20. Given the probability $p$, is the standard deviation of the binomial distribution directly proportional to the number of trials $n$ ?

## Section 2.10 Standard Normal Distributions and z-scores <br> Practice Problems 2.10

For Problem 1-4, if $x$ is a continuous random variable and $p(x)$ is the probability that $x$ occurs, use the diagram to choose the probability it represents.

1. $p(0<x<a)$ or $p(x>a)$

2. 

$p(c>x>a)$ or $p(a<x<b)$

2. $p(0<x<b)$ or $p(a<x<b)$

4. $p(x>a)$ or $p(a<x<b)$


For Problem 5 and 6, tell whether the statement is true or false.
5. As the number of trials $(n)$ increases, the graph of a binomial probability distribution with a fixed probability spreads out.
6. As the number of trials $(n)$ increases, the graph of a binomial probability distribution approaches a normal distribution.

For Problem 7-9, use the graph below to solve the problem.

For Problem 7-9, use the graph below to solve the problem.

7. What is $p(z<1.3)$ ?
8. What is $p(0<z<1.3)$ ?

What is $p(z>1.3)$ ?

For Problem 10-20, solve the word problem. Use $z=\frac{x-m}{s}$ for Problem 10-11.
10. When $m=0.6$ and $s=0.03$, find the standard $z$-score for $x=0.495$ where $x$ is normally distributed.
11. When $m=0.5$ and $s=0.01$, find the standard $z$-score for $x=0.535$ where $x$ is normally distributed.
12. Is the statistical measure of $m$ (the axis of symmetry) the mean or standard deviation?
13. Is the statistical measure of $s$ (the measure of spread) the mean or standard deviation?
14. What is the horizontal asymptote of the normal curve?
15. What is the horizontal asymptote of the standard normal curve?
16. Sketch the graph of the normal distribution of $x$ where $m=4$ and $s=2$.
17. If the height of adult females in the US is normally distributed at approximately 66 inches, what is the median?
18. Given the equation $f(x)=\frac{1}{3 \sqrt{2 \pi}} e^{-\frac{1}{2}\left(\frac{x-6}{3}\right)^{2}}$, what is $m$ (the axis of symmetry) and what is $s$ (the spread)?
19. Graph the equation from Problem 18 on a graphing calculator. What is the area under the curve?
20. For normal distributions, almost all data falls within three standard deviations of the mean. To find the standard deviation for a normally distributed variable, what must the range be divided by in the approximate equation shown below?

$$
s \approx \frac{\text { range }}{\square}
$$

Section 2.11 Null and Alternative Hypothesis Testing
Practice Problems 2.11
For Problem 1-10, use the information below to solve the problem.

A business has 3 male salespersons and 4 female salespersons. At the end of the year, 3 people will become managers and only 1 will be a female.
Are these promotions discriminatory against women?

1. Null hypothesis $\left(H_{0}\right)$ : the probability of becoming manager is equal for each individual State the alternative hypothesis $\left(H_{1}\right)$.
2. What is the total number of combinations for choosing 3 managers from a total of 7 people? (Hint: ${ }_{7} \mathrm{C}_{3}$ )
3. What is the total number of combinations for choosing 2 out of 3 males to be managers?
4. What is the total number of combinations for choosing 1 out of 4 females to be manager?
5. What is the probability of choosing 2 males and 1 female out of 7 total employees to be managers? (Hint: $p$ ( 2 males, 1 female): this is the number of combinations of 2 out of 3 males and the number of combinations of 1 out of 4 females. Remember, "and" means multiply. Then divide by the total number of combinations of 3 out of 7 people.)
6. Why are we looking for the probability of no more than 1 female becoming manager?
7. What is the probability that all 3 managers chosen are males and 0 are females? ( $p$ ( 3 males, 0 females))
8. What is the probability that no more than 1 female out of 7 employees is chosen to be manager?
9. If the significance level is 0.10 , does the evidence contradict the null hypothesis?
10. If the significance level is 0.01 , does the evidence support the alternative hypothesis?

Section 2.12 Central Limit Theorem and Confidence Intervals<br>Practice Problems 2.12<br>For Problem 1-6, fill in the blank.

1. Using the normal distribution curve, $\qquad$ percent of a sample will fall 3 standard deviations above the mean.
2. The percent from Problem 1 is also $\qquad$ deviations from the mean in a normal distribution.
3. $\quad$ The Central Limit Theorem says that when $n>$ $\qquad$ the mean of the sample population approximates a normal distribution, even if the population is not normal.
4. The means of the Central Limit Theorem are calculated from a uniform distribution, but the means themselves are $\qquad$ distributed.
5. The Central Limit Theorem also states that the means from a sample population will approximate the means of a population regardless of the $\qquad$ of the population.
6. Most population data falls within $\qquad$ standard deviations of the mean in a normal distribution.

