Explaining Debt Covenant Amendments: a Structural Approach*

Irina Luneva†

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Abstract

Financial covenants, which use financial information to determine control rights in corporate loan contracts, are frequently amended (Roberts (2015)). In this paper, I offer a model of debt covenant amendments and structurally estimate it on novel data on amendments of financial covenants in U.S. companies' loan contracts. The results suggest that covenants are amended, first, because contracting parties face a high degree of contractual incompleteness, and second, because the cost of amendments is considerably smaller than the cost of misallocating control rights.

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†The Wharton School, University of Pennsylvania. E-mail: iluneva@wharton.upenn.edu

Introduction

Corporate loan contracts are frequently amended. The most common outcome of these amendments is a long-term change of financial covenants – the state-contingent allocation of control rights between the lender(s) and the borrower.[1] Despite the documented commonality of contract and covenant amendments (Roberts and Sufi (2009), Denis and Wang (2014), Roberts (2015)), we have a limited understanding of which economic forces prompt contracting parties to amend how they allocate control rights. In this study, I offer a theory to explain debt covenant amendments, apply this theory to the data, and measure key economic parameters describing amendment behavior.

It is important to note the difference between amendments – long-term changes in control rights – and waivers – short-term deviations from a control rights allocation prescribed by a contract.[2] A waiver can be an intertemporal alternative to an amendment in the sense that instead of changing the contract long-term, the parties can leave a suboptimal contract in place, wait until it misallocates control rights – a covenant gets violated when it should not be, – and then waive the misallocation. While waivers have been heavily studied (see, among others, Rajan and Winton (1995), Gârleanu and Zwiebel (2008), Guttman and Marinovic (2018), Lu et al. (2018), Gao and Liang (2019)), amendments remain not well understood even though quite prominent in the data.[3] Prior empirical research (e.g., Roberts and Sufi (2009), Denis and Wang (2014), Roberts (2015), Nikolaev (2018), Dyreng et al. (2023)) treats waivers and amendments monolithically as a broader category – renegotiations. However, amendments are different because they are independent of covenant violations. The forward-looking nature of amendments and their low correlation with borrowers' financial health[4] suggest that amendments may have different economic drivers than waivers and should be studied separately.

I propose a model of debt contract amendments and evaluate whether its economic forces can explain the amendment behavior of U.S. firms. I build a multiperiod debt contracting model where the bank and the firm write a contract on a contractible signal and can amend financial covenants when they receive new information about a non-contractible state of the world. Amending financial covenants improves contracting efficiency by allowing contracting parties to use the most recent non-contractible information and (partially) complete the inherently incomplete debt contract. The model offers three non-mutually-exclusive explanations for why lenders and borrowers amend covenants: (1) the degree of contractual incompleteness is high, (2) the current non-contractible state of the world

[1] In a random sample of US public companies, about 29% of loans get a financial covenant amended at least once before maturity.

[2] For example, an amendment changes the debt-to-EBITDA ratio that the borrower has to satisfy multiple years going forward until the contract's maturity date. A waiver either waives a violation of the debt-to-EBITDA ratio covenant or increases the threshold for a short period of time, such as a month.

[3] In my sample, about 92.6% of covenant renegotiations are long-term – amendments.

[4] In my sample, the correlation of amendments with covenant violations is 0.241 and with borrowers' debt-to-EBITDA ratio -0.003.

is highly predictive of the future non-contractible state, and (3) the cost of amending financial covenants is significantly less than the cost of misallocating control rights. To quantify the economic importance of these channels in explaining the amendment behavior of U.S. firms, I hand-collect data on financial covenant changes for a sample of American companies and structurally estimate the model using this data. The estimates mostly support the first and the third explanations. In the U.S., financial covenants are amended because contracting parties face a high level of contractual incompleteness, and the cost of misallocating control rights is considerably higher than the amendment cost.

I choose the structural estimation approach for two reasons. First, multiple interactions between lenders and borrowers and the information that they receive are unobservable to researchers: the data on meetings or phone calls is hardly available, and a lot of information is exchanged through in-person interactions during banks' monitoring process. The structural method can use observable statistics, such as frequency and magnitudes of amendments, to back out the characteristics of unobserved information received by the contracting parties. Second, I am interested in the economic and informational characteristics that explain current amendment behavior in the US in static. A reduced-form approach can speak to how a change in an economic parameter, e.g., the degree of contractual incompleteness, changes amendment behavior. The structural estimation, in contrast, can measure the economic parameter itself and then conclude whether its value is consistent with the amendment behavior that we observe. In turn, the magnitudes of economic parameters can be used to measure contracting parties' welfare and make implications relevant for policymakers.

The debt contracting model features a non-contractible state and a contractible signal which is imperfectly correlated with the state, and a trade-off between amending the contract ex ante and misallocating control rights ex post. The non-contractible state represents which party should be in control of the firm – the manager or the bank. The contractible signal is imperfectly correlated with the state and sometimes does not coincide with it, allocating control rights to the "wrong" party. If control rights are misallocated, the contracting parties can (1) leave the wrong party in control and destroy a part of the total surplus or (2) bear a deadweight loss and waive the misallocation. In an attempt to reduce the cost of misallocating control rights, the parties can amend their contract in the preceding period, incorporating the non-contractible information learned at the time. The parties thus trade off the cost of amending the contract in advance with the reduction in the expected cost of control rights misallocation in the future.

I model the contract in a general way. Rather than choosing a particular metric(s) and a threshold(s), the parties choose the information system – the probability that the covenant makes an error in each state of the world. The parties choose the contract to minimize the total expected costs of amending and misallocating control rights. When the cost of contract amendment is less than the expected reduction in future control misallocation cost, the parties choose to amend the contract. Amendments thus naturally occur in the multiperiod setting and improve contracting efficiency by saving control rights misallocation costs.

In equilibrium, the contracting parties amend their contract whenever the contract they designed originally is considerably distant from the optimal contract suggested by the parties' updated beliefs given the new non-contractible state. Thus, the decision to amend is not a function of the borrower's initial financial health or its propensity to violate a covenant. Amendments happen when the parties receive considerably new information that they could not account for when writing the original contract.

Comparative statics suggest three non-mutually-exclusive economic explanations for why financial covenants are amended. The first is the high degree of contractual incompleteness. Contractual incompleteness is the extent to which a signal(s) used in a contract **can not** capture the actual state of the world, which is not contractible. When the contractible signal is imprecise, covenants make overall more errors, and the parties gain more from the amendment option. For example, a company whose interest coverage ratio more poorly represents the company's actual ability to pay interest would more likely have its interest coverage ratio covenant amended. The second explanation is the persistence of the non-contractible state of the world: as the current state becomes more predictive of the future state, the amended contract is likely more suitable for the future, increasing the parties' motivation to amend. Intuitively, if a firm is more certain about whether it will stay financially healthy based on today's state, the firm will more probably amend its contract. The last reason for amendments is low amendment cost compared to the cost of misallocating control rights. In an environment where lenders and borrowers face smaller frictions when changing the contract in advance compared to losses once a wrong party is in control covenants are amended more often.

To quantify the economic importance of the three reasons explaining financial covenant amendments, I first gather detailed data on original and amended financial covenant schedules. The most commonly used loan contract database, Thomson Reuters Dealscan, does not record amendments to financial covenants and original covenants in sufficient detail. For original covenant schedules, the database only contains levels and directions for dynamic covenants, without specifying how the covenant levels change every year.[5] As for amendments, even though the instances are recorded, whether and how financial covenants were amended is missing. I manually collect data on dynamic covenant schedules and amended financial covenants for a random sample of about one hundred U.S. firms from the loan contracts filed with the SEC.

[5]For example, Dealscan would record the same way a covenant that provides a borrower a few months grace period and a covenant that evenly becomes stricter every year. The first covenant represents a short-term adjustment, potentially a waiver, while the second regulates an agency problem that is expected to worsen as time passes.

Next, I structurally estimate the model using the data on financial covenant amendments. The algorithm minimizes the distance between theoretical moments – functions of parameters – and their data counterparts: initial and amended covenant strictness and probabilities of an amendment unconditional and conditional on no covenant violation. The estimated parameters help me evaluate the explanations for debt covenant amendments. The estimated degree of contractual incompleteness is high: if a covenant metric always represents the state correctly in one state, it will be wrong in another state about 31.3% of the time. Thus, banks and firms often change covenants because the information they can contract on is very imprecise. The second explanation – the persistence of non-contractible information – finds limited support in the data. If a company does not (does) need lender's intervention this year, it likely will not (will) need it next year with only a 65.4% (50.1%) chance. Finally, the cost of amending the contract is about 1.9% of the average cost of misallocating control rights, supporting the third explanation. To summarize, the estimates suggest that U.S. companies amend financial covenants in their loan contracts for two reasons. First, they face a high degree of contractual incompleteness, and second, adjusting contracts in advance is considerably cheaper than giving control to a wrong party later by mistake.

I also evaluate contracting efficiency gain from the amendment option for the firms in my sample. Ex ante, the option to amend financial covenants saves the contracting parties about 2.32% of control misallocation cost. Ex post, conditional on an amendment, the parties save 15.32% of cost in the state when the manager is the best party-in-control and 12.62% in the state when the bank needs to be in control. These numbers are economically meaningful. Control misallocation cost captures the cost of having in control the party that can make a suboptimal decision. For instance, a manager can take excessive risks,[6] or a bank can forgo positive-NPV investment opportunities and reduce the company's ultimate value.[7]

One of the benefits of structural estimation is the ability to predict economic agents' behavior in different counterfactual scenarios. I investigate how changes to the main economic parameters of the model – the quality of contractible information, the importance of non-contractible information, and renegotiation cost – would alter amendment behavior. I begin by analyzing how sensitive the frequency of amendments and contractual efficiency gain from amendments are to different model parameters. The most influential parameter is the persistence of the non-contractible

[6]For example, Malmendier and Tate (2005) demonstrate that companies with overconfident executive managers exhibit (likely excessive) high sensitivity of their investments to cash flows. When a CEO is overconfident, the company's investment increases by about 23.4% of assets more in response to a rise in cash flows. Because overconfident CEOs overestimate returns to some investment projects, firm value can be destroyed significantly.

[7]Chava and Roberts (2008) find that a covenant violation leads to about 13% decline in capital expenditures for the violating firm, suggesting considerable loss in company value if this violation is by mistake. Dyreng et al. (2022) compute that companies violating financial covenants experience between 6% and 11% lower abnormal stock returns compared to their non-violating counterparts. The numbers indicate that stockholders perceive the cost of violation, including both real and reputation costs, to be quite high.

state of the world. In a more stable economy where a current good (bad) non-contractible state is 10% more predictive of the future non-contractible state, the contracting parties would gain 91.3% (66.5%) more contracting efficiency because of the amendment option and amend their contracts 13.6% (22.1%) more. The next factor that alters amendment behavior a lot is the degree of contractual incompleteness. If metrics used in loan contracts were 10% more accurate at representing the true state of the world, the gain from amendments would be 49.3% less and we would observe amendments by 29.4% less often. The third factor by its marginal effect is the amendment cost. A regulation that makes it 10% harder for contracting parties to change their contracts will reduce contracting efficiency gains from amendments by about 19.6%.

Next, I make more substantial changes to the model and study how two aspects of contract renegotiations – borrower's financial health and the possibility of a waiver – affect amendment behavior. The first and second scenarios look at completely financially healthy and completely financially unhealthy companies, respectively.[8] Perhaps counter-intuitively, as time goes on, the firms are similarly likely to amend their financial covenants. The reason is that for amendments, it matters how far the existing contract is from the optimal one suggested by the new information. Financially healthy and unhealthy companies set the most suitable covenants in the beginning, but then amendments are determined by new unexpected information about the firms. The results are consistent with covenant amendments not being correlated with borrowers' financial health and some findings by Roberts (2015): the author documents that neither firms' EBITDA-to-assets ratio nor their stock returns predict that a covenant will be amended.

The third counterfactual scenario introduces an additional waiver opportunity. In reality, waivers most plausibly happen (and can be observed by a researcher) only when a covenant is violated but the company management stays in control. In terms of the model, this means control rights are misallocated such that the bank gets control by mistake, and the parties waive the misallocation and return control to the manager. Theoretically, the opposite case is also possible: when the control rights are allocated to the manager by mistake but should be given to the bank. This case is hard to observe in the data because it would imply the lender intervenes in a firm without any formal technical default. For that reason, in the main estimation, I prohibit waivers of this type. Nevertheless, to look at how amendment behavior would look if we believe the parties can waive control rights misallocation in the case when the bank is the best party-in-control, I consider a counterfactual scenario where waivers are allowed in both control misallocation cases. I find that when the parties can resolve control rights misallocation in both states of the world, the amendment option becomes less valuable: the ex ante gain from amendments drops by 1 percentage point, and

[8] In the counterfactual scenario for a completely financially healthy (unhealthy) company, the probability that the company will not (will) need the lender's intervention in the first time period is set to 0.999.

conditional on an amendment in the state when bank should have control, by 4 percentage points.

The final part of the paper studies how corporations' and loans' characteristics are associated with amendment behavior and contractual efficiency gains from amendments. I consider three characteristics: financial accounting quality, relationship lending, and lender coordination cost. The results suggest a substantial gain from amendments for firms with less informative earnings numbers and in close relationships with their lenders. As for coordination costs, when it becomes harder for lenders to renegotiate, higher waiver friction leads to more likely amendments for larger syndicates.

My study aims to bridge the gap between empirical observations and theoretical literature on debt contracting. I develop a general framework that captures key economic conflicts present in real-world debt contracting. First, agency problems are described in reduced form as a loss of total surplus from the company-bank relationship. These parameters can conveniently capture multiple issues described by prior theories (e.g., Aghion and Bolton (1992), Rajan and Winton (1995), Sridhar and Magee (1996), Gârleanu and Zwiebel (2008), Guttman and Marinovic (2018), Gao and Liang (2019), Laux (2020)): asset substitution, debt overhang, bargaining power, etc., in a single theoretical construct. Second, financial covenants in my model are not specific thresholds for specific metrics (such as in Guttman and Marinovic (2018) or Gao and Liang (2019)) but a broader construct – type-1 and type-2 errors. Finally, because my model is more tractable, it can be brought into a dynamic setting to study questions, such as amendment behavior, that may be harder to answer with classic frameworks.

I contribute to the empirical literature on debt contracting and renegotiations (e.g., Beatty et al. (2008), Bharath et al. (2009), Christensen and Nikolaev (2012), Choi and Triantis (2012), Hong et al. (2016), Li et al. (2016), Saavedra (2018), Armstrong et al. (2020), Christensen et al. (2022), Wang and Wang (2023)) by clearly distinguishing two types of renegotiations: short-term deviations – waivers – and long-term changes – amendments, and demonstrate that these types may have different economic drivers, suggesting caution to empirical research treating them as the same (e.g., Roberts (2015), Nikolaev (2018), Dyreng et al. (2023)). In particular, waivers happen when control rights are allocated inefficiently in a current period, while amendments represent a revision of beliefs about the borrower or the lender and do not necessarily occur when control rights are misallocated currently. In addition, amendments are not associated with companies' financial health at loan initiation but depend on how strongly the state of the world changes over time.

Finally, my study makes a first attempt to quantify important economic constructs in the debt contracting setting. The estimate of contractible signal's precision is Aghion and Bolton (1992)'s contractual incompleteness; the costs of misallocating control rights and frictions preventing renegotiations measure total surplus losses due to various frictions in the economy (Coase (1937)).

The rest of the paper is organized as follows. Section 1 presents the debt covenant amendments model and discusses the equilibrium. In section 2, I describe the data collection process and main properties of my dataset. Section 3 shows the estimated parameters, analyses model fit, and provides estimates of contracting efficiency gains from amendments. Section 4 discusses counterfactual scenarios, and section 5 shows parameter estimates for different subsamples of the data. Section 6 concludes.

1 Model of debt covenant amendments

In this section, I present a model of debt covenant amendments and discuss its departure from existing debt contracting literature. Next, I discuss the equilibrium, welfare implications, and comparative statics that imply testable empirical predictions. In the final part, I present theoretical moments used to identify model parameters.

1.1 Model setup and its difference from prior models

I study a classic debt contracting problem. A company, governed by a manager, has a positive NPV investment opportunity and has decided to borrow funds from a bank. The bank, operating in a perfectly competitive credit market, has money and can provide funding. However, the bank is worried that in some states of the world, its money can be diluted: because the manager obtains profit when the project turns out successful but the bank bears the cost if the project fails, the manager has incentives to choose actions that are too risky, destroying the value of the firm and the bank's claim. The bank addresses this issue by signing a contract that specifies when the manager stays in control of the firm and when control rights are transferred to the bank.

However, the contracting parties face the problem of non-contractibility (Aghion and Bolton (1992)). Even though the state of the world is observed by the bank and the manager, it can not be contracted upon in the debt contract either because it is impossible to describe the state in sufficient detail or the state is not verifiable (Christensen et al. (2016)). Instead, the contract must be written on an imperfect signal about the state of the world. For example, a company's financial health is a broad concept that partially relies on difficult-to-verify soft information (e.g. growth opportunities) and can not be described in a contract, but the company's debt-to-EBITDA ratio is a hard metric that is correlated with financial health and can be contracted upon.

In the classic framework (e.g. Rajan and Winton (1995), Sridhar and Magee (1996), Gârleanu and Zwiebel (2008), Guttman and Marinovic (2018), Lu et al. (2018), Gao and Liang (2019)), outlined in figure 1a, the bank-company relationship proceeds as follows. The contract, specifying control rights as a function of the contractible signal, is signed and the company makes the investment. In the next period, the bank and the company's manager observe the state of the world and the signal that determines which party has control rights. In cases when control rights are misallocated – or the signal does not agree with the state – the parties can renegotiate, compensate the party that got control inefficiently and return control rights to the optimal party-in-control. The process repeats itself in the next periods until the debt contract matures.

When the classic framework is brought into the world where company-bank relationships last for more than a single period, one important possibility is missing. If the states of the world are somewhat persistent, after the parties have arrived at time $t = 1$, they have more information at hand than they did at time $t = 0$ when they signed the initial contract. Specifically, they know the state at time $t = 1$, which is correlated with the state at time $t = 2$. As a result, the contracting parties adjust their beliefs about the expected optimal party-in-control at time $t = 2$. If the parties had the option to write a new contract, this contract would be closer to the optimal because it would result in a lower expected probability and cost of misallocating control rights at time $t = 2$.

I depart from the classic framework and allow the parties to amend their contract at time $t = 1$. The new timeline is shown in figure 1b. Even though the state of the world can not be contracted upon, the parties can adjust their beliefs after observing it and amend the contract. The possibility to amend the contract as the bank and the manager learn new non-contractible information allows the parties to (partially) dynamically complete the inherently incomplete debt contract.

Consider an example of a change to a loan agreement during the COVID-19 pandemic. Imagine a restaurant that has a bank loan is forced to close its restaurants for dine-in, loses revenue, and thus trips its debt-to-EBITDA ratio covenant. The loan contract prescribes the bank to intervene in the chain's operations, however, both the bank and the chain realize that the restaurant manager is still the most skilled party to manage the restaurant. Because the parties may not always be able to waive this control misallocation due to fundamental frictions (e.g., the bank derives private benefits from having power at the firm), the company value is destroyed when the bank intervenes.

If the parties expect COVID restrictions to stay in place for the foreseeable future, they can also amend the contract. In the extreme, the debt-to-EBITDA ratio covenant can be completely removed. As long as amending the contract results in a greater surplus from the bank-company relationship, the parties can always find an interest rate or a fee schedule to make each party at least indifferent between agreeing to amend and not.

Now imagine that the restaurant has not violated any covenants today. However, the restaurant and the bank expect the closures to continue and believe there is a possibility that the restaurant will trip a covenant. The parties would also likely use the amendment option and change their contract now to avoid the likely cost of misallocating control rights a year from now.

The example demonstrates how the amendment is different from waivers and that it is not correlated with them. The contract can be amended if the parties **expect** potential misallocation of control rights and costs associated with it in the future, based on new non-contractible information received, while the waiver is a way to resolve a **certain** misallocation when it has already occurred and is not related to the parties' beliefs about the future.

Next I provide a detailed description of the model, which has two main building blocks. The first is the perfectly observable but non-contractible state of the world and a non-contractible signal that is imperfectly correlated with the state. The second block is the trade-off between the deadweight loss if the contract is amended now and the cost of potentially misallocating control rights in the future.

The state of the world at time $t \in \{1, 2\}$ is described by a set of two variables: $\{r_t, s_t\}$. $r_t \in \{m, b\}$ is a binary random variable representing which party should have control rights: m implies the optimal party-in-control is the manager, and b implies the optimal party-in-control is the bank. The probability that at time $t = 1$ the state of the world is $r_1 = m$ is μ_0 and that $r_1 = b$ is $(1 - \mu_0)$. Conditional on $r_1 = m$, $r_2 = m$ with probability μ and $r_2 = b$ with probability $(1 - \mu)$; conditional on $r_1 = b$, $r_2 = b$ with probability β and $r_2 = m$ with probability $(1 - \beta)$, where $\mu, \beta > \frac{1}{2}$. I assume $\mu_0 > \frac{1}{2}$ to focus on the cases when the company is more likely than not to be financially healthy and not in need of the lender's intervention. The state of the world is observed by both the manager and the bank but is non-contractible. Even though the state of the world can not be contracted upon, there exists a contractible signal $s_t \in \{m, b\}$ about the optimal party-in-control that is imperfectly correlated with the state r_t.

The parties trade-off the cost of dealing with control rights misallocation if it occurs ex post against the cost of amending their contract in advance to reduce expected control misallocation cost. When control is already misallocated, part of the firm value is destroyed. This loss is captured by the continuous random variable c_t. c_1 is a random draw from a distribution with support $\tilde{c}_1 \sim [0, \bar{c}]$. If $r_1 = r_2$, the cost of misallocating control rights persists: $\tilde{c}_2 = \rho c_1 + (1 - \rho)\tilde{c}$, where $\tilde{c} \sim [0, \bar{c}]$. If $r_1 \neq r_2$, the cost at time $t = 2$ is a new random draw, $\tilde{c}_2 \sim [0, \bar{c}]$. For instance, if the manager runs the company without restrictions during financial instability, she may take excessive risk and reduce the ultimate total value of the firm. The size of this loss is likely related to the loss if the situation continues for another year. But if the next year the situation is the opposite – financial conditions are promising, but the bank steps in and limits the firm's investment – the size of the firm value loss due to forgone investment opportunities is not necessarily comparable to the loss in the case the manager takes excessive risk. Sometimes the parties may be able to waive a control misallocation if it occurs and avoid bearing large losses. However, due to various frictions (Coase (1937)), such as parties' private benefits from being in control or bargaining power, a waiver is not always possible (see, for example, Rajan and Winton (1995)). I introduce variables η_w^m and η_w^b to capture the size of the frictions preventing waivers in the states $r_t = m$ and $r_t = b$, respectively.[9] If the cost c_t is below the waiver friction, c_t has to be borne. If the cost c_t is higher than $\eta_w^i, i \in \{m, b\}$, the value loss that exceeds the parties' private benefits can be avoided, and the total surplus only suffers the loss of η_w^i.

An alternative to suffering a loss if control rights are misallocated is an amendment – an ex ante alteration of the contract. After signing the initial contract at time $t = 0$, the parties have the option to write a new contract at time $t = 1$. An amendment involves a fixed cost η_a, which is a deadweight loss. η_a can capture fees to externally hired lawyers, tax consequences (Dyreng et al. (2023)), coordination cost (Becker and Ivashina (2016)), or time and effort that the manager and the bank spend writing a new contract. η_a can be thought of as friction (Coase (1937)) preventing the parties from always updating the contract in response to new non-contractible information.

I model the contract in a general form. The financial covenants allocate the imperfection of the signal s_t between the two states, m and b. The probability that a covenant chosen at time $\tau < t$ makes a type-1 error, or is violated when should not be, at time t, is $\pi_{mt}^\tau = Pr\left[s_t = b | r_t = m\right]$. The probability that this covenant makes a type-2 error, or is not violated when should be, at time t, is $\pi_{bt}^\tau = Pr\left[s_t = m | r_t = b\right]$. The precision of the covenants in states m and b is $1 - \pi_{mt}^\tau$ and $1 - \pi_{bt}^\tau$, respectively. Note that the subscripts of the probabilites denote the realizations of the state r_t and no of the signal s_t.

The quality of the contractible signal s_t is captured by a parameter $q \in (0.5, 1]$. The possibility frontier for the precision of the contract is described by the equation $(1 - q - \pi_{mt}^\tau)^2 + (1 - q - \pi_{bt}^\tau)^2 = (1 - q)^2$ and illustrated in figure 2. The parties have to trade-off the type-1 error against the type-2 error, and as one error gets closer to zero, the covenant has to sacrifice a lot of precision in another state to further reduce the error. If the covenants never make an error in one state, the maximum possible probability of not making an error in the other state is q.

Below I formally define the contract in the model.

Definition 1. *A contract, chosen at time τ, is the set of probabilities that the signal disagrees with the state at times $t = 1, 2$ in states m and b: $\{\{\pi_{m1}^\tau, \pi_{b1}^\tau\}, \{\pi_{m2}^\tau, \pi_{b2}^\tau\}\}$.*

The timeline of the events in the model is shown in figure 3. At time $t = 0$, the initial contract is chosen. I allow

[9] Different cost of a waiver in different states allows the model to capture potentially different levels of frictions arising when control rights are misallocated in different situations in the real world. In particular, waivers of covenant violations – misallocations of control rights when the contract grants control to the bank but the manager should be in control – have been widely documented (e.g., Dichev and Skinner (2002)), while the opposite case – where no covenant was violated but the bank still demands control – has not been observed and may not be plausible as the bank could face litigation.

for dynamic covenant schedules: covenants for time $t = 1$, $\{\pi^0_{m1}, \pi^0_{b1}\}$, and time $t = 2$, $\{\pi^0_{m2}, \pi^0_{b2}\}$, chosen at time $t = 0$, can be different. At time $t = 1$, the parties observe the state of the world r_1, the cost of misallocating control rights in this state, c_1, and the signal s_1. If the state disagrees with the signal, $r_1 \neq s_1$, the parties either can bear the cost c_1 or waive the violation or the non-violation and bear the cost $\eta^i_w, i \in \{m, b\}$. Additionally, because the parties have learned new information about the likely state at time $t = 2$ and the likely cost of misallocating control at time $t = 2$, they may choose to amend the contract for time $t = 2$ from the initial one, $\{\pi^0_{m2}, \pi^0_{b2}\}$, to a new one $\{\pi^1_{m2}, \pi^1_{b2}\}$. At time $t = 2$, the parties observe the new state of the world r_2, the cost of misallocating control rights c_2, and the signal s_2. If $r_2 \neq s_2$, the parties can either incur c_2 or waive and bear η^i_w. At time $t = 3$, the contract matures.

Besides the distinction between waivers and amendments, the model has three innovations relative to the existing literature. First, the model does not have an explicit agency conflict between the borrower and the lender but focuses on the optimal choice of control rights given the information environment. I make this modeling choice because my goal is to estimate contracting efficiency and the impact of the information environment on it using real-world data. In reality, multiple agency conflicts can be present at the same time, each of them involving losses in the total value of the lender-borrower relationship. My model captures them in a reduced form through parameters c_t and η^i_w, helping me get to the aggregate estimates of efficiency losses in debt contracting. As discussed above, η^i_w can capture private benefits that either party derives from having control rights. Alternatively, it may be a measure of the parties' bargaining power: the larger the outside option, the higher amount has to be offered to a party to accept a contract or an amendment. c_t describes different moral hazard issues, such as asset substitution or the debt overhang problem. When the manager is in control suboptimally, she may take excessive risks (asset substitution); when the bank gets unjustified control, it may force the firm to underinvest and forego profitable projects (debt overhang).

The second innovation is that the contract in the model consists solely of control rights, or covenants, and does not include an interest rate. It can be shown that the problem where one party chooses a contract, consisting of a covenant and an interest rate, to offer to another party condenses to two steps. In the first step, a covenant that maximizes the total surplus from the parties' relationship is chosen. In the second step, given the optimal covenant, the interest rate is chosen to maximize the offering party's payoff conditional on the other party accepting the offer. In this setting, the interest rate just splits the total surplus between the parties and does not affect the overall contracting efficiency gain.[10] Since the focus of my study is aggregate contracting efficiency, I can abstract away from the interest rate's role and limit the analysis to the choice of optimal covenant schedules. For that reason, in the remainder of the paper,

[10] Gigler et al. (2009) arrive at the same conclusion when examining the role of conservative accounting in debt contracts.

I use the terms contract and covenant schedule interchangeably.

Third, I abstract away from the specific structure of the signal used to determine control rights. This general notion of the signal allows me to capture diverse metrics used in real contracts (e.g., different financial ratios and amounts, upper and lower thresholds) using one theoretical construct. The covenant in the model is not a specific statistic, but broadly the probability that a signal(s) used in the contract makes an error.

1.2 Benchmark: a model without amendments

Before stating the equilibrium of the full model, I analyze a benchmark model where the parties do not have the option to amend the contract at time $t = 1$. The benchmark is useful to understand how the dynamic covenant schedule works and why the amendment option helps the parties save the cost of control rights misallocation.

In the benchmark model, the only choice that the parties make is whether to waive control misallocation when this misallocation occurs. At time $t = 2$, if $s_2 \neq r_2$, the parties will waive if $c_2 \geq \eta_w^i$ and will not waive otherwise. Similarly, at time $t = 1$, the waiver will happen iff $c_1 \geq \eta_w^i$. Therefore, at any point in time, if control rights are misallocated, the deadweight loss will be $\min\{c_t, \eta_w^i\}$.

At time $t = 0$, the dynamic covenant schedule, $\{\{\pi_{m1}^0, \pi_{b1}^0\}, \{\pi_{m2}^0, \pi_{b2}^0\}\}$, is chosen. Let us denote the expected deadweight loss due to control misallocation for the case when the parties do not have any information about c_t by $e^i \equiv E\left[\min\{\tilde{c}_t, \eta_w^i\}\right], i \in \{m, b\}$. The expected loss of control rights misallocation equals e^i at time $t = 1$ and at time $t = 2$ when $r_2 \neq r_1$. At at time $t = 2$ when $r_2 = r_1$, the parties will have some information about c_2 because they will have observed c_1. Let us denote by a random variable $\tilde{f}_j^i \equiv E\left[\min\{\tilde{c}_2, \eta_w^i\}|\tilde{c}_1, r_2 = r_1\right] = E\left[\min\{\rho c_1 + (1-\rho)\tilde{c}, \eta_w\}|\tilde{c}_1, r_2 = r_1\right]$ the expected cost of misallocating control at time $t = 2$ in case the time-1 state persists, where $j \in \{m, b\}$ denotes the state at time $t = 1$ and $i \in \{m, b\}$ denotes the state at time $t = 2$. This variable is random at time $t = 0$ because it is a function of the realization of c_1, which is unknown at time $t = 0$. Using this notation, the problem at time $t = 0$ is

$$\min_{\{\pi^0_{m1},\pi^0_{b1},\pi^0_{m2},\pi^0_{b2}\}} \quad \pi^0_{m1}\mu_0 e^m + \pi^0_{b1}(1-\mu_0)e^b +$$

$$\pi^0_{m2}\left(\mu_0\mu E[f^m_m] + (1-\mu_0)(1-\beta)e^m\right) + \pi^0_{b2}\left(\mu_0(1-\mu)e^b + (1-\mu_0)\beta E[f^b_b]\right)$$

$$\text{s.t.} \quad (1-\pi^0_{m1}-q)^2 + (1-\pi^0_{b1}-q)^2 = (1-q)^2$$

$$(1-\pi^0_{m2}-q)^2 + (1-\pi^0_{b2}-q)^2 = (1-q)^2$$

$$\pi^0_{kt} \in [0, 1-q], \text{ for } k \in \{m, b\} \text{ and } t \in \{1, 2\}$$

The optimal covenant schedule minimizes the cost of misallocating control rights at all times and states subject to the restriction on the total precision of the signals. The cost of misallocating control at time $t = 1$ is e^m in the state $r_1 = m$ and e^b in the state $r_1 = b$, and the probability of misallocating control is $\pi^0_{m1}\mu_0$ in the state $r_1 = m$ and $\pi^0_{b1}(1-\mu_0)$ in the state $r_1 = b$. At time $t = 2$, in the state $r_2 = m$ ($r_2 = b$), the covenant makes an error with probability π^0_{m2} (π^0_{b2}). If $r_2 = m = r_1$ ($r_2 = b = r_1$), which happens with probability $\mu_0\mu$ ($(1-\mu_0)\beta$), the cost of the covenant making an error is $E[f^m_m]$ $\left(E[f^b_b]\right)$. If $r_2 = m \neq r_1 = b$ ($r_2 = b \neq r_1 = m$), which is the case with the $(1-\mu_0)(1-\beta)$ $(\mu_0(1-\mu))$ chance, the cost of an error is e^m $\left(e^b\right)$.

Proposition 1. *The optimal covenant schedule chosen at time* $t = 0$ *in the model without amendments,* $\{\{\pi^0_{m1}, \pi^0_{b1}\}, \{\pi^0_{m2}, \pi^0_{b2}\}\}$*, is unique and as follows*

$$\pi^{0*}_{m1} = 1 - q - \frac{(1-q)\mu_0 e^m}{\sqrt{(\mu_0 e^m)^2 + ((1-\mu_0)e^b)^2}} \tag{1}$$

$$\pi^{0*}_{b1} = 1 - q - \frac{(1-q)(1-\mu_0)e^b}{\sqrt{(\mu_0 e^m)^2 + ((1-\mu_0)e^b)^2}} \tag{2}$$

$$\pi^{0*}_{m2} = 1 - q - \frac{(1-q)(\mu_0\mu E[f^m_m] + (1-\mu_0)(1-\beta)e^m)}{\sqrt{(\mu_0\mu E[f^m_m] + (1-\mu_0)(1-\beta)e^m)^2 + (\mu_0(1-\mu)e^b + (1-\mu_0)\beta E[f^b_b])^2}} \tag{3}$$

$$\pi^{0*}_{b2} = 1 - q - \frac{(1-q)(\mu_0(1-\mu)e^b + (1-\mu_0)\beta E[f^b_b])}{\sqrt{(\mu_0\mu E[f^m_m] + (1-\mu_0)(1-\beta)e^m)^2 + (\mu_0(1-\mu)e^b + (1-\mu_0)\beta E[f^b_b])^2}} \tag{4}$$

The optimal covenant schedule is determined by two components: the probabilities of each state occurring and the expected costs of misallocating control rights in each of the states. For example, for the covenant in the state m at time 2, π^0_{m2}, if this state m persists from $t = 1$, which happens with probability $\mu_0\mu$, the cost is $E[f^m_m]$. If the time-1 state is b and the time-2 state m occurs, which happens with probability $(1-\mu_0)(1-\beta)$, the cost is e^m. As the total expected cost of misallocating control rights in the state $r_2 = m$, $\mu_0 \mu E[f_m^m] + (1-\mu_0)(1-\beta)e^m$, increases, the optimal error that the covenant makes in this state, π_{m2}^0, decreases.

When amendments to the covenant schedule are prohibited, the contracting parties ignore valuable non-contractible information that they learn over time. At time $t = 1$, the parties have observed the state r_1 – a projection for the state at time $t = 2$ – and the cost of control misallocation c_1 – a projection for the cost at time $t = 2$ if the state of the world persists. This information can be used to adjust the contract to redistribute covenant error between the states at time $t = 2$ and reduce the cost of misallocating control rights and waivers at time $t = 2$.

In the next section, I introduce the option to amend the contract, which helps avoid the loss of non-contractible information and dynamically complete the incomplete debt contract.

1.3 Full model equilibrium

In the full model, the parties can amend the contract at time $t = 1$ at a fixed cost η_a. The timeline is shown in figure 3. I solve the model by backward induction.

At time $t = 2$, the only decision is whether to waive a potential misallocation of control rights. The parties will waive iff $c_2 \geq \eta_w^i$.

At time $t = 1$, two decisions are made. First, the parties choose whether to waive a misallocation of control rights if it occurs. The waiver is chosen iff $c_1 \geq \eta_w^i$. Second, the parties may decide to amend the covenant schedule. In this case, they bear a deadweight loss of η_a and instead of having the initial contract, $\{\pi_{m2}^0, \pi_{b2}^0\}$, sign a new contract, $\{\pi_{m2}^1, \pi_{b2}^1\}$, for time $t = 2$. When deciding whether to amend the contract, the parties first find out the new optimal covenant schedule and then compare the cost of amendment η_a with the expected saved cost of control misallocation and waivers at time $t = 2$. The problem of choosing the new covenant schedule is different in different states. In the state $r_1 = m$, the new optimal contract minimizes the expected cost at time $t = 2$ given new non-contractible information received at time $t = 1$:[11]

$$\begin{aligned} \min_{\{\pi_{m2}^{1m}, \pi_{b2}^{1m}\}} \quad & \mu f_m^m \pi_{m2}^{1m} + (1-\mu) e^b \pi_{b2}^{1m} \\ \text{s.t.} \quad & (1 - \pi_{m2}^{1m} - q)^2 + (1 - \pi_{b2}^{1m} - q)^2 = (1-q)^2 \\ & \pi_{k2}^{1m} \in [0, 1-q] \text{ for } k \in \{m, b\} \end{aligned}$$

[11]The problem for $r_1 = b$ is the same except that the cost making an error if $r_2 = r_1 = b$ is f_b and the conditional probabilities of the covenant making an error are different, π_{m2}^{1b} and π_{b2}^{1b}.