For Problem 7-10, use the information below to solve the problem.
In a sample size of 275 teachers, the mean of their salaries was $\$ 51,500$ with a standard deviation of $\$ 13,200$.
7. What is the mean salary for the population calculated with a $90 \%$ confidence interval?
8. What is the mean salary for the population calculated with a $95 \%$ confidence interval?
9. What is the mean salary for the population with a $99 \%$ confidence interval?
10. If the average salary for a teacher is 2 standard deviations above the mean, what is the salary? What percentage of the population makes this salary?

## Section 2.13 Probability and Two-Way Tables

## Practice Problems 2.13

For Problem 1-4, use the information below to solve the problem.

In Example 3 from the Lesson Notes, you found the marginal relative frequency of each row to find the conditional relative frequencies for the 120 community college students surveyed. These students were asked if they played a sport or had a STEM major.

1. Complete the table using the marginal relative frequency of each column to find the conditional relative frequency.

|  | Have a STEM Major | Do not Have a STEM Major |
| :---: | :---: | :---: |
| Play a Sport |  |  |
| Do not Play a Sport |  |  |

2. Given the student has a STEM major, what is the probability he/she does play a sport?
3. Given the student has a STEM major, what is the probability he/she does not play a sport?
4. What does 0.625 represent in the table?

For Problem 5, use the information below to solve the problem.

The table below displays a tally of the test predictions and results for the new virus. For each row, a positive prediction is true, and a negative prediction is false. For each column, positive ( + ) means the subject has the virus and negative $(-)$ means the subject does not have the virus.

|  | Virus (Yes) Actual Positives | $\begin{gathered} \hline \text { Virus (No) } \\ \text { Actual Negatives } \\ \hline \end{gathered}$ |
| :---: | :---: | :---: |
| Test: True (Positive Predictions) | Aff II | dit |
| Test: False (Negative Predictions) | dHT III | $111$ |

5. Complete the two-way table using numerical frequencies.

|  | Virus + | Virus - | Total |
| :---: | :---: | :---: | :---: |
| Test (+) |  |  |  |
| Test (-) |  |  |  |
| Total |  |  |  |

For Problem 6-10, use the information below to solve the problem.

Dr. Novick and Dr. Fedrizzi are collaborating on research for a test regarding a new strain of virus. The two-way table below demonstrates that if the test predicts the virus and the patient does not have the virus, it is a false positive. A healthy person will test negative $(-)$ for the virus whereas a person with the virus (an unhealthy person) will test positive (+).
6. Find the joint relative frequencies and the marginal relative frequencies for the table.

|  | Virus + | Virus - | Total |
| :---: | :---: | :---: | :---: |
| Test + |  |  |  |
| Test - |  |  |  |
| Total |  |  |  |


|  | Actual <br> Positives | Actual <br> Negatives |  |
| :---: | :---: | :---: | :---: |
| Positive <br> Predictions | True Positive <br> (TP) | False Positive <br> (FP) | TP and FP <br> Total Positive <br> Predictions |
| Negative <br> Predictions | False Negative <br> (FN) | True Negative <br> (TN) | FN and TN <br> Total Negative <br> Predictions |
| Total | TP and FN <br> Total with the <br> Virus | FP and TN <br> Total without <br> the Virus |  |

7. What is the probability that an unhealthy person is correctly identified as an unhealthy person?
8. What is the probability that a healthy person is incorrectly identified as an unhealthy person?
9. What is the probability that an unhealthy person is incorrectly identified as a healthy person given the person is unhealthy?
10. What is the probability that a healthy person is correctly identified as a healthy person given the person is healthy?

## Section 2.14 Module Review

For Problem 1-3, use the information below to solve the problem.

The distance in miles that ten employees drive to get to work is shown below:

$$
7,12,15,8,8,10,13,14,15,9
$$

1. What is the mean of the set of data?
2. Find the variance of the set of data.
3. Find the standard deviation of the set of data.