Proposition 2. *When the covenant schedule is amended at time $t = 1$, the new optimal covenant schedule is unique and as follows:*

- *When $r_1 = m$,*

$$\pi_{m2}^{1m*} = 1 - q - \frac{\mu f_m^m(1-q)}{\sqrt{(\mu f_m^m)^2 + ((1-\mu)e^b)^2}} \tag{5}$$

$$\pi_{b2}^{1m*} = 1 - q - \frac{(1-\mu)e^b(1-q)}{\sqrt{(\mu f_m^m)^2 + ((1-\mu)e^b)^2}} \tag{6}$$

- *When $r_1 = b$,*

$$\pi_{m2}^{1b*} = 1 - q - \frac{(1-\beta)e^m(1-q)}{\sqrt{\left(\beta f_b^b\right)^2 + ((1-\beta)e^m)^2}} \tag{7}$$

$$\pi_{b2}^{1b*} = 1 - q - \frac{\beta f_b^b(1-q)}{\sqrt{\left(\beta f_b^b\right)^2 + ((1-\beta)e^m)^2}} \tag{8}$$

where $f_m^m = E[\min\{c_2, \eta_w^m\}|c_1, r_1 = m]$, $f_b^b = E[\min\{c_2, \eta_w^b\}|c_1, r_1 = b]$, $e^m = E[\min\{c_2, \eta_w^m\}]$, and $e^b = E[\min\{c_2, \eta_w^b\}]$.

Similar to the equilibrium in the benchmark model, the optimal error allocated to a state decreases in the expected cost of misallocating control rights in this state. The following proposition describes the parties' decision to amend the contract.

Proposition 3. *At time $t = 1$ in the state $r_1 = j$, the parties amend the covenant schedule iff the expected cost of misallocating control rights at time $t = 2$ in case the time-1 state persists, f_j^j, is low enough or high enough:*

$$f_j^j \leq x_j^l \text{ or } f_j^j \geq x_j^u \tag{9}$$

x_m^l, x_m^u, x_b^l, and x_b^u are defined in the Appendix.

The intuition for the decision to amend is as follows. When choosing whether to amend or not, the parties compare the cost of amendment η_a against the expected reduction in control misallocation cost under a new contract. If the state from $t = 1$ persists in $t = 2$, an estimate of this expected reduction is f_j^j. If the estimate is low enough, the optimal contract puts much more error into the more likely state than the initial contract. Similarly, if the estimate is high, the new contract removes more error from the more likely state than the original contract. For intermediate levels of f_j^j, the initial covenant schedule is not in much disagreement with the optimal covenant schedule to justify an amendment.

Corollary 1. *After an initial contract, $\{\{\pi_{m1}^0, \pi_{b1}^0\}, \{\pi_{m2}^0, \pi_{b2}^0\}\}$, is chosen, the probability of amendment at time $t = 1$ is*

- *decreasing in the quality of contractible information, q,*
- *increasing in the persistence of non-contractible information, μ and β,*
- *decreasing in the cost of amendments, η_a,*
- *weakly increasing in the waiver friction, $\eta_w^i, i \in \{m, b\}$, except for the case $\eta_w^i \in \Big(\max\{x_i^u, x_i^l + \frac{1-\rho}{2}\},$ $x_i^u + \frac{1-\rho}{2}\Big)$.*

Two main factors determining whether to amend a contract are the degree of an error made by covenants and the cost of making an error. If the covenants generally make fewer errors (q is higher), there is no strong need to amend them. If the non-contractible information about the state is very predictive of the future (μ or β is high), the parties can eliminate errors more efficiently if they adjust the contract, increasing the likelihood of amendments. As for the cost, η_w^i is a lower bound for the deadweight loss if a covenant makes a mistake. When η_w^i increases, amendments are preferred more often. Finally, when amending the contract gets more expensive (η_a increases), the contracting parties are less likely to choose this option.

Finally, at time $t = 0$, the parties choose the optimal covenant schedule, anticipating that the contract will be amended in some cases. The problem at time $t = 0$ is

$$\begin{aligned}
\min_{\{\pi_{m1}^0,\pi_{b1}^0,\pi_{m2}^0,\pi_{b2}^0\}} \; & \mu_0\times\Big(\pi_{m1}^0 e^m + E\Big[\min\{\mu f_m^m \pi_{m2}^0+(1-\mu)e^b\pi_{b2}^0, \eta_a+(1-q)(\mu f_m^m+(1-\mu)e^b)-(1-q)\sqrt{(\mu f_m^m)^2+\big((1-\mu)e^b\big)^2}\}\Big]\Big) \\
& +(1-\mu_0)\times\Big(\pi_{b1}^0 e^b + E\Big[\min\{\beta f_b^b \pi_{b2}^0+(1-\beta)e^m\pi_{m2}^0, \eta_a+(1-q)(\beta f_b^b+(1-\beta)e^m)-(1-q)\sqrt{\big(\beta f_b^b\big)^2+((1-\beta)e^m)^2}\}\Big]\Big) \\
\text{s.t.} \quad & (1-\pi_{m1}^0-q)^2+(1-\pi_{b1}^0-q)^2=(1-q)^2 \\
& (1-\pi_{m2}^0-q)^2+(1-\pi_{b2}^0-q)^2=(1-q)^2 \\
& \pi_{kt}^0\in[0,1-q] \text{ for } k\in\{m,b\} \text{ and } t\in\{1,2\}
\end{aligned}$$

The equilibrium initial covenants for time $t = 1$, $\{\pi^0_{m1}, \pi^0_{b1}\}$, are the same as in the benchmark case:

$$\pi^{0*}_{m1} = 1 - q - \frac{(1-q)\mu_0 e^m}{\sqrt{(\mu_0 e^m)^2 + ((1-\mu_0)e^b)^2}} \tag{10}$$

$$\pi^{0*}_{b1} = 1 - q - \frac{(1-q)(1-\mu_0) e^b}{\sqrt{(\mu_0 e^m)^2 + ((1-\mu_0)e^b)^2}} \tag{11}$$

The covenants for time $t = 2$, $\{\pi^{0*}_{m2}, \pi^{0*}_{b2}\}$, however, are different because they play a dual role: they determine when the contract is amended and, if it is not amended, stay as the covenants at time $t = 2$. When estimating the model, I solve the problem of finding $\{\pi^{0*}_{m2}, \pi^{0*}_{b2}\}$ numerically. Because the set of potential solutions is convex and the objective function is generally concave, the solution is unique. The details are provided in Appendix.

1.4 Ex ante contractual efficiency gain

Amendments bring a weak welfare gain to the bank-firm relationship. We have demonstrated how the option to amend the debt contract allows the contracting parties to incorporate the most recent non-contractible information and, as a result, avoid bearing some cost of misallocating control rights. Because amendments are made only if they cost less than the expected gain, and never amending the contract is still an option, the possibility to amend always weakly increases contracting efficiency.

I define contracting efficiency gain due to amendments as the expected reduction in the cost of misallocating control rights:

$$\min_{\{\pi^{0a}_{m1}, \pi^{0a}_{b1}, \pi^{0a}_{m2}, \pi^{0a}_{b2}\}} \Big[\mu_0 \times \Big(\pi^{0a}_{m1} e^m + E\Big[\min\{\mu f^m_m \pi^{0a}_{m2} + (1-\mu)e^b \pi^{0a}_{b2}, \mu f^m_m \pi^{1m}_{m2} + (1-\mu)e^b \pi^{1m}_{b2}\}\Big]\Big) \tag{12}$$

$$+(1-\mu_0) \times \Big(\pi^{0a}_{b1} e^b + E\Big[\min\{\beta f^b_b \pi^{0a}_{b2} + (1-\beta)e^m \pi^{0a}_{m2}, \beta f^b_b \pi^{1b}_{b2} + (1-\beta)e^m \pi^{1b}_{m2}\}\Big]\Big)\Big] \tag{13}$$

$$- \min_{\{\pi^{0na}_{m1}, \pi^{0na}_{b1}, \pi^{0na}_{m2}, \pi^{0na}_{b2}\}} \Big[\mu_0 \times \Big(\pi^{0na}_{m1} e^m + E\Big[\mu f^m_m \pi^{0na}_{m2} + (1-\mu)e^b \pi^{0na}_{b2}\Big]\Big) \tag{14}$$

$$+(1-\mu_0) \times \Big(\pi^{0na}_{b1} e^b + E\Big[\beta f^b_b \pi^{0na}_{b2} + (1-\beta)e^m \pi^{0na}_{m2}\Big]\Big)\Big], \tag{15}$$

where ^{a} and ^{na} denote optimal covenant schedules in the model with and without amendments, respectively.

In figure 4, I plot ex ante contracting efficiency gain as a function of model parameters: the quality of contractible information, q, the persistence of the non-contractible state m, μ, the waiver friction in the state m, η^m_w, and the cost of amendments, η_a.

When the signal used in the contract becomes more precise, the total covenant error rate decreases, making the option to amend less valuable. In the extreme, if the signal perfectly represents the actual state ($q = 1$), it is unnecessary to amend it.

The importance of non-contractible information affects efficiency gain in the opposite direction. As the actual state at time $t = 1$ becomes more predictive of the state at time $t = 2$, amending at time $t = 1$ helps avoid more error and cost of control misallocation at time $t = 2$.

Finally, contracting efficiency gain is increasing in the waiver friction and decreasing in the cost of amendments. When it is more expensive for a covenant to make an error (η_w^m increases), an amendment saves more control misallocation cost. In contrast, when amendments themselves take more resources (η_a increases), the net gain from changing the contract decreases.

1.5 Theoretical moments and identification

In the last part of this section, I list theoretical moments that are used to identify the model's parameters from the data. There are eight parameters. Parameters $\bar{c}$ and ρ govern the distribution and persistence of the control misallocation cost; parameters $\eta_w^i, i \in \{m, b\}$ and η_a are costs of waivers and amendments, respectively. The uncertainty about the optimal party-in-control is described by the initial probability that the firm's management should stay in control, μ_0, and by how likely the manager and the bank are to remain the optimal party, μ and β, respectively. Finally, the precision of contractible information is captured by the parameter q.

First of all, I introduce an assumption about waivers to make the model descriptive of real-world waiver situations. Theoretically, waivers are symmetric: the contracting parties can waive a control misallocation both when the bank and the manager get control suboptimally. In reality, however, the first situation – when a covenant is violated but the violation gets waived and control returns to the manager – is plausible and has been studied in prior work, while the symmetric situation – when the bank obtains control without any covenant violations – is hard to track and seems implausible because the bank may face litigation if it attempts to press the borrower without formal contract breaches. This disconnect between the model and the data may lead to biased estimates of the model. To avoid the potential misspecification, I manually preclude waivers when control is suboptimally granted to the manager in the model and set η_w^b prohibitively high in the estimation.

I make two additional assumptions to estimate the model. First, the nature of the model is such that the parameters describing the cost of misallocating control rights, $\bar{c}$ and η_w^m, can not be separated. The decision to waive or keep a "wrong" party in control is determined by comparing the control misallocation cost and the waiver friction and choosing the smaller one. I can not observe every instance when control rights are misallocated[12] and thus can not know whether the parties waived or kept the wrong party in control. For that reason, in estimation, I fix parameter and $\bar{c} = 1$ and estimate η_w^m. The obtained estimated waiver cost, η_w^m, and amendment cost, η_a, thus are interpreted relative to the control misallocation cost.

Second, the parameters governing the persistence of the control misallocation cost, ρ, and of non-contractible states, μ and β, affect the amendment behavior in the same way. If either control misallocation cost or the state of the world is more persistent, the parties have more motivation to amend the contract because they are more certain of what the cost of misallocation and the state will be in the future. To identify one of the parameters describing the persistence of the parties' new information, I need to fix one of the two parameters. I assume that the cost of having a wrong party in control is stable over time, conditional on the wrong party staying the same, and set $\rho = 0.9$. The parameters μ and β are estimated. I examine the estimates' robustness to the choice of cost persistence ρ in Appendix 7.12.1.

To identify the six remaining parameters, I use six moments listed in Appendix 7.5, describing initially set and amended covenant schedules and probabilities of amending financial covenants. Because the number of parameters equals the number of moments, the model is just identified.

The first moment is the strictness of financial covenants in the first period after loan initiation. I define strictness as the probability of violating a covenant within the first period, consistent with the empirical definition of covenant strictness I use (Demerjian and Owens (2016)). The probability of violation is a function of, first, covenant levels and, second, the firm's financial performance: a firm is more likely to violate when its covenant levels are more restrictive and when it is less financially healthy. The first moment is thus a function of and helps identify the precision of contractible information q and the cost of waivers η_w^m, which jointly determine the level of initial covenants, and the probability that the best party-in-control will initially be company management, μ_0. When the cost of waivers η_w^m increases, the costs of the covenant's error in different states become closer to each other, and the parties allocate more of their signal's precision to the good state m, loosening the covenant. A higher probability of the state m, μ_0, implies the parties are more likely to be in the state m in the first period, prompting them to put more signal's precision in this state, or loosen the covenant.

The second theoretical moment is the strictness of financial covenants in the second period after loan initiation. Similarly to the strictness of initial covenants, they depend on the level of covenants set for the second period and on the company's performance in the second period. Because the contracting parties anticipate that the covenants

[12] Even though I can observe that a waiver happened, I can not observe situations when a waiver was considered but dismissed.

may be amended in the second period when they write the initial contract, another factor in addition to q and η_w^m determining how they choose covenants is the potential amendment cost η_a. Intuitively, since the parties understand they will amend whenever their current contract is too far from the new optimal contract given new information, they try to anticipate the new information and set the original contract as close to the expected new contract as possible. Another difference from the strictness of covenants for the first period is that the company's financial performance in the second period is determined by its performance in the first period, μ_0, and by how likely either good or bad state is to persist in the second period, μ and β.

The third moment describes the unconditional probability that the covenant schedule will be amended. First, because the probability of amendment may be different in different states, this moment is a function of the probabilities of these states μ_0. Second, the decision to amend in each state depends on all parameters listed in corollary 1: the probability of amending is higher for higher waiver cost η_w^m and lower amendment cost η_a, and for more persistent non-contractible states μ and β.

The fourth moment is the probability of an amendment conditional on no covenant being violated. This moment is similar to the third but only describes amendments in a subset of cases. Besides the factors listed for the third moment, the probability of an amendment conditional on no violation depends on the likelihood of no violation in the first period, which is a function of q, η_w^m, and the company's initial financial performance μ_0. Therefore, the fourth moment is more sensitive to these parameters.

The fifth moment describes the strictness of amended financial covenants. Similarly to initial financial covenants for the first period after loan origination, the strictness of amended covenants depends on the level of covenants, determined by q and η_w^m, and the company's financial performance, μ if the current state is m and β if the current state is b.

The last, sixth, moment is the strictness of amended covenants conditional on no violation in the period when the covenants are amended. The strictness is described by the same parameters as in the fifth moment, but in addition, it depends on the probability of no violation, a function of covenant levels in the original contract, q and η_w^m, and the firm's financial performance in the first period μ_0.

2 Data and descriptive statistics

In this section, I first describe data sources and the process of collecting detailed financial covenant schedules and amendments to financial covenants. Second, I discuss a measure of financial covenant strictness. Finally, I present descriptive statistics.

2.1 Data sources and collection

To estimate the model, I need three series of data: (1) initial financial covenant schedules, (2) amendments of financial covenants, and (3) violations of financial covenants. Because the data on exact dynamic initial covenant schedules[13] and amendments of financial covenants are not available in databases that I am aware of, I manually collect detailed covenant schedules and financial covenant amendments from loan contracts that companies are required to file with the SEC under Regulation S-K.

For loans originated before 2012, I start data collection with the dataset created by M. Roberts (Roberts (2015)). Roberts' sample includes 114 randomly selected companies from the intersection of Compustat and Thomson Reuters Dealscan databases starting from 1984. The dataset includes all loan paths – from origination to every amendment, amendment and restatement, or rollover until maturity or refinancing – for every company in the sample. Specifically, for each event (Origination, Amendment, Amended & Restated, Rollover, Refinanced), the dataset contains binary information on whether interest rate, loan size, maturity, any covenant, investment, collateral, distribution, or financial covenant was amended, and percent changes in interest rate, loan amount, or maturity. The dataset also includes information sources: SEC filings, specifying the form, the filing date, and the exhibit where a contract can be found. In the first half of the data collection procedure, I read the filings recorded in Roberts' dataset and add detailed financial covenant schedules and amendments of financial covenants.

For loans originated after 2012, I use Dealscan to find all loans for the companies in the Roberts' dataset. Next, I use the EDGAR search tool to search for original loan agreements and all amendments to them. The search for a given company is based on the name of the lead arranger and the loan origination date provided in Dealscan.

After the filings are collected, I record data on financial covenant schedules and amendments to them in the following way. For initial loan agreements, I manually search for financial covenants, typically found in the "Negative covenants" or "Financial covenants" sections. For each financial covenant, I record dynamic schedules on a yearly basis, starting from the contract effective date for the first five years of the loan. For example, for a debt-to-EBITDA covenant that starts with 5:1, falls to 4.5:1 in 10 months, then falls to 4:1 in another ten months, and then falls to 3:1

[13] The Dealscan database contains information on whether a financial covenant is monotone or changes over time. The latter are recorded as "Increasing from XXX to YYY" without specifying when the threshold changes exactly and how fine the steps from XXX to YYY are. For instance, Dealscan would record two covenant schedules provided in Appendix 7.6 in the same manner, while they are economically different: the first one represents a short-term adjustment with a fixed covenant for the remaining life of the loan (Li et al. (2016)), while in the second one, the covenant changes gradually over the entire period, probably to regulate a fundamental agency problem that is expected to worsen with time. My paper models dynamic covenant schedules explained by long-term fundamental considerations and not short-term adjustments, making the two cases different from this study's perspective.

in another ten months and stays the same until maturity, I record the schedule as "4:1 in 1 year, 3:1 in 2 years, 3:1 in 3 years, 3:1 in 4 years". Static covenant schedules are recorded as constant for all years. Examples of dynamic covenant schedules and their records are in Appendix 7.7.

For amendments, I first check whether an amendment to a loan contract contains changes to financial covenants. These changes are typically made in two ways: the numeric level of a financial covenant is amended, or the definition of a variable used in a financial covenant is amended. For the latter, I record an amendment as containing a financial covenant amendment but do not specify the new level of a financial covenant. For the former, I record a new level of a financial covenant in the same yearly manner as for original loans, starting from the amendment effective date. Appendix 7.8 contains examples of the two types of amendments to financial covenants.

It is essential to discuss which amendments to numeric levels of financial covenants are included in the sample and which are not. In the context of this study, an amendment is a permanent change to a covenant schedule driven by new material information learned by the lender and the borrower, and a waiver is a temporary change of control rights once the parties find themselves in an inefficient situation. Some amendments to loan contracts, in reality, change financial covenants short-term to avoid an anticipated violation of these financial covenants. Such adjustments are waivers rather than amendments in my model because they are alterations of current control rights ex post, after the parties realize that a violation is happening. I define short-term changes as those that only alter financial covenant schedules for within a year after the amendment effective date but keep the schedule unchanged for the rest of time until the contract's maturity.[14] These instances are not recorded as amendments for this study. An example of an amendment excluded from the sample and an amendment included are in Appendix 7.9.

The hand-collection process has limitations. Some loan contracts and amendments could not be accessed because EDGAR only contains filings up to a specific year in the past. When the data was collected for this paper, the most recent year available was 2001. To avoid throwing away all observations before this year, I made the following assumptions. First, for dynamic contracts, I assume that covenants evolve linearly over time. For instance, for a covenant "Increasing from 1:1 to 1.2:1" in a 4-year loan, I assumed that the covenant is "1:1" in one year, "1.1:1" in two years, and "1.2:1" in three years. Second, when an amendment contract could not be found but was in the Roberts' dataset, I used an indicator created by Roberts (2015) for changes in financial covenants instead of using my indicators. In these cases, the data on how exactly financial covenants were changed was recorded as missing.

Other data comes from two databases. Original loan contract terms and borrower and lender characteristics are

[14]Even though I choose a one-year cutoff to distinguish short-term from long-term changes to financial covenants, I have not observed instances when an amendment changes covenants for slightly less than a year. A typical short-term change amends covenants for the next month or quarter.

from Thomson Reuters Dealscan, and companies' financial information is from Compustat.

2.2 Measure of financial covenant strictness

There exists a disconnect between the notion of financial covenant strictness in the model and the data. I model financial covenants as an aggregate signal, while real-world loan contracts often use more than one financial covenant. To bridge the gap between theory and empirical setting, I use the measure of financial covenant strictness proposed by Demerjian and Owens (2016).

Demerjian and Owens (2016) develop a non-parametric measure of how likely a company is to violate any of its financial covenants in the next quarter after the contract's effective date. Because my study concentrates on longer-term financial covenant schedules and amendments, I adapt the Demerjian and Owens (2016) measure to an annual basis. My measure captures the probability of violation in the next year after the contract is signed or amended.

The measure is computed as follows. First, all companies available in Compustat are matched into twelve groups based on size and profitability. For a given year, the firms are sorted into size quartiles based on average total assets, and then within each size quartile, they are sorted into terciles based on their return-on-assets (ROA).

Next, to compute financial covenant strictness for a company i, another company j is randomly drawn from the bins matched to the company i one and two years before the loan initiation date. For company j, a rate of change in its financial variables is calculated and company i's financial variables are multiplied by these rates. Finally, company i's projected changes are compared with its financial covenants, and an indicator for whether any covenant is violated is recorded. In other words, I check whether the company i would have violated at least one of its covenants if it evolved exactly as the company j. The procedure is repeated 1000 times. The resulting probability of violating a covenant within a year for the company i is the fraction of simulations that are recorded as violations:

$$PVIOL_i = \frac{\sum_{j \neq i} VIOL_j}{1000} \tag{16}$$

Both empirical measure and theoretical construct of financial covenant strictness are functions of two variables: the level of financial covenants and the company's financial performance. Demerjian and Owens (2016) measure would increase if either financial covenant levels were more restrictive or if companies similar to the company i performed worse financially. As can be seen from theoretical moments 1-2 and 5-6, the probability of violating a covenant is a product of covenant levels and probabilities of states where the borrower or the lender are the optimal party-in-control.

2.3 Descriptive statistics

Below I present the summary statistics of my sample. First, I discuss descriptive statistics about the companies and loans and compare them to the total Compustat and Dealscan population. Next, I present summary statistics for the variables used in estimation: financial covenant schedules and amendments to them, – and their relations with each other and companies' and loans' characteristics.

The sample comprises 1,068 loans issued to 110 medium-sized companies between 1994 and 2021.

I present the summary statistics for my sample and juxtapose them against the summary for the whole Compustat universe in table 1. The companies in this paper's sample are among larger firms: the median firm in my sample has about $1,000 million in total assets, while the median firm in total Compustat has only about $236 million. My firms have slightly better growth opportunities (median market-to-book is 2.14, compared to 1.71 in the Compustat universe) and are more profitable (median return-on-assets is 0.04 compared to 0.01 in the Compustat universe), which is expected for companies that borrow large amounts from multiple reputable lenders. Because I study firms that are financed with a considerable amount of bank loans, my firms are more leveraged: the median leverage and debt-to-EBITDA ratios are 0.48 and 2.66, respectively, compared to 0.26 and 0.84 in the total Compustat data.

The loans in my sample are large longer-term corporate loans provided by lending syndicates. The summary statistics for my sample and the total Dealscan universe are shown in tables 2 and 3. My sample differs from the whole Dealscan in the ways consistent with my firms being publicly traded companies with their data available in the Compustat database. The median loan in my sample is a $200 million loan with about a 5-year maturity; the median loan in Dealscan is a $130 million loan with the same maturity. The spreads on the loans in this paper's sample and in the Dealscan data are very similar. I have a higher fraction of secured loans than overall Dealscan: 62% compared to 35%. Finally, consistent with my companies having larger loans, the median number of members in a syndicate is 7 in my sample, greater than 4 in the Dealscan universe. More than half of my sample are revolving facilities for longer than a year, and about a third of the sample are term loans. Dealscan has a lower proportion of revolvers, only around 30%. This difference in composition resembles evidence in prior studies that revolvers are generally taken by larger borrowers (see summary statistics in Berg et al. (2016)).

Table 4 shows the average number of loans a borrower has in each year. Borrowers often get a package (deal) of loans (tranches) from the same syndicate of lenders. In this paper, I treat each tranche as a separate loan because sometimes tranches within the same deal have different financial covenants that are modified independently of changes in other tranches. In my sample, an average borrower has between one and two deals in a given year, and about two or three tranches, implying that some deals comprise more than one tranche.

Summary statistics of the data used to estimate the model are presented in tables 5 and 6. The first two rows of table 5 show the strictness of covenant schedules in initial loan agreements: the first row of covenants for one year and the second row for two years after the contract is signed. On average, for original loan terms, financial covenants in one and in two years are of similar strictness. A median company is about 24% likely to violate one of its covenants within the first or the second year after loan origination.

In the third and fourth rows of table 5, I present the distribution of strictness of amended covenants and changes in covenant strictness after amendments, respectively. Amended covenants are stricter than original more often than not, but sometimes financial covenants are relaxed when contracts are amended. As the last row suggests, a firm in the 25th percentile has about 20% looser covenants after an amendment.

The most important statistics for this study are in table 6, which shows the frequency of financial covenant amendments in general and for cases when no covenants are violated. About 28.9% of loans in my sample have a financial covenant amended at least once, and about 20.3% have a covenant amended within the first year after signing the loan agreement. Out of these 28.9% and 20.3%, 42.6% and 42.0%, respectively, are amendments when the borrower did not violate or did not expect to violate a financial covenant. These instances of amendments not due to violations highlight that amendments are not made simply to avoid a technical default but may have different economic drivers.

I provide correlations between variables describing financial covenants and amendment behavior and other firms' and loans' characteristics in tables 7 and 8. A few observations are worth discussing. First, the strictness of financial covenants in original contracts is associated with the firm's and loan's characteristics in an intuitive way. Larger firms (measured by total assets) with less leverage (measured by leverage and debt-to-EBITDA ratios) generally obtain looser covenants from their lenders, in line with findings in prior studies (e.g., Bradley and Roberts (2015)). Covenant strictness is positively correlated with loan spreads (all-in-drawn spread) and with the loan involving collateral, consistent with riskier borrowers obtaining worse credit conditions on all dimensions. Note that if we control for the borrowers' risk, we would see a negative correlation between covenant strictness and interest rate, suggesting the trade-off between the two ways of granting control to the lender (Bradley and Roberts (2015)).

Second, amendment behavior seems not strongly correlated with the strictness of initial covenants or with the borrower's financial health. The correlation between a binary variable for amendment and the strictness of financial covenants is less than 10% and the binary variable for being close to a covenant violation is only 24%. Similarly, amendments are not correlated with borrowers' leverage ratio and are only weakly correlated with the debt-to-EBITDA ratio. These facts suggest that amendments may not serve to resolve violations, as waivers do, but have different economic purposes.

3 Model estimation and main results

This section describes the model's estimation procedure, presents parameter estimates for the full sample, and discusses the model's fit.

3.1 Estimation procedure

I estimate the model using the Generalized Method of Moments (GMM). Intuitively, the method searches for the set of model's parameters – the quality of contractible information, q, the persistence of non-contractible information, μ and β, companies' financial health, μ_0, and waiver friction, η_w, and amendments, η_a – that minimize the distance between model-implied theoretical moments and their empirical counterparts. Since the number of moments equals the number of parameters (6), the model is just-identified. I describe the estimation procedure in detail in Appendix 7.11.

3.2 Main results

In this section, I discuss the main estimates of the model. They allow me to tell which of the theoretical mechanisms explain financial covenant amendments observed in the data: low precision of contractible information, high persistence of non-contractible information, low amendment cost compared to the cost of misallocating control rights, or more than one of the above. Table 9 shows parameter estimates for the full sample. All the parameters are statistically significant.

The estimated parameters suggest that in in my sample financial covenants are amended primarily because (1) the precision of contractible information is low and (2) the cost of amending covenants is considerably less than the cost of misallocating control rights. The estimated precision of contractible information is quite low: if covenants do not make any errors in one state, they will make errors in the other state about 31.3% of the time. The first explanation for amendments in the data seems to be that the parties face a high level of contractual incompleteness: the accounting numbers used in financial covenants do not capture the true state of the world well.

The second explanation for amendments – persistent non-contractible information – finds some support in the data: the estimated persistences of states when the manager and the bank are the best parties-in-control are low. Conditional on a company being financially healthy, i.e., not needing lender's intervention, in one year after the contract is signed, the company will stay healthy next year only with a 65.4% chance. The state when the bank needs to take control is even less persistent: if the bank should have control rights in one year, this will be the case in two years only with a probability of 50.1%. Overall, the data suggest that the non-contractible state of the world is not very persistent, implying that contract amendments happen often not because of the persistence of non-contractible information.

The final theoretical reason why financial covenants are amended is that amendments are cheap compared to the misallocation of control rights. The empirical estimate of the amendment cost is about 1.8% of the average cost of control misallocation, while the waiver cost is around 140%,[15] suggesting that amending financial covenants in advance is indeed much cheaper from the total surplus perspective than facing control misallocation later when it occurs.

Finally, I estimate how likely a typical company is to be financially healthy in the next year after it signs a contract with a syndicate. The evaluated probability is $\mu_0 = 61.4\%$. In 38.6% of the cases, a lender needs to intervene in some way within the first year of the lender-borrower relationship.

To summarise, the data suggests two most likely explanations for U.S. firms' amendment behavior. Financial covenants are changed often because contractible information used in debt contracts is imprecise, and amending the contracts in advance is cheaper than misallocating control rights ex post. The persistence of non-contractible information as a reason for amendments finds limited empirical support.

3.3 Contracting efficiency gain

I estimate the efficiency gain from amendments – the amount of control misallocation cost saved ex ante because the parties can amend the contract – and interpret it economically. The estimated number ranges between 2.29% and 2.36%, or the option to amend in expectation saves about 232.3 basis points of control misallocation cost. Ex post, conditional on an amendment, the parties save 15.32% of cost in the state when the manager is the best party-in-control and 12.62% in the state when the bank needs to be in control. These amounts represent economically significant savings. Recall that the cost of a wrong party-in-control, parameter c, captures deadweight loss in firm value when the wrong party obtains control rights. For example, when the manager gets control during unstable financial conditions, she may take excessive risk and destroy company value. When the bank is given the right to intervene in the firm's operations without the need, it may inefficiently reduce investment and R&D (Chava and

[15]These interpretations are calculated by multiplying the respective estimates by 2 such that the percentages are not of the upper bar of the cost, $\bar{c} = 1$, but of the mean of the cost, $\frac{\bar{c}}{2} = 0.5$.

Roberts (2008), Nini et al. (2009)), fire executives (Nini et al. (2012)), or shape the company's disclosure policy (Vashishtha (2014)). These actions, if taken when they should not be, may substantially hurt the firm's long-term performance. Although the parties may waive these inefficiencies ex post when the bank gets control, the cost of doing that is also high, as the estimate of η_w^m suggests. To sum up, saving even about two percent of the loss in firm value may be a lot in monetary terms. The fact that we observe contracting parties amending their contracts implies the parties' revealed preference to save these control misallocation costs.

3.4 Model fit and analysis of identification

To evaluate how well the theoretical model fits empirical observations, I examine the distance between estimated theoretical and data moments. The values of moments and t-statistics of differences are shown in table 10. All of the model-generated moments are statistically indistinguishable from their counterparts at the 1% significance level.

I provide more intuition of how each parameter is identified from data moments by plotting the moments' sensitivities to parameters in figure 5. The plots show that different moments are important to identify different parameters. Generally, the moments move with parameters in the directions consistent with the theoretical discussion in section 1.5.

The precision of contractible information, q, is primarily identified from the parties' decisions to amend the contract. When the contractible information better captures the non-contractible state – q is higher – the probability of amendment unconditional and conditional on no violation decreases (figure 5a).

Companies' financial health at the time of loan origination, μ_0, mostly comes from the strictness of financial covenants in the original loan contract. Healthier firms, i.e., firms that are less likely to need the lender's intervention within the first year after loan origination, obtain less strict financial covenants (figure 5b).

Persistences of the non-contractible states, μ and β, are identified, first, from initial and amended covenant levels for the period more than one year after loan origination, and second, from the probabilities of amendment. As can be seen in figure 5c (figure 5d), if the good (bad) state is more likely to persist after it occurs in the first period, the covenants for the second period are set less (more) strict. The probability of amendment increases in persistences of both states.

The cost of amendments primarily is obtained from the probabilities of amendments (figure 5e). As the cost of amending a contract goes up, the parties are less likely to use this option. The cost of amendment can be disentangled from the precision of contractible information, q, by using the strictness of amended financial covenants. Amended financial covenants are less strict when the precision of contractible information is higher, but are more strict when the cost of amendment is higher.

Finally, the cost of waiving a covenant violation, η_w^m, is identified from the strictness of financial covenants, mostly from amended. If waivers are more expensive, the parties try to avoid them more and set covenants looser (figure 5f).

4 Counterfactual analyses

One of the benefits of structural estimation is the ability to predict changes in economic agents' behavior in different scenarios. These analyses, in turn, can inform policy decisions without costly experiments in the real world.

In this section, I conduct several counterfactual analyses. First, I study how the contracting efficiency gain from amendments changes when model parameters change slightly, i.e., within 10%. Second, I consider substantial changes in economic conditions and show how the efficiency of loan agreements and the likelihood of amendments evolve.

4.1 Sensitivities of the likelihood of amendments and contracting efficiency gain to small parameter changes

To understand how sensitive the likelihood of amendments and contractual efficiency gain are to various forces in the model, I vary each estimated parameter by 10% up and down and calculate by how much the likelihood of amendments and efficiency gain from amendments change. Figure 6 shows changes in the probability of debt contract amendments, and figure 7 plots changes in the expected contracting efficiency gains from amendments.

Given the current parameter estimates, the most moving factor for the contractual efficiency gain is the persistence of non-contractible information ((third and fourth bars of the histogram in figure 7)). When the current good (bad) non-contractible state becomes 10% more predictive of the future good (bad) non-contractible state, the parties' gain from the option to amend their contract increases by 91.31% (66.52%). This result suggests that empirically, amendments should be observed much more often for stable companies or in periods with less volatile macroeconomic conditions.

The second most crucial factor affecting contracting parties' decision to amend and gain from amendments is the precision of contractible information. If the signals that can be contracted upon got 10% more precise, we would observe about 29.4% fewer amendments (first bar of the histogram in figure 6) because banks and companies would expect to gain about 49.3% less from changing their contracts (first bar of the histogram in figure 7).

The next significant factor is the cost of amending a financial covenant schedule. If the deadweight loss when a contract is changed increases (for instance, lawyers become more expensive), the contracting parties lose about one-fifth of contracting efficiency gains and prefer to amend their contract about 8.72% less.

Finally, companies' financial health at the beginning of their relationship with the bank and the cost of waiving financial covenant violations influence contracting parties' choices the least (second and sixth bars of the histograms in figures 6 and 7).

The sensitivities of amendments and contractual efficiency to small parameter changes inform us about how firms and banks would respond to minor perturbations in the economic environment. The counterfactual analyses demonstrate that all of the determinants of debt covenant amendments – the quality of contractible information, the persistence of non-contractible states, and the cost of amending in advance and waiving ex post – can alter the parties' behavior quite significantly.

4.2 Changes to economic environment & their effects on the likelihood of amendments and contracting efficiency

In this section, I consider substantial changes to economic conditions and analyze implications for financial covenant amendments and contracting efficiency. I investigate five scenarios. The first scenario looks at an extremely financially healthy company that almost never needs a lender's intervention in the first year after loan origination. This case demonstrates that covenant amendments are not a feature of just poorer performing firms but are an option that financially stable companies find useful. In the second scenario, I flip the analysis and look at a company that almost surely needs to give control to the lender within one year after loan origination. By comparing the first and the second scenarios, we can understand to what extent frequent debt contract changes can be attributed to companies' financial health. The third scenario introduces the option to waive control rights misallocation in the state when the manager gets control suboptimally. This analysis shows how amendment behavior would look if we believe that the parties can grant control to the bank even if no financial covenants were violated.

4.2.1 Completely financially healthy and completely financially unhealthy companies

In the first counterfactual scenario, I analyze a very financially healthy company and demonstrate that this company still amends financial covenants in the contract with its lender(s). To make a firm in the model completely healthy, I set the initial probability that the company manager will be the optimal party-in-control close to one: $\mu_0 = 0.999$. The first scenario in table 11 shows that even such a stable company will still have its financial covenants amended about 12.53% of the time. The covenants will be amended because the lender and the borrower still learn new information about the state further in the future and about the cost of a covenant making an error in the future. This learning does not disappear if a company becomes healthier: even if the parties are almost certain what will happen in the near future, they still learn valuable information about the future that is more distant.

The second scenario looks at the opposite case – a very financially unhealthy company – and shows that this company is not substantially more likely to amend its debt contract than a healthy firm. In the second scenario in table 11, I show the likelihood of an amendment for a company that very likely will need the lender's intervention in the near future, $\mu_0 = 0.001$. In expectation, a financially unstable firm negotiates with its lender and changes financial covenants about 15.86% of the time, similar to the healthy company.

The two counterfactual scenarios suggest that firms' general financial health does not have to be correlated with how often firms amend their debt contracts. However, this fact may be non-trivial to confirm empirically. Financial covenant amendments are determined by the amount of information learned over time about the state of the world, and this amount does not change with companies' financial health per se. In reality, firms' financial stability may be correlated with various characteristics of information systems. For example, companies that do not need lenders' intervention may also have higher accounting quality – the precision of contractible information – which would translate into fewer amendments compared to less healthy firms. A researcher who properly controls for characteristics of contractible and non-contractible information should expect to find no association between financial health and the likelihood of debt contract amendments. Roberts (2015) does find that the probability of amending a covenant is largely unrelated to borrowers' initial performance: neither EBITDA-to-assets ratio nor stock performance predict the likelihood of covenant amendments.