For Problem 4-9, solve the word problem.
4. Using the expression for the Fundamental Counting Principle, $n$ !, find how many ways are there to arrange four books on a shelf?
5. There are 12 books on a shelf. How many ways are there to arrange the 12 books if only 4 go on the shelf at a time? Use the permutation equation ${ }_{\mathrm{n}} P_{r}=\frac{n!}{(n-r)!}$.
6. Suppose you order an entrée at a restaurant. Two side dishes come with an entrée. There are 10 side dishes to choose from. Use the equation ${ }_{\mathrm{n}} C_{r}=\frac{n!}{(n-r)!r!}$ to figure out how many combinations of side dishes there are.
7. According to a survey, $45 \%$ of 16-19-year-olds have a What's Up app on their cellphone. Suppose you randomly ask five people if they have the What's Up app. If a "yes" is a success, draw a probability distribution for the binomial experiment. Use the equation $p(k$ successes $)={ }_{\mathrm{n}} C_{k} p^{k}(1-p)^{n-k}$.
8. What is the probability of drawing a vowel from a bowl full of each letter of the alphabet?
9. A family has three children and all of them are girls. Therefore, they believe their fourth child will be a girl. Is this highly likely?

For Problem 10-13, match the term to the correct description.
10. Normal Distribution
a. $\quad{ }_{\mathrm{n}} C_{k} p^{k}(1-p)^{n-k}$
11. Binomial Distribution
b. Mean of a Population
12. $\mu$
c. Bell-shaped Curve
13. Standard Deviation
d. Square Root of the Variance

For Problem 14-16, tell whether the statement is true or false.
14. A binomial probability distribution with a bell-shaped curve is called a normal distribution.
15. For $n>5$, the mean of the sample approaches the mean of the population.
16. When $p=0.5$, as $n$ increases, the graph flattens out more and more.

For Problem 17-20, match the term to the correct equation.
17. Standard Deviation a. $f(z)=\frac{1}{\sqrt{2 \pi}} e^{-\frac{z^{2}}{2}}$
18. Standard Normal Curve
b. $\quad \mu=n p$
19. Standard Score of a Normal Distribution
c. $\quad z=\frac{x-m}{s}$ or $z=\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$
20. Mean Score of a Binomial Distribution
d. $\quad \sigma=\sqrt{n p q}$

## Section 2.15 Module Test

For Problem 1-3, use the information below to solve the problem.

A set of test scores for ten students is shown below:
$77,68,82,83,82,84,88,92,89,76$

1. What is the mean of the set of data?
2. Find the variance of the set of data.
3. Find the standard deviation of the set of data.

For Problem 4-9, solve the word problem.
4. Using the expression for the Fundamental Counting Principle, $n!$, find how many ways there are to arrange 3 flags on a flagpole?
5. There are 8 flags on a cruise ship, but only 3 of them can hang on a flagpole at any one time. How many ways are there to arrange 3 of the 8 flags at a time using the equation ${ }_{\mathrm{n}} P_{r}=\frac{n!}{(n-r)!}$.
6. Of a total of 16 campus photos, 2 will be chosen for the cover of the campus magazine. How many ways can the 2 photos be chosen for the cover? Use the equation ${ }_{\mathrm{n}} C_{r}=\frac{n!}{(n-r)!r!}$ to figure out how many combinations of photos there are for the cover of the campus magazine.
7. According to a survey, $32 \%$ of all middle school students have personal computers. Suppose you randomly ask six people whether or not they have a personal computer. Draw a probability distribution for the binomial experiment.
8. What is the probability of drawing a consonant out of a bowl full of each letter of the alphabet?
9. A married couple has 5 boys. They assume their next child will be a boy. Is it more probable their next child will be a boy rather than a girl?

For Problem 10-13, match the term with the correct description.
10. $\bar{x} \pm m$
11. $H_{0}$
12. $m=z_{c} \frac{\sigma}{\sqrt{n}}$
13. $H_{a}$
a. Margin of Error
b. Alternative Hypothesis
c. Null Hypothesis
d. Confidence Interval

For Problem 14-16, tell whether the statement is true or false according to the Central Limit Theorem for sample size $n$ selected from a population with mean $\mu$ and standard deviation $\sigma$.
14. As $n$ increases, the distribution approaches a normal curve.
15. The mean of the distribution of the sample, $\mu_{\bar{x}}$, deviates from $\mu$.
16. The standard deviation of the distribution of the sample mean approaches $\frac{\sigma}{\sqrt{n}}$.

For Problem 17-20, match each definition with the correct formula.
17. Coefficient of Correlation
a. $\quad \sigma^{2}=n p q$
18. Coefficient of Determination
b. $\quad \sqrt{r^{2}}$
19. Variance of a binomial distribution
c. $\quad \frac{S S_{r e s}-S s_{\text {dev }}}{S S_{\text {dev }}}$
20. $\quad p(k$ successes $)$ in a binomial experiment
d. $\quad{ }_{\mathrm{n}} C_{k} p^{k}(1-p)^{n-k}$