4.2.2 A possibility of a waiver when no covenants are violated

The third counterfactual scenario introduces an additional waiver opportunity. In reality, waivers most plausibly happen (and can be observed by a researcher) only when a covenant is violated but the company management stays in control. In terms of the model, this means control rights are misallocated such that the bank gets control by mistake, and the parties waive the misallocation and return control to the manager. Theoretically, the opposite case is also possible: when the control rights are allocated to the manager by mistake but should be given to the bank. This case is hard to observe in the data because it would imply the lender intervenes in a firm without any formal technical default. For that reason, in the main estimation, I prohibit waivers of this type. Nevertheless, to look at how amendment behavior would look if we believe the parties can waive control rights misallocation in the case when the bank is the best party-in-control, I consider a counterfactual scenario where waivers are allowed in both control misallocation cases. I set the cost of giving control to the bank when no covenant was violated equal to the estimated cost of waivers of violations, $\eta_w^b = \eta_w^m = 0.700$.

When the parties can waive a misallocation of control rights in another state, they find it cheaper to misallocate control rights overall. As scenario 3 in table 11 suggests, as a result, the amendment option becomes less valuable: the ex ante contracting efficiency gain drops by 1 percentage point, and conditional on an amendment in the state b – exactly when the state suggests the bank might need to obtain control in the future, – by about 4 percentage points. The frequency of amendments does not change.

5 How does the amendment behavior and efficiency gain vary with borrowers' and loans' characteristics?

In this section, I evaluate how the economic parameters and efficiency gains from amendments vary with borrowers' and loans' characteristics. I estimate the model on subsamples of my dataset.

For subsample analysis, I identify three firms' and loans' characteristics that could affect amendment behavior: the quality of contractible information, the importance of non-contractible information, and renegotiation cost. As a proxy for the quality of borrowers' contractible information, I choose the earnings response coefficient (Lang and McNichols (1990), Imhoff and Lobo (1992)). Prior researchers have used this measure to proxy for the amount of relevant financial performance information in the companies' accounting numbers. Intuitively, if stock market investors react more strongly to earnings releases, earnings should contain less noise and more useful information. Even though earnings reported to stock market investors are rarely used in loan contracts (Dyreng et al. (2017)), measures of earnings in loan agreements and for the stock market have many components in common. I expect the quality of stock market earnings to be correlated with the quality of earnings used in loan contracts.

To measure the importance of non-contractible information in lender-borrower pairs, I use indicators for relationship loans. In contrast to the stock market investors, banks invest heavily in monitoring (Diamond (1984), Winton (1995)) and screening (Ramakrishnan and Thakor (1984), Allen (1990)) their borrowers, as a result obtaining relevant, often soft information about the company that is not available to outsiders (Bhattacharya and Chiesa (1995), Yosha (1995)). I assume that if a lead lender already issued a loan to this borrower, the lender must know more specific non-contractible information and understand how to use it to predict the borrower's future performance. Thus, for relationship loans, the degree of contractual incompleteness may be higher as more soft information is used to regulate the lender-borrower relationship.

The characteristic related to the renegotiation cost is the number of lenders in a syndicate. Literature on syndicate lending posits that one of the factors affecting how loan agreements are structured is the cost of coordination among investors (Becker and Ivashina (2016)). Coordination costs are higher for more dispersed and larger syndicates. If it is difficult to get all decision-making investors together and make them reach an agreement, the contracts will be written to reduce the need to coordinate in the first place, and if there is a need to renegotiate, this action will cost a high price. In my framework, high renegotiation costs can imply both high amendment costs and high waiver friction. The empirical association between syndicate size and the likelihood of amendments will be determined by which of the costs dominates. If amendment costs are much higher, but waiver frictions do not differ for larger syndicates, we would observe fewer amendments for loans with more lenders, and vice versa. There might be no association between coordination cost and amendment probability if greater amendment cost is offset by greater waiver frictions.

I construct the three variables as follows. The earnings response coefficient is estimated from regressing abnormal returns on earnings surprises for eight quarterly earnings announcements preceding the loan initiation date. Abnormal returns are computed using the market model. An indicator for relationship loans is set to one if at least one of the lead arrangers issued at least one loan to this borrower in the past. The number of lenders in a lending syndicate is directly taken from the Dealscan database.

Table 12 shows summary statistics for the proxies. Most companies in my sample have positive responses to their earnings surprises, with a median company increasing abnormal returns by 0.310 per $1 of an earnings surprise. About 23% of the loans in my sample are issued to borrowers who have already had some interaction with lenders. An average syndicate consists of 9 lenders, but sizes vary considerably.

The estimated parameters, the likelihood of amendments, and contractual efficiency gains for different subsamples are presented in table 13. For companies whose earnings are perceived by the market as less informative, financial performance information used in loan contracts also seems to be of low quality, resulting in a larger gain these firms obtain when they can amend their loan contracts. The aggregate financial covenants' error for firms with low ERC is about 45%, almost 30 percentage points higher than for firms with high ERC. Because low-earnings-quality borrowers' contractible information is so imprecise, they amend covenant schedules more than twice more often than their high-earnings-quality peers and obtain an ex ante 2.188% contracting efficiency gain due to the amendment option. In addition to having more precise contractible information, high-ERC companies are healthier and more stable: the initial probability that a high-ERC (low-ERC) company will not need a lender's intervention is 72%

(59%), and conditional on no lender intervention in the first year, 82% (59%). Firms with better financial reporting quality also face lower renegotiation frictions than their lower-quality peers, consistent with more precise accounting numbers attracting alternative capital sources and thus improving corporate borrowers' bargaining position.

Relationship and non-relationship loans differ in the degree to which contractible information can capture the actual state of the world and in negotiation frictions that contracting parties face. First, when lenders have had a prior relationship with the borrower, contractible signals capture the borrower's non-contractible state worse. Second, in relationship loans, the costs of both amendments and waivers are higher than in non-relationship loans. This difference may be due to a hold-up problem when lenders have acquired enough information about the borrower that can not be transferred to other parties, reducing the bargaining power of the borrower.

Syndicates with more lenders seem to face higher waiver frictions but not amendment costs. Because harder waivers dominate, larger syndicates are slightly more likely to amend their contracts. In addition, loans by larger syndicates seem to use more precise contractible information, perhaps because they are typically provided to larger and more established borrowers.

6 Conclusion

This study attempts to understand why financial covenants in corporate loan contracts are amended. The option to adjust a loan agreement as contracting parties learn new information allows lenders and borrowers to (partially) dynamically complete the inherently incomplete debt contract. I show that contracting efficiency, and thus incentives to amend, are more significant for companies with less precise contractible information, more important non-contractible information, and when amending the contract in advance is considerably cheaper than resolving control rights misallocation ex post. In the U.S., the first and the last explanations seem true.

The paper is only one of the first steps toward sufficiently understanding why and how corporations and their investors change the contracts regulating their relationship. Many dimensions of contract renegotiations remain completely opaque or understudied. For example, in this paper, I treat companies' information systems as fixed, while in reality, they are not static. Properties of firms' hard and soft information and contracting parties' learning what this information represents evolve over the life of the lender-borrower relationship, which can potentially affect contract structure.

A broader limitation of my framework is its focus solely on financial covenants. Even though financial covenants are the most common feature that gets amended, this is not the only term determining control rights,[16] nor are control rights the sole dimension that loan contracts regulate. Other features are also not static and get amended before contracts mature (Roberts (2015)). Studying why these are changed and why sometimes the parties amend multiple features at a time can uncover new trade-offs that lenders and borrowers face.

Overall, corporate loan contract amendments are an intriguing phenomenon inviting a lot of future research.

[16] Control rights in loan agreements can be regulated by other "action" covenants, such as negative pledge clauses or payout covenants, cov-lite financial covenants (Bräuning et al. (2022)), sweep provisions, collateral, and other terms.

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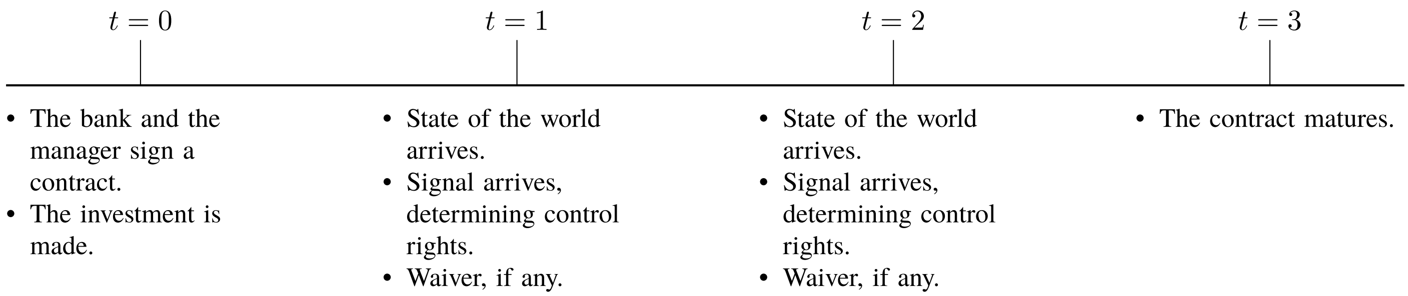
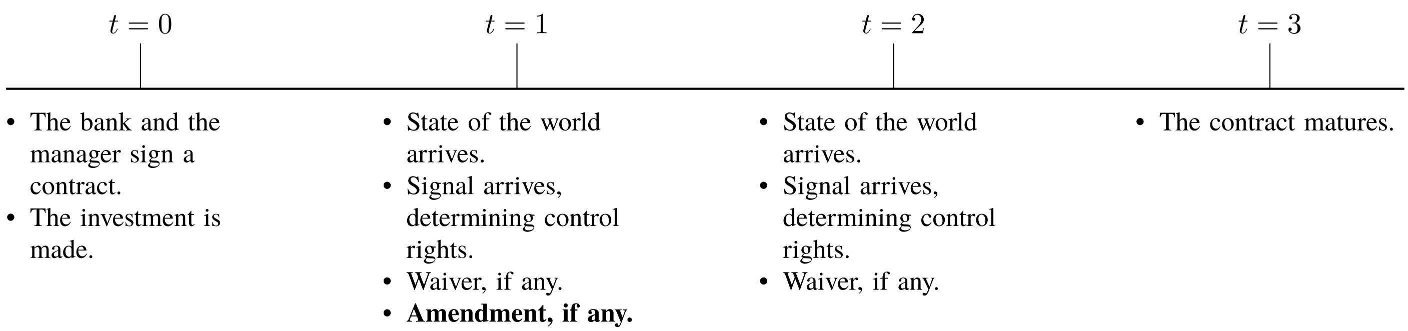
Figures and tables

Figure 1: Timelines in the classic and adapted debt contracting frameworks

(a) Classic framework



(b) Adapted dynamic framework



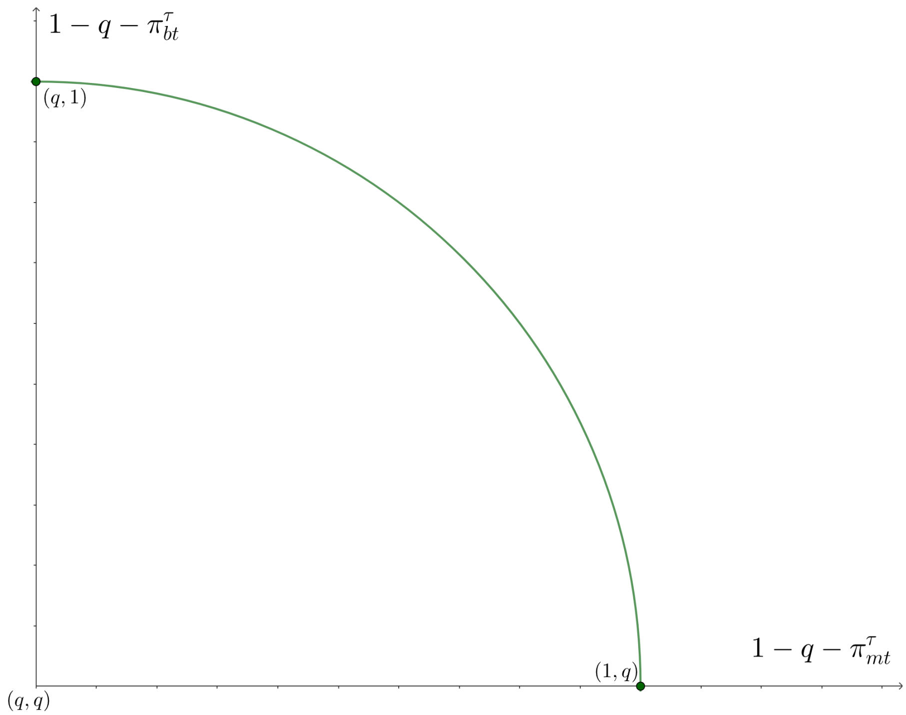


Figure 2: Possibility frontier for the precision of the covenants in states m and b. The precision in the state m is on the horizontal axis, the precision in the state b is on the vertical axis. $q = 0.8$.

Figure 3: Timeline of events in the model

$t = 0$

- The initial contract, $\{\{\pi_{m1}^0, \pi_{b1}^0\}, \{\pi_{m2}^0, \pi_{b2}^0\}\}$, is chosen.

$t = 1$

- The manager and the bank observe $\{r_1, s_1, c_1\}$.
- Waiver, if any, takes place.
- Amendment of the contract to a new one, $\{\pi_{m2}^1, \pi_{b2}^1\}$, if any, takes place.

$t = 2$

- The manager and the bank observe $\{r_2, s_2, c_2\}$.
- Waiver, if any, takes place.

$t = 3$

- The contract matures.

Figure 4: Saved cost of control rights misallocation as a function of model parameters. $q = 0.6, \eta_a = 0.02, \eta_w^m = 0.95, \eta_w^b = 1000, \bar{c} = 1, \mu = 0.65, \beta = 0.6, \rho = 0.9, \mu_0 = 0.6$.

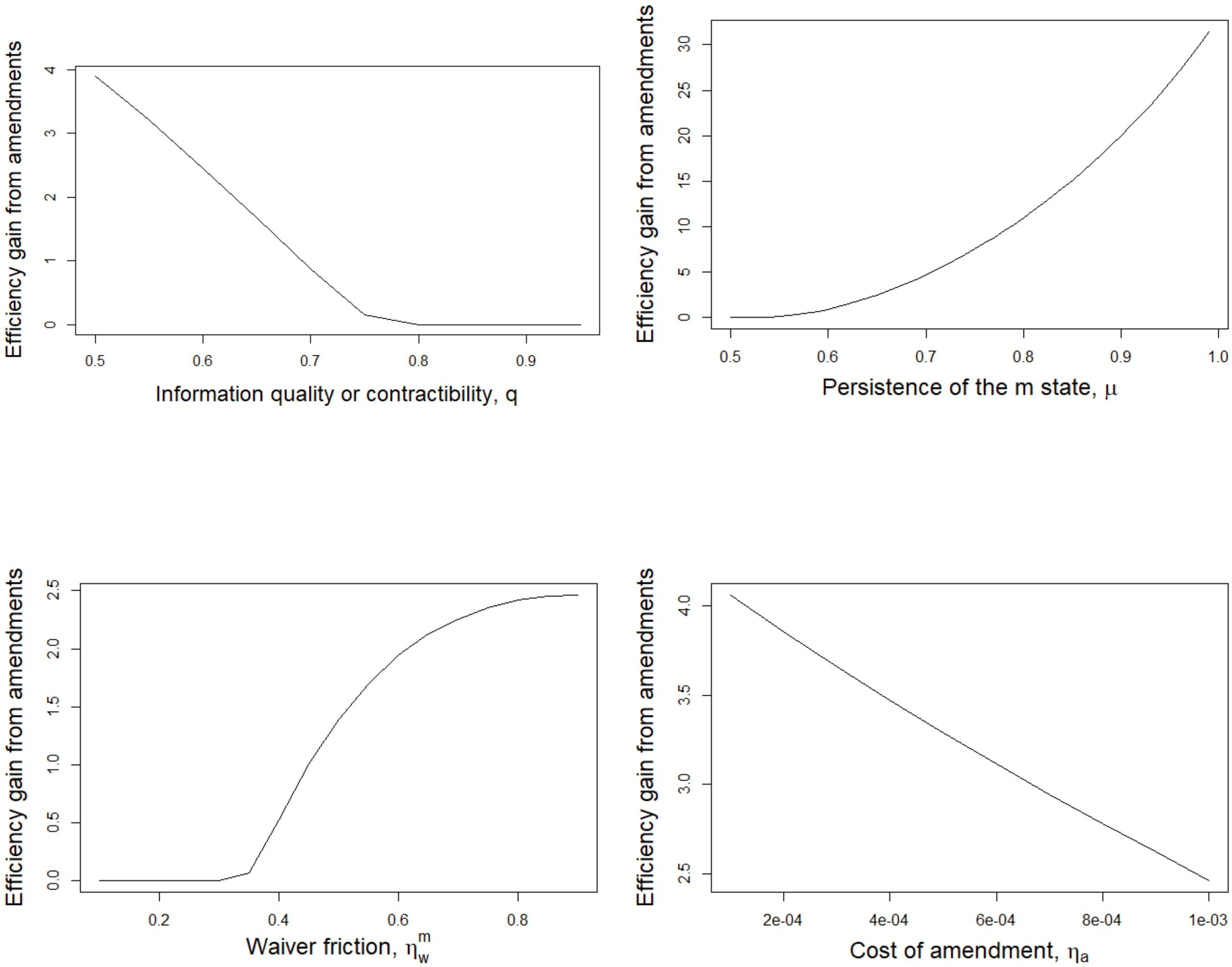
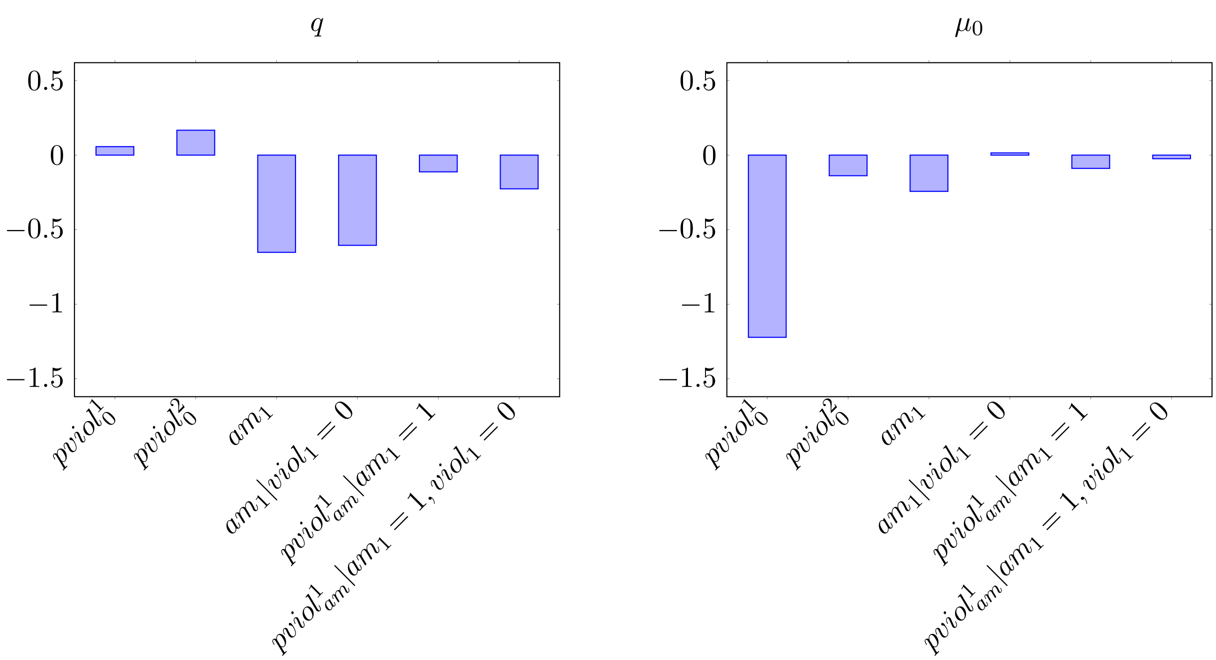
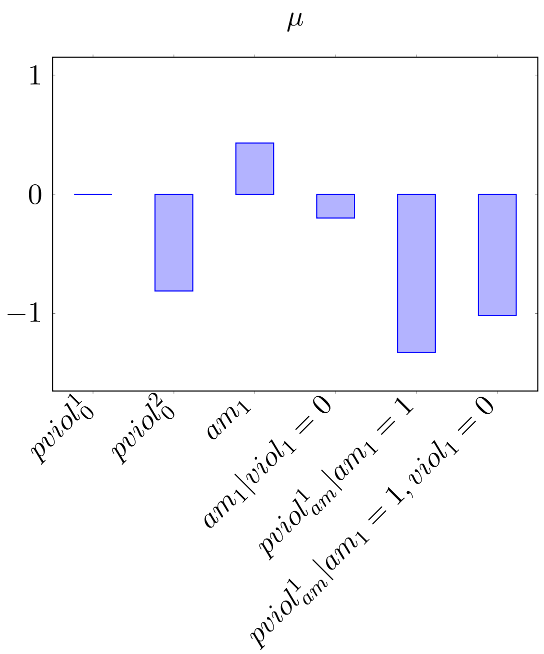


Figure 5: Sensitivities of moments to the model's parameters, at the estimated parameters. $pviol_0^1$ is the strictness of initial financial covenants for the first year after loan origination. $pviol_0^2$ is the strictness of initial financial covenants for the second year after loan origination. am_1 is the probability of amending a covenant within the first year after loan origination. $am_1|viol_1 = 0$ is the probability of amending a covenant within the first year after loan origination, conditional on no violation within the first year after origination. $pviol_{am}^1|am_1 = 1$ is the strictness of amended financial covenants for the first year after an amendment. $pviol_{am}^1|am_1 = 1, viol_1 = 0$ is the strictness of amended financial covenants for the first year after an amendment, conditional on no violation within the first year after origination. The sensitivity of a moment with respect to a parameter is computed by varying the parameter 1% up and down (keeping other parameters constant) and dividing the difference between the new value of the moment at the 1% higher parameter and the new value of the moment at the 1% lower parameter by 2% of the parameter value.

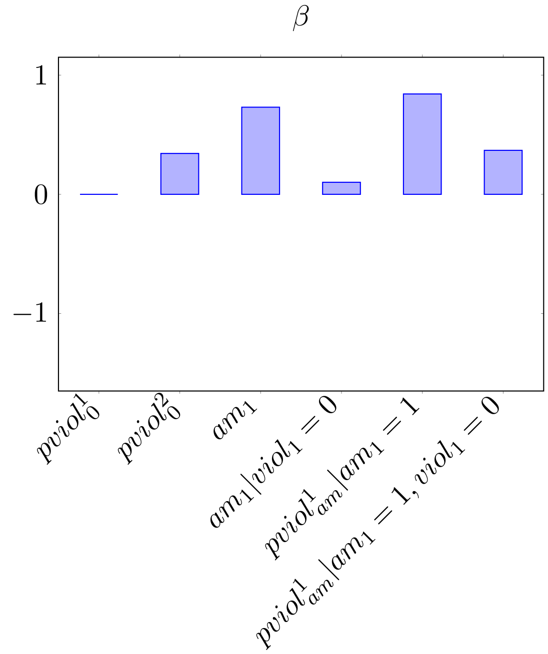


(a) Sensitivities of the model parameters to the precision of contractible information, q.

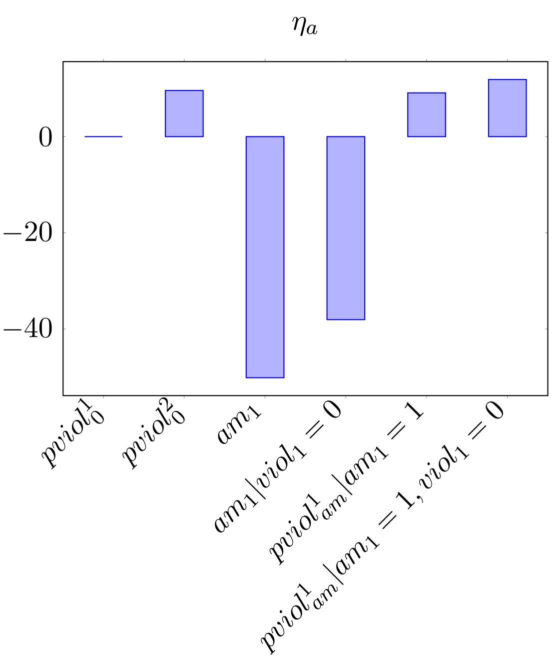
(b) Sensitivities of the model parameters to the initial probability of the manager being the optimal party-in-control, μ_0.



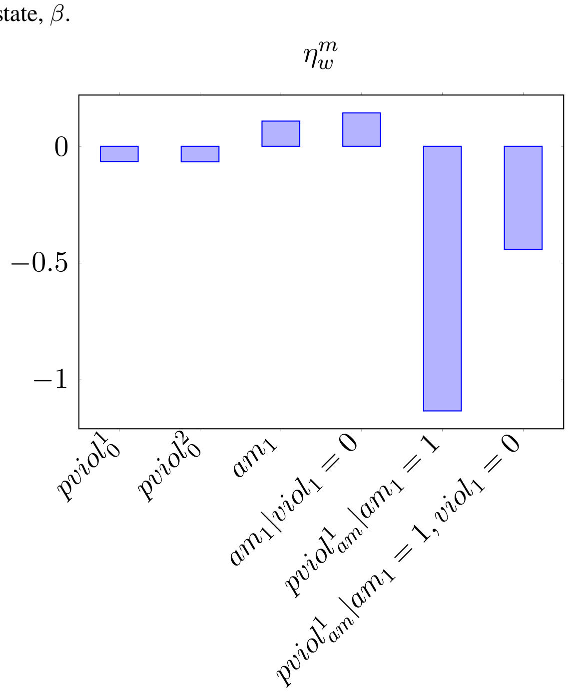
(c) Sensitivities of the model parameters to the persistence of the m state, μ.



(d) Sensitivities of the model parameters to the persistence of the b state, β.



(e) Sensitivities of the model parameters to the amendment cost, η_a.



(f) Sensitivities of the model parameters to the waiver cost, η_w^m.

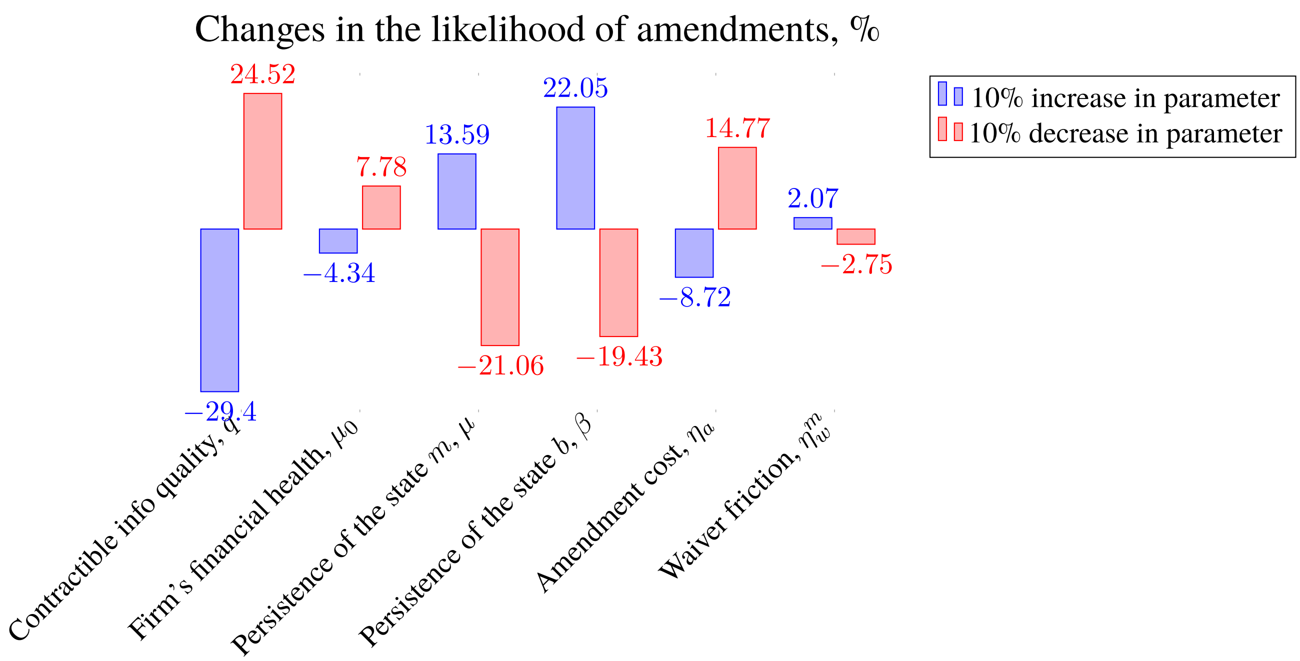


Figure 6: Sensitivity of the likelihood of amendments to model parameters. The values of parameters are as estimated (see Table 9).

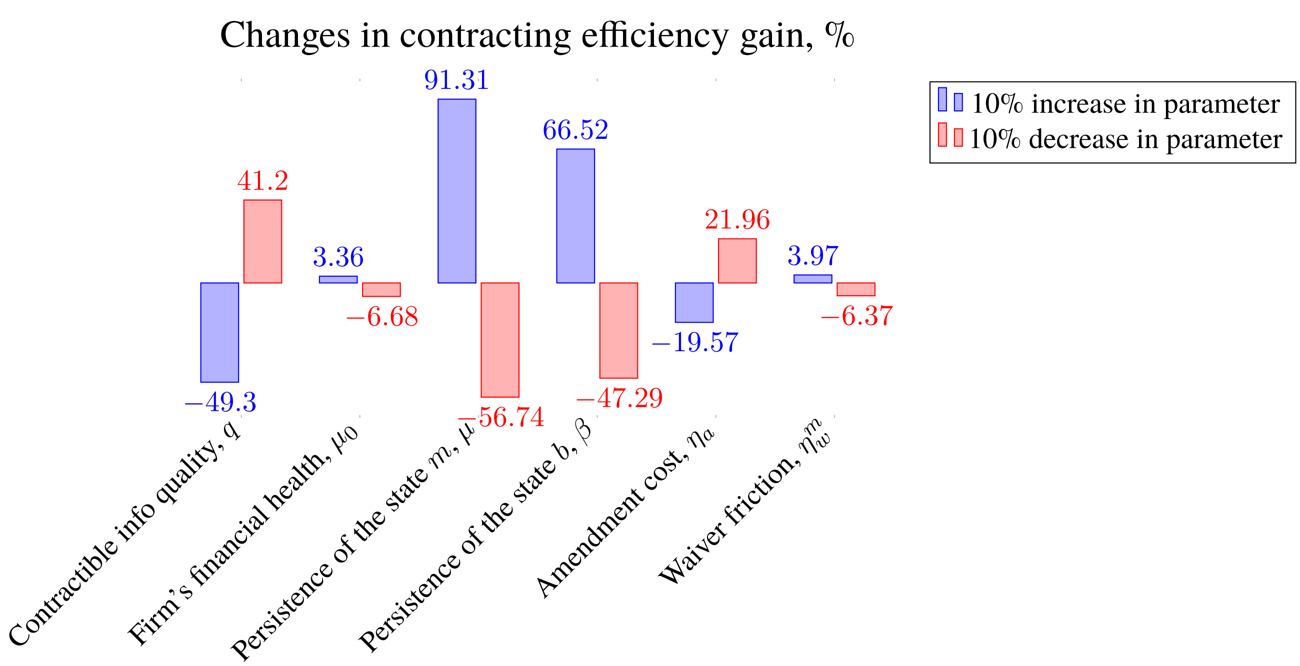


Figure 7: Sensitivity of contracting efficiency gain to model parameters. The values of parameters are as estimated (see Table 9).

Table 1: Summary statistics: companies' characteristics in my sample and in the Compustat universe. For the Compustat data, I truncate all the variables at 1% from above and below and leave only observations that have no missing data on any of the variables. The market-to-book ratio is the close price multiplied by the number of common shares outstanding and divided by the total value of common/ordinary equity. The return on assets is income before extraordinary items divided by total assets. The leverage ratio is the sum of the total long-term debt and the total debt in current liabilities, divided by the sum of the total long-term debt, the total debt in current liabilities, and the total value of common/ordinary equity. The debt-to-EBITDA ratio is the sum of the total long-term debt and the total debt in current liabilities, divided by the operating income before depreciation.

	This paper's data						Compustat data					
	N	Mean	SD	P25	P50	P75	N	Mean	SD	P25	P50	P75
Total assets ($mil)	1,003	3,496.74	9,108.32	401.01	1,049.30	2,803.15	208,668	3,899.90	15,353.75	37.64	236.41	1,365.63
Market-to-book ratio	952	3.42	11.25	1.43	2.14	3.20	208,668	2.69	5.74	0.96	1.71	3.23
Return on assets	1,002	0.03	0.09	0.01	0.04	0.07	208,668	-0.19	0.97	-0.10	0.01	0.05
Leverage ratio	1,003	0.50	0.30	0.30	0.48	0.65	208,668	0.31	0.41	0.01	0.26	0.52
Debt-to-EBITDA ratio	998	3.34	3.82	1.28	2.66	4.34	208,668	1.97	5.10	0.00	0.84	3.20

Table 2: Summary statistics: loans' characteristics in my sample and in the Dealscan universe.

	This paper's data						Dealscan data					
	N	Mean	SD	P25	P50	P75	N	Mean	SD	P25	P50	P75
Loan amount ($mil)	956	365.74	599.38	75.00	200.00	400.00	332,110	5,076.11	208,182.05	40.00	130.00	570.00
Maturity (months)	940	52.52	21.23	36.00	59.00	60.00	302,546	58.52	47.23	35.00	59.00	71.00
All-in-drawn spread (bps)	904	226.77	167.06	125.00	200.00	300.00	179,894	242.50	176.97	112.50	220.00	325.00
Secured	957	0.62	0.49	0.00	1.00	1.00	332,786	0.35	0.48	0.00	0.00	1.00
Number of lenders	957	9.09	7.87	4.00	7.00	12.00	332,786	5.62	6.25	2.00	4.00	7.00

Table 3: Summary statistics: types of loans in my sample and in the Dealscan universe.

Loan type	% in this paper's sample	% in Dealscan data
Revolver/Line >= 1 year	50.797	30.422
Term Loan	29.709	35.860
Other	19.494	33.718

Table 4: Average number of loans per borrower in a given year

Year	Number of deals an average borrower has	Number of tranches an average borrower has
1994	1.00	1.00
1995	1.00	1.33
1996	1.19	1.94
1997	1.45	2.15
1998	1.84	2.92
1999	2.01	3.37
2000	2.04	3.35
2001	2.11	3.42
2002	2.00	3.11
2003	2.05	3.10
2004	1.98	2.96
2005	2.11	3.17
2006	2.08	2.84
2007	1.99	2.77
2008	1.71	2.36
2009	1.60	2.06
2010	1.58	2.24
2011	1.59	2.30
2012	1.67	2.54
2013	1.91	3.06
2014	1.91	3.11
2015	1.84	2.95
2016	1.95	2.98
2017	1.78	2.83
2018	1.83	2.98
2019	1.70	2.70
2020	1.69	2.59
2021	1.81	2.58
2022	1.63	2.50

Table 5: Summary statistics: financial covenants. The definitions of the variables are provided in Appendix 7.10.

Variable	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Probability of violating a covenant during the 1st year after origination	0.369	0.355	0.042	0.245	0.692
Probability of violating a covenant during the 2nd year after origination	0.366	0.356	0.035	0.237	0.682
Probability of violating a covenant within the 1st year after an amendment	0.549	0.366	0.174	0.718	0.875
Change in covenant strictness after an amendment, %	NA	NA	-20.067	2.340	60.056

Table 6: Summary statistics: financial covenants amendments. The definitions of the variables are provided in Appendix 7.10.

Event	Frequency
Amend a covenant at least once	0.289
of them not in violation	0.426
Amend a covenant within the 1st year	0.203
of them not in violation	0.420

Table 7: Correlation matrix of variables used in the estimation. The definitions of the variables are provided in Appendix 7.10.

	Probability of violating a covenant during the 1^{st} year after origination	Probability of violating a covenant during the 2^{nd} year after origination	A financial covenant got amended during the 1^{st} year after origination	A financial covenant was violated during the 1^{st} year after origination
Probability of violating a covenant during the 1^{st} year after origination	1	0.926	0.097	0.394
Probability of violating a covenant during the 2^{nd} year after origination	0.926	1	0.111	0.390
A financial covenant got amended during the 1^{st} year after origination	0.097	0.111	1	0.241
A financial covenant was violated during the 1^{st} year after origination	0.394	0.390	0.241	1

Table 8: Correlation of variables used in the estimation with firms' and loans' characteristics. The definitions of the variables used in the estimation are provided in Appendix 7.10. The market-to-book ratio is the close price multiplied by the number of common shares outstanding and divided by the total value of common/ordinary equity. The return on assets is income before extraordinary items divided by total assets. The leverage ratio is the sum of the total long-term debt and the total debt in current liabilities, divided by the sum of the total long-term debt, the total debt in current liabilities, and the total value of common/ordinary equity. The debt-to-EBITDA ratio is the sum of the total long-term debt and the total debt in current liabilities, divided by the operating income before depreciation.

	Probability of violating a covenant during the 1st year after origination	Probability of violating a covenant during the 2nd year after origination	A financial covenant got amended during the 1st year after origination	A financial covenant was violated during the 1st year after origination	Change in covenant strictness after the amendment
Total assets	-0.063	-0.070	-0.050	-0.104	0.020
Market-to-book ratio	-0.054	-0.057	0.089	-0.040	-0.040
Return on assets	-0.238	-0.188	-0.122	-0.071	0.148
Leverage ratio	0.182	0.250	0.020	0.029	-0.316
Debt-to-EBITDA ratio	0.335	0.371	-0.003	0.128	-0.344
Loan amount	-0.180	-0.167	-0.072	-0.159	-0.150
Maturity	-0.047	-0.031	-0.063	-0.061	0.170
All-in-drawn spread	0.110	0.113	0.075	0.003	-0.052
Secured	0.149	0.169	0.032	0.114	0.083
Number of lenders	-0.031	0.001	-0.034	0.028	-0.108

Table 9: Parameter estimates for the full sample

Parameter	Estimate
Contractible information precision, q	0.687 (0.002)
Initial probability of the state m, μ_0	0.614 (0.000)
Persistence of the state m, μ	0.654 (0.001)
Persistence of the state b, β	0.501 (0.001)
Amendment cost, η_a	0.009 (0.000)
Waiver friction, η_w	0.700 (0.001)
Contracting efficiency gain, %	2.323 (0.033)

Standard errors are in parentheses. In estimation, the parameters are set $\bar{c} = 1$, $\rho = 0.9$.

Table 10: Model fit: theoretical and empirical moments. Calculations of the moments are presented in Appendix 7.11.1.

Moment	Theoretical value	Empirical value	t-statistic [p-value]
Strictness of initial financial covenants for the first year after loan origination	0.368	0.370	−0.170 [0.865]
Strictness of initial financial covenants for the second year after loan origination	0.366	0.367	−0.088 [0.930]
Probability of amending a covenant within the first year after loan origination	0.212	0.194	1.475 [0.141]
Probability of amending a covenant within the first year after loan origination, conditional on no violation within the first year after origination	0.124	0.125	−0.015 [0.988]
Strictness of amended financial covenants for the first year after an amendment	0.547	0.558	−0.722 [0.471]
Strictness of amended financial covenants for the first year after an amendment, conditional on no violation within the first year after origination	0.501	0.487	0.611 [0.543]

Standard errors are in parentheses. In estimation, the parameters are set $\bar{c} = 1$, $\rho = 0.9$.

Table 11: Likelihood of amendments and contracting efficiency gain from amendments in counterfactual scenarios

Scenario	Likelihood of amendments, %	Contracting efficiency gain, %	Contracting efficiency gain in state m conditional on amendments, %	Contracting efficiency gain in state b conditional on amendments, %
Current estimates	21.303	2.323	15.383	12.702
1. A completely financially healthy company, $\mu_0 = 0.999$.	12.532	1.393	17.067	15.149
2. A completely financially unhealthy company, $\mu_0 = 0.001$.	15.856	1.434	14.581	17.843
3. An economy where waivers without violations are allowed, $\eta_w^b = \eta_w^m = 0.700$.	21.833	1.363	14.365	8.772

Table 12: Summary statistics for accounting quality, relationship loans, and syndicate size

Statistic	N	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Earnings response coefficient[17]	562	0.704	1.218	0.082	0.704	0.953
Relationship loan	1,067	0.228	0.420	0	0	0
Number of lenders	957	9.088	7.867	4	7	12

Table 13: Parameter estimates for different data subsamples

Parameter	Quality of contractible info		Importance of non-contractible info		Renegotiation cost	
	ERC above median	ERC below median	Relationship loan	Non-relationship loan	Number of lenders above the 3rd quartile	Number of lenders below the 1st quartile
Contractible information precision, q	0.861	0.548	0.507	0.666	0.623	0.536
	(0.000)	(0.009)	(0.067)	(0.001)	(0.003)	(0.003)
Initial probability of the state m, μ_0	0.722	0.594	0.607	0.603	0.619	0.610
	(0.001)	(0.001)	(0.011)	(0.000)	(0.000)	(0.001)
Persistence of the state m, μ	0.820	0.592	0.650	0.637	0.641	0.648
	(0.000)	(0.002)	(0.001)	(0.000)	(0.000)	(0.000)
Persistence of the state b, β	0.500	0.500	0.500	0.502	0.500	0.500
	(0.012)	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)
Amendment cost, η_a	0.0003	0.009	0.016	0.011	0.012	0.014
	(0.000)	(0.000)	(0.005)	(0.000)	(0.000)	(0.000)
Waiver friction, η_w	0.024	0.590	0.991	0.774	0.743	0.522
	(0.005)	(0.007)	(3.115)	(0.003)	(0.001)	(0.009)
Amendment frequency, %	11.021	26.368	19.564	19.307	20.488	19.157
Efficiency gain, %	0.063	2.188	1.812	1.590	1.858	1.725

Standard errors are in parentheses. In estimation, the parameters are set $\bar{c} = 1$, $\rho = 0.9$.

7 Appendix

7.1 Proof of Propositions 2 and 3

At time 2, if $r_2 = m$ ($r_2 = b$) and $s_2 = b$ ($s_2 = m$), the parties waive and m (b) regains the $t = 2$ control rights iff $\eta_w^b \leq c_2$ ($\eta_w^m \leq c_2$).

At time 1, if $r_1 = m$ ($r_1 = b$) and $s_1 = b$ ($s_1 = m$), the parties waive and m (b) regains the $t = 1$ control rights iff $\eta_w^b \leq c_2$ ($\eta_w^m \leq c_2$). To determine whether the covenant schedule is amended, first consider the optimal covenant schedule $\{\pi_{m2}^1, \pi_{b2}^1\}$ had the parties amend. When $r_1 = m$, the problem is

$$\begin{aligned} \min_{\{\pi_{m2}^1, \pi_{b2}^1\}} \quad & \mu f_m^m \pi_{m2}^1 + (1-\mu) e^b \pi_{b2}^1 \\ \text{s.t.} \quad (1 - \pi_{m2}^1 - q)^2 + (1 - \pi_{b2}^1 - q)^2 &= (1-q)^2 \\ \pi_{mt}^\tau &\in [0, 1-q] \\ \pi_{bt}^\tau &\in [0, 1-q] \end{aligned}$$

where

$$\begin{aligned} f_m^m &= E[\min\{c_2, \eta_w^m\} | c_1, r_2 = r_1 = m] \\ e^b = E[\min\{c_2, \eta_w^b\} | c_1, r_2 = b \neq r_1] &= E[\min\{c_2, \eta_w^b\}] \end{aligned}$$

f_m^m is a random variable with a pdf $h_m(f)$ and a cdf $H_m(f)$ and the support $[0, \eta_w^m]$, and e^b is a constant. Lagrangian for the problem with only the first constraint is

$$\mathcal{L} = \mu e^b \pi_{m2}^1 + (1-\mu) f_m^m \pi_{b2}^1 + \lambda \left((1-q)^2 - (1 - \pi_{m2}^1 - q)^2 - (1 - \pi_{b2}^1 - q)^2 \right)$$

The derivatives with respect to π_{m2}^1, π_{b2}^1, and λ are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \pi_{m2}^1} &= \mu e^b + 2\lambda (1 - \pi_{m2}^1 - q) \\ \frac{\partial \mathcal{L}}{\partial \pi_{b2}^1} &= (1-\mu) f_m^m + 2\lambda (1 - \pi_{b2}^1 - q) \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= (1-q)^2 - (1 - \pi_{m2}^1 - q)^2 - (1 - \pi_{b2}^1 - q)^2 \end{aligned}$$

Equating the derivatives above to zero and solving the system yields:

$$\pi_{m2}^{1m*} = 1 - q - \frac{\mu f_m^m (1-q)}{\sqrt{(\mu f_m^m)^2 + ((1-\mu)e^b)^2}}$$
$$\pi_{b2}^{1m*} = 1 - q - \frac{(1-\mu)e^b (1-q)}{\sqrt{(\mu f_m^m)^2 + ((1-\mu)e^b)^2}}$$

The determinant of the matrix of second derivatives of the Largangian is

$$\begin{vmatrix} -2\lambda & 0 \\ 0 & -2\lambda \end{vmatrix} = 4\lambda^2 > 0,$$

suggesting that the solution found is a minimum.

The parties will amend the contract iff

$$\begin{aligned} \eta_a + \mu f_m^m \pi_{m2}^{1*} + (1-\mu)e^b \pi_{b2}^{1*} &\leq \mu f_m^m \pi_{m2}^0 + (1-\mu)e^b \pi_{b2}^0 \\ \eta_a + (1-q)(\mu f_m^m + (1-\mu)e^b) - (1-q)\sqrt{(\mu f_m^m)^2 + ((1-\mu)e^b)^2} &\leq \mu f_m^m \pi_{m2}^0 + (1-\mu)e^b \pi_{b2}^0 \\ (f_m^m)^2 \left[\mu^2((1-q)^2 - (1-q-\pi_{m2}^0)^2)\right] - f_m^m \left[2(1-q-\pi_{m2}^0)\mu(\eta_a + (1-q-\pi_{b2}^0)(1-\mu)e^b)\right] & \\ + \left((1-q)(1-\mu)e^b\right)^2 - (\eta_a + (1-q-\pi_{b2}^0)(1-\mu)e^b)^2 &\geq 0 \end{aligned}$$

The quadratic expression of f_m^m on the left-hand side has two roots. Therefore, the parties will not amend the contract iff $f_m^m \in (x_m^l, x_m^u)$, and amend otherwise, where

$$x_m^l = \frac{(1-q-\pi_{m2}^0)(\eta_a + (1-q-\pi_{b2}^0)(1-\mu)e^b) - (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b}}{\mu((1-q)^2 - (1-q-\pi_{m2}^0)^2)}$$
$$x_m^u = \frac{(1-q-\pi_{m2}^0)(\eta_a + (1-q-\pi_{b2}^0)(1-\mu)e^b) + (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b}}{\mu((1-q)^2 - (1-q-\pi_{m2}^0)^2)}$$

When $r_1 = b$,

$$\pi_{m2}^{1b*} = 1 - q - \frac{(1-\beta)e^m (1-q)}{\sqrt{\left(\beta f_b^b\right)^2 + ((1-\beta)e^m)^2}}$$
$$\pi_{b2}^{1b*} = 1 - q - \frac{\beta f_b^b (1-q)}{\sqrt{\left(\beta f_b^b\right)^2 + ((1-\beta)e^m)^2}}$$

and the parties will amend the covenant schedule iff $f_b^b \in (x_b^l, x_b^u)$, where

$$x_b^l = \frac{(1-q-\pi_{b2}^0)(\eta_a + (1-q-\pi_{m2}^0)(1-\beta)e^m) - (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{m2}^0)(1-\beta)e^m}}{\beta((1-q)^2 - (1-q-\pi_{b2}^0)^2)}$$
$$x_b^u = \frac{(1-q-\pi_{b2}^0)(\eta_a + (1-q-\pi_{m2}^0)(1-\beta)e^m) + (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{m2}^0)(1-\beta)e^m}}{\beta((1-q)^2 - (1-q-\pi_{b2}^0)^2)}$$

7.2 Problem of finding the optimal contract at time 0 in the main model

At time 0, the problem is

$$\min_{\{\pi_{m1}^0,\pi_{b1}^0,\pi_{m2}^0,\pi_{b2}^0\}} \quad \mu_0 \times \Big(\pi_{m1}^0 e^m$$
$$+E\left[\min\{\mu f_m^m \pi_{m2}^0 + (1-\mu)e^b\pi_{b2}^0, \eta_a + (1-q)(\mu f_m^m + (1-\mu)e^b) - (1-q)\sqrt{(\mu f_m^m)^2 + ((1-\mu)e^b)^2}\}\right]\Big)$$
$$+(1-\mu_0) \times \Big(\pi_{b1}^0 e^b$$
$$+E\left[\min\{\beta f_b^b \pi_{b2}^0 + (1-\beta)e^m\pi_{m2}^0, \eta_a + (1-q)(\beta f_b^b + (1-\beta)e^m) - (1-q)\sqrt{\left(\beta f_b^b\right)^2 + ((1-\beta)e^m)^2}\}\right]\Big)$$
$$\text{s.t.} \quad (1-\pi_{m1}^0-q)^2 + (1-\pi_{b1}^0-q)^2 = (1-q)^2$$
$$(1-\pi_{m2}^0-q)^2 + (1-\pi_{b2}^0-q)^2 = (1-q)^2$$

The problems of finding $\{\pi_{m1}^0, \pi_{b1}^0\}$ and $\{\pi_{m2}^0, \pi_{b2}^0\}$ are separable. First, we can find $\{\pi_{m1}^0, \pi_{b1}^0\}$:

$$\pi_{m1}^{0*} = 1-q-\frac{(1-q)\mu_0 e^m}{\sqrt{(\mu_0 e^m)^2 + ((1-\mu_0)e^b)^2}}$$
$$\pi_{b1}^{0*} = 1-q-\frac{(1-q)(1-\mu_0)e^b}{\sqrt{(\mu_0 e^m)^2 + ((1-\mu_0)e^b)^2}}$$

$\{\pi_{m2}^0, \pi_{b2}^0\}$ are computed numerically.

7.3 Proof of Proposition 1

In the model where amendments are prohibited, the problem at time 0 is

$$\min_{\{\pi^0_{m1},\pi^0_{b1},\pi^0_{m2},\pi^0_{b2}\}} \quad \mu_0\left(\pi^0_{m1}e^m + \mu\pi^0_{m2}E[f^m_m] + (1-\mu)\pi^0_{b2}e^b\right) + (1-\mu_0)\left(\pi^0_{b1}e^b + \beta\pi^0_{b2}E[f^b_b] + (1-\beta)\pi^0_{m2}e^b\right)$$

$$\text{s.t.} \quad (1-\pi^0_{m1}-q)^2 + (1-\pi^0_{b1}-q)^2 = (1-q)^2$$

$$(1-\pi^0_{m2}-q)^2 + (1-\pi^0_{b2}-q)^2 = (1-q)^2$$

π^{0*}_{m1} and π^{0*}_{b1} are the same as in the main model. The optimal π^0_{m2} and π^0_{b2} are

$$\pi^{0*}_{m2} = 1-q-\frac{(1-q)(\mu_0\mu E[f^m_m] + (1-\mu_0)(1-\beta)e^m)}{\sqrt{(\mu_0\mu E[f^m_m] + (1-\mu_0)(1-\beta)e^m)^2 + (\mu_0(1-\mu)e^b + (1-\mu_0)\beta E[f^b_b])^2}}$$

$$\pi^{0*}_{b2} = 1-q-\frac{(1-q)(\mu_0(1-\mu)e^b + (1-\mu_0)\beta E[f^b_b])}{\sqrt{(\mu_0\mu E[f^m_m] + (1-\mu_0)(1-\beta)e^m)^2 + (\mu_0(1-\mu)e^b + (1-\mu_0)\beta E[f^b_b])^2}}$$

7.4 Proof of Corollary 1

In this section, I present proofs for the case $r_1 = m$. The case $r_1 = b$ can be proven analogously.

$$\frac{\partial x^l_m}{\partial \eta_a} \propto (1-q-\pi^0_{m2}) - (1-q)\frac{\eta_a + (1-q-\pi^0_{b2})(1-\mu)e^b}{\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi^0_{b2})(1-\mu)e^b}} \propto$$

$$(1-q-\pi^0_{m2})\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi^0_{b2})(1-\mu)e^b} - (1-q)(1-q-\pi^0_{b2})(1-\mu)e^b - (1-q)\eta_a <$$

$$-\pi^0_{m2}(1-q-\pi^0_{b2})(1-\mu)e^b - \pi^0_{m2}\eta_a < 0$$

$$\frac{\partial x^u_m}{\partial \eta_a} \propto (1-q-\pi^0_{m2}) + (1-q)\frac{\eta_a + (1-q-\pi^0_{b2})(1-\mu)e^b}{\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi^0_{b2})(1-\mu)e^b}} \propto$$

$$(1-q-\pi^0_{m2})\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi^0_{b2})(1-\mu)e^b} + (1-q)(1-q-\pi^0_{b2})(1-\mu)e^b + (1-q)\eta_a > 0$$

Since x^l_m is decreasing in η_a and x^u_m is increasing in η_a, the probability of amendment ($f^m_m \leq x^l_m$ or $f^m_m \geq x^u_m$) is decreasing in η_a.

The no-amendment region is

$$x_m^u - x_m^l = \frac{2(1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b}}{\mu(1-q-\pi_{b2}^0)^2}$$

$$\frac{\partial\left(x_m^u - x_m^l\right)}{\partial\mu} \propto -\frac{2(1-q)\eta_a(1-q-\pi_{b2}^0)e^b\mu}{\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b}} - 2(1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b} \propto$$
$$-(1-q)\eta_a(1-q-\pi_{b2}^0)e^b\mu - (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b} < 0$$

The no-amendment region is decreasing in μ, thus the probability of amendment is increasing in μ.

$$\frac{\partial\left(x_m^u - x_m^l\right)}{\partial q} \propto \left(-2\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b} - \frac{(1-q)2\eta_a(1-\mu)e^b}{\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b}}\right) \times$$
$$\mu(1-q-\pi_{b2}^0)^2 + 4\mu(1-q-\pi_{b2}^0)(1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b} \propto$$
$$2\eta_a^2\mu(1-q-\pi_{b2}^0)\left[2(1-q)-(1-q-\pi_{b2}^0)\right] + 2\eta_a(1-\mu)\mu e^b(1-q-\pi_{b2}^0)^2\left[(1-q)-(1-q-\pi_{b2}^0)\right] > 0$$

The no-amendment region is increasing in q, thus the probability of amendment is decreasing in q.

To prove that the no-amendment region is decreasing in η_w^m, let us first derive the cumulative distribution function of f_m^m, $H_m(f_m^m)$.

$$f = \begin{cases} \rho c_1 + (1-\rho)\frac{1}{2}, & \text{if } c_1 < \frac{\eta_w^m-(1-\rho)}{\rho} \\ \eta_w^m - \frac{1}{2}\frac{(\eta_w^m-\rho c_1)^2}{1-\rho}, & \text{if } c_1 \in \left[\frac{\eta_w^m-(1-\rho)}{\rho}\ ,\ \frac{\eta_w^m}{\rho}\right) \\ \eta_w^m, & \text{if } c_1 \geq \frac{\eta_w^m}{\rho} \end{cases}$$

$$H_m(x) = Pr\left[f < x\right] = \begin{cases} Pr\left[\rho c_1 + (1-\rho)\frac{1}{2} < x\right], & \text{if } x < \eta_w^m - \frac{1-\rho}{2} \\ \frac{\eta_w^m-(1-\rho)}{\rho} + \frac{1-\rho}{\rho} \times Pr\left[\eta_r - \frac{1}{2}\frac{(\eta_w^m-\rho c_1)^2}{1-\rho} < x\right], & \text{if } x \in \left[\eta_w^m - \frac{1-\rho}{2}\ ,\ \eta_w^m\right) \\ 1, & \text{otherwise} \end{cases}$$

$$H_m(x) = \begin{cases} \frac{x - \frac{1}{2}(1-\rho)}{\rho}, & \text{if } x < \eta_w^m - \frac{1-\rho}{2} \\ \frac{\eta_w^m - (1-\rho)}{\rho} + \frac{1-\rho}{\rho}\frac{1}{\rho}\left(\eta_w^m - \sqrt{2(1-\rho)(\eta_w - x)}\right), & \text{if } x \in \left[\eta_w^m - \frac{1-\rho}{2}\ ,\ \eta_w^m\right) \\ 1, & \text{otherwise} \end{cases}$$

If both x_m^u and x_m^l are less than $\eta_w^m - \frac{1-\rho}{2}$ or both x_m^u and x_m^l are greater than or equal to η_w^m, the probability of no amendment, $H(x_m^u) - H(x_m^l)$, does not change with η_w^m.

If both x_m^u and x_m^l are in the region $\left[\eta_w^m - \frac{1-\rho}{2}\ ,\ \eta_w^m\right)$,

$$\frac{\partial\left(H(x_m^u) - H(x_m^l)\right)}{\partial \eta_w^m} = \frac{1-\rho}{\rho^2}\left(-\frac{1-\rho}{\sqrt{2(1-\rho)(\eta_w^m - x_m^u)}} + \frac{1-\rho}{\sqrt{2(1-\rho)(\eta_w^m - x_m^l)}}\right)$$

Function $\frac{1-\rho}{\sqrt{2(1\rho)(\eta_w^m - y)}}$ is increasing in y. Since $x_m^u > x_m^l$, $\frac{\partial\left(H(x_m^u) - H(x_m^l)\right)}{\partial \eta_w^m} < 0$.

If x_m^l is in the region $\left[\eta_w^m - \frac{1-\rho}{2}\ ,\ \eta_w^m\right)$ and x_m^u is greater than or equal to η_w^m,

$$\frac{\partial\left(H(x_m^u) - H(x_m^l)\right)}{\partial \eta_w^m} = -\left(\frac{1}{\rho} + \frac{1-\rho}{\rho^2}\left(1 - \frac{1-\rho}{\sqrt{2(1-\rho)(\eta_w^m - x_m^l)}}\right)\right)$$

For $x_m^l \geq \eta_w^m - \frac{1-\rho}{2}$, $1 - \frac{1-\rho}{\sqrt{2(1-\rho)(\eta_w^m - x_m^l)}} > 0$, and thus $\frac{\partial\left(H(x_m^u) - H(x_m^l)\right)}{\partial \eta_w^m} < 0$.

The only case when $\frac{\partial\left(H(x_m^u) - H(x_m^l)\right)}{\partial \eta_w^m} > 0$ is when $\eta_w^m \in \left(\max\{x_m^u, x_m^l + \frac{1-\rho}{2}\}, x_m^u + \frac{1-\rho}{2}\right)$.

7.5 Theoretical moments

The first set of moments describes the initially set covenant schedule:

1. Ex ante probability of violating a covenant at time $t = 1$, as set at time $t = 0$:

$$\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=1})] = \\ \mu_0 \times \pi_{m1}^{0*} + (1 - \mu_0) \times (1 - \pi_{b1}^{0*}) \tag{17}$$

2. Ex ante probability of violating a covenant at time $t = 2$, as set at time $t = 0$:

$$\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=2})] = \\ \mu_0 \times \left(\mu \times \pi_{m2}^{0*} + (1 - \mu) \times (1 - \pi_{b2}^{0*})\right) + (1 - \mu_0) \times \left(\beta \times (1 - \pi_{b2}^{0*}) + (1 - \beta) \times \pi_{m2}^{0*}\right) \tag{18}$$

The second set of moments is about the frequency of amendments to the covenant schedule:

3. Ex ante unconditional probability of amending the covenant schedule:

$$\mathbf{E}_0[\mathbf{1}(\text{amendment}_{t=1})] = \\ \mu_0 \times (1 - H_m) + (1 - \mu_0) \times (1 - H_b) \tag{19}$$

4. Ex ante probability of amending the contract at time $t = 1$, conditional on no covenant violation:

$$\mathbf{E}_0[\mathbf{1}(\text{amendment}_{t=1})|\mathbf{1}(\text{violation}_{t=1}) = 0] = \\ \frac{\mu_0 \times (1 - \pi_{m1}^{0*}) \times (1 - H_m) + (1 - \mu_0) \times \pi_{b1}^{0*} \times (1 - H_b)}{\mu_0 \times (1 - \pi_{m1}^{0*}) + (1 - \mu_0) \times \pi_{b1}^{0*}} \tag{20}$$

The last set of moments captures the distribution of amended covenant schedules:

5. The amended probability of violating a covenant at time $t = 2$, conditional on amendment at time $t = 1$:

$$\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=2})|\mathbf{1}(\text{amendment}_{t=1})] = \frac{1}{\mu_0 \times (1 - H_m) + (1 - \mu_0) \times (1 - H_b)}$$
$$\times \left(\mu_0 \times (1 - H_m) \times \left(\mu \times E\left[\pi_{m2}^{1m*}|f_m^m \notin \left(x_m^l, x_m^u\right)\right] + (1 - \mu) \times \left(1 - E\left[\pi_{b2}^{1m*}|f_m^m \notin \left(x_m^l, x_m^u\right)\right]\right)\right)\right.$$
$$\left. + (1 - \mu_0) \times (1 - H_b) \times \left(\beta \times \left(1 - E\left[\pi_{b2}^{1b*}|f_b^b \notin \left(x_b^l, x_b^u\right)\right]\right) + (1 - \beta) \times E\left[\pi_{m2}^{1b*}|f_b^b \notin \left(x_b^l, x_b^u\right)\right]\right)\right) \tag{21}$$

6. The amended probability of violating a covenant at time $t = 2$, conditional on no covenant violation and amendment at time $t = 1$:

$$\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=2})|\mathbf{1}(\text{amendment}_{t=1}) \times (1 - \mathbf{1}(\text{violation}_{t=1}))] = \frac{1}{\mu_0 \times (1 - \pi_{m1}^{0*}) \times (1 - H_m) + (1 - \mu_0) \times \pi_{b1}^{0*} \times (1 - H_b)}$$
$$\times \left(\mu_0 \times (1 - H_m) \times (1 - \pi_{m1}^{0*}) \times \left(\mu \times E\left[\pi_{m2}^{1m*}|f_m^m \notin \left(x_m^l, x_m^u\right)\right] + (1 - \mu) \times \left(1 - E\left[\pi_{b2}^{1m*}|f_m^m \notin \left(x_m^l, x_m^u\right)\right]\right)\right)\right.$$
$$\left. + (1 - \mu_0) \times (1 - H_b) \times \pi_{b1}^{0*} \times \left(\beta \times \left(1 - E\left[\pi_{b2}^{1b*}|f_b^b \notin \left(x_b^l, x_b^u\right)\right]\right) + (1 - \beta) \times E\left[\pi_{m2}^{1b*}|f_b^b \notin \left(x_b^l, x_b^u\right)\right]\right)\right) \tag{22}$$

$\mathbf{1}()$ above denotes the indicator function, and H_j denotes the probability that the random variable f_j^j falls into the corresponding non-amendment region.

7.6 An example of economically different financial covenant schedules that would be recorded the same way by Dealscan

(c) Leverage Ratio. Permit the Leverage Ratio at any time during any period of four fiscal quarters of Limited set forth below to be greater than the ratio set forth opposite such period:

Four Fiscal Quarters Ending:	
November 30, 2005	4.00 to 1.00
Each fiscal quarter thereafter	3.50 to 1.00

Figure 8: The first example of a covenant that would be recorded by Dealscan as "Decreasing from 4:1 to 3.5:1". The contract effective date is November 15, 2005. The screenshot is taken from the Third Amendment to a Credit Agreement filed by Helen of Troy Limited in Exhibit 10.2 of the 10-Q on January 9, 2006.

SECTION 6.08. Maximum Consolidated Total Leverage Ratio. The Borrower will cause the Consolidated Total Leverage Ratio to be less than (a) 4.00 to 1.00 at all times during the period from the Effective Date to and including December 30, 2009, (b) 3.75 to 1.00 at all times during the period from December 31, 2009 to and including December 30, 2010 and (c) less than 3.50 to 1.00 at all times thereafter.

Figure 9: The second example of a covenant that would be recorded by Dealscan as "Decreasing from 4:1 to 3.5:1". The contract effective date is August 25, 2008. The screenshot is taken from the Amended and Restated Credit Agreement filed by The Manitowoc Company, Inc. in Exhibit 4.1 of the 10-Q on November 10, 2008.

7.7 Examples of financial covenant schedules and how they are recorded

7.6 **Financial Covenants**.

A. **Minimum Interest Coverage Ratio**. Company shall not permit the ratio of (i) Consolidated EBITDA to (ii) Consolidated Interest Expense for any four-Fiscal Quarter period to be less than 4.00:1.00.

Figure 10: An example of a static financial covenant schedule. This excerpt is taken from the Credit Agreement between Hexcel Corporation and a lending syndicate with Bank of America Securities LLC as a Lead Arranger, dated as of July 9, 2010. This financial covenant schedule will be recorded as "4:1 in 1 year, 4:1 in 2 years, 4:1 in 3 years, 4:1 in 4 years".

8.09 Consolidated Leverage Ratio.

The Borrower will not permit the Consolidated Leverage Ratio as of the end of any fiscal quarter of the Borrower ending during a period set forth below to exceed the ratio set forth below opposite such period:

Period	**Consolidated Leverage Ratio**
Effective Date through December 31, 2011	5.00:1.00
January 1, 2012 through September 30, 2012	4.50:1.00
October 1, 2012 through September 30, 2013	4.00:1.00
October 1, 2013 and thereafter	3.75:1.00

Figure 11: An example of a dynamic financial covenant schedule. This excerpt is taken from the Credit Agreement between Hanger Orthopedic Group, Inc. and a lending syndicate with Merrill Lynch, Pierce, Fenner & Smith Incorporated and Jefferies Finance LLC as Joint Lead Arrangers, dated as of December 1, 2010. This financial covenant schedule will be recorded as "5:1 in 1 year, 4:1 in 2 years, 3.75:1 in 3 years, 3.75:1 in 4 years".

(b) Fixed Charge Coverage Ratio. Permit the Fixed Charge Coverage Ratio to be less than (i) 1.75 to 1.00 as of April 30, 2003 or July 31, 2003 for the period of four fiscal quarters most recently then ended, (ii) 1.50 to 1.00 as of October 31, 2003 or January 31, 2004 for the period of four fiscal quarters most recently then ended, (iii) 1.75 to 1.00 as of April 30, 2004 for the period of four fiscal quarters most recently then ended, (iv) 1.50 to 1.00 as of July 31, 2004, October 31, 2004, or January 31, 2005 for the period of four fiscal quarters most recently then ended, or (v) 1.75 to 1.00 as of April 30, 2005 or any fiscal quarter ending thereafter, for the period of four fiscal quarters most recently then ended.

Figure 12: An example of a dynamic financial covenant schedule. This excerpt is taken from the Credit Agreement between Gerber Scientific, Inc., and a lending syndicate with Ableco Finance LLC as Agent, dated as of May 9, 2003. This financial covenant schedule will be recorded as "1.5:1 in 1 year, 1.75:1 in 2 years, 1.75:1 in 3 years, 1.75:1 in 4 years".

7.8 Types of amendments to financial covenants

(x) Section 1.10 of the Credit Agreement is hereby amended by deleting the text of such Section in its entirety and replacing such deleted text with the following:

> Notwithstanding anything to the contrary herein, the Total Leverage Ratio and the Total Senior Secured Leverage Ratio shall be calculated (including, but not limited to, for purposes of determining the Maximum Incremental Amount with respect to Sections 2.14(a), 2.16(a), and 2.17(a), on a Pro Forma Basis with respect to each Specified Transaction occurring during the applicable four quarter period to which such calculation relates, or subsequent to the end of such four-quarter period but not later than the date of such calculation; provided that notwithstanding the foregoing, when calculating the Total Leverage Ratio and the Total Senior Secured Leverage Ratio for purposes of (i) determining the applicable percentage of Excess Cash Flow set forth in Section 2.05 and (ii) determining actual compliance (and not Pro Forma Compliance or compliance on a Pro Forma Basis) with any applicable covenant pursuant to Section 7.11, any Specified Transaction and any related adjustment contemplated in the definition of Pro Forma Basis (and corresponding provisions of the definition of Consolidated EBITDA) that occurred subsequent to the end of the applicable four quarter period shall not be given pro forma effect.

Figure 13: An amendment to the definition of Leverage Ratio used in financial covenants. The excerpt is taken from the First Amendment and Refinancing Agreement dated as of March 1, 2011 amending the Credit Agreement between NBTY, Inc., a Delaware Corporation and Barclays Capital, Merrill Lynch, Pierce, Fenner & Smith Incorporated, and Credit Suisse Securities (USA) LLC as Joint Lead Arrangers dated as of October 1, 2010.

1. Amendment to Credit Agreement – Section 1. The definition of EBITDAR in Section 1 of the Credit Agreement is hereby restated as follows:

> **EBITDAR** means, as determined, on a rolling twelve month basis and in respect of any Person the sum of (a) the Net Income of such Person, plus (b) the interest expense of such Person for such period as determined in accordance with GAAP and as such item is reported on such Person's financial statements, plus (c) the income tax expense of such Person for such period, plus (d) the amount reported as the depreciation of the assets of such Person for such period, computed in accordance with GAAP, and as such item is used in the computation of such Person's Net Income for such period, plus (e) the amount reported as the amortization of intangibles for such Person for such period, computed in accordance with GAAP, and as such item is used in the computation of such Person's Net Income for such period, minus (f) Rental Payments related to Capitalized Leases, plus (g) Rental Payments.

Figure 14: An amendment to the definition of EBITDAR used in financial covenants. The excerpt is taken from the Amendment No. 1 dated as of August 26, 2016 to Credit Agreement between Monro Muffler Brake, Inc. and Citizens Bank, N.A. as Administrative Agent dated as of January 25, 2016.

6. Amendment to Section 5.20. Section 5.20 of the Credit Agreement hereby is amended by deleting it in its entirety, and substituting the following therefor:

SECTION 5.20. Minimum Consolidated Net Worth. Consolidated Net Worth will at no time be less than $240,000,000, less the amount of any non-cash loss from the sale of Gerber Coburn in an Approved Gerber Coburn Sale and less the amount of restructuring charges recorded in the fourth Fiscal Quarter of the 2001 Fiscal Year, not to exceed $30,000,000, and excluding any change in the "accumulated other comprehensive income/(loss)" component of shareholders' equity after January 31, 2001, plus the sum of: (i) 75% of the cumulative Consolidated Net Income of the Borrower and its Consolidated Subsidiaries during any period after the Closing Date (taken as one accounting period), calculated quarterly at the end of each Fiscal Quarter but excluding from such calculations of Consolidated Net Income for purposes of this clause (i), any Fiscal Quarter in which the Consolidated Net Income of the Borrower and its Consolidated Subsidiaries is negative; and (ii) 100% of the cumulative Net Proceeds of Capital Stock received during any period after the Closing Date, calculated quarterly at the end of each Fiscal Quarter.

Figure 15: An amendment to the Minimum Net Worth Covenant. The excerpt is taken from the Second Amendment dated as of January 31, 2002 amending the Credit Agreement between Gerber Scientific, Inc. and Wachovia Bank, N.A. as Agent dated as of March 14, 2001.

8.09 Consolidated Leverage Ratio.

The Borrower will not permit the Consolidated Leverage Ratio as of the end of any fiscal quarter of the Borrower ending during a period set forth below to exceed the ratio set forth below opposite such period:

Period	Consolidated Leverage Ratio
Effective Date through December 31, 2011	5.00:1.00
January 1, 2012 through September 30, 2012	4.50:1.00
~~**October 1, 2012 through September 30, 2013**~~	~~4.00:1.00~~
October 1, ~~2013~~2012 and thereafter	~~3.75~~4.00:1.00

Figure 16: An amendment to the Leverage Ratio Covenant. The excerpt is taken from Amendment No. 1 dated as of March 11, 2011 amending the Credit Agreement between Hanger Orthopedic Group, Inc., a Delaware corporation and Bank of America, N.A. as Administrative Agent dated as of December 1, 2010.

7.9 An example of an amendment and waivers in the data

c. Leverage Ratio. Section 6.10 of the Credit Agreement is amended in its entirety to read as follows:

"Section 6.10 Leverage Ratio. The Company will not permit its Leverage Ratio at the end of any fiscal quarter to be greater than the levels indicated below for the corresponding periods:

Period	Ratio
January 1, 2002 through March 31, 2002	2.95 to 1.00
April 1, 2002 through June 30, 2002	2.25 to 1.00
July 1, 2002 through September 30, 2002	2.00 to 1.00
October 1, 2002 through March 31, 2003	1.75 to 1.00
April 1, 2003 through September 30, 2003	2.50 to 1.00
October 1, 2003 through December 31, 2003	4.50 to 1.00
January 1, 2004 through March 31, 2004	2.50 to 1.00
April 1, 2004 through June 30, 2004	2.25 to 1.00
July 1, 2004 and thereafter	2.00 to 1.00"

Figure 17: An example of a precautionary waiver. The level of leverage ratio is lifted up for three months (October-December 2003) and then goes back to its original level. This excerpt is taken from the Amendment No. 5, dated as of December 29, 2003, amending the Second Amended and Restated Credit Agreement, dated as of April 30, 2002, between Global Industries, Ltd., a Louisiana Corporation, and Bank One, NA, as administrative agent.

a. Leverage Ratio.

Period Ratio

January 1, 2002 through March 31, 2002 2.95 to 1.00

April 1, 2002 through June 30, 2002 2.25 to 1.00

July 1, 2002 through September 30, 2002 2.00 to 1.00

October 1, 2002 through March 31, 2003 1.75 to 1.00

April 1, 2003 through March 31, 2004 2.50 to 1.00

April 1, 2004 through June 30, 2004 2.25 to 1.00

July 1, 2004 and thereafter 2.00 to 1.00

Figure 18: The leverage covenant in the original Credit Agreement, dated as of April 30, 2002, between Global Industries, Ltd., a Louisiana Corporation, and Bank One, NA, as administrative agent.

2.1. WAIVER. As of the Effective Date and solely for the period through January 31, 2002 (the "Waiver Period"), the Majority Lenders signatory hereto waive any Default or Event of Default under Section 15(c) of the Credit Agreement resulting from the Borrower's failure to comply with any one or more of the covenants set forth in Section 14.1 of the Credit Agreement for the periods ending on December 31, 2001 and as of December 31, 2001, as the case may be.

Figure 19: An example of an ex post waiver. The violation of a financial covenant by the borrower is waived. This excerpt is taken from the Waiver, dated as of December 31, 2001, to the Second Amended and Restated Credit Agreement, dated as of September 15, 1998, between Hexcel Corporation and Citibank, N.A., as Documentation Agent, and Credit Suisse First Boston, as Administrative Agent.

6. Amendment to Section 5.20. Section 5.20 of the Credit Agreement hereby is amended by deleting it in its entirety, and substituting the following therefor:

SECTION 5.20. Minimum Consolidated Net Worth. Consolidated Net Worth will at no time be less than $240,000,000, less the amount of any non-cash loss from the sale of Gerber Coburn in an Approved Gerber Coburn Sale and less the amount of restructuring charges recorded in the fourth Fiscal Quarter of the 2001 Fiscal Year, not to exceed $30,000,000, and excluding any change in the "accumulated other comprehensive income/(loss)" component of shareholders' equity after January 31, 2001, plus the sum of: (i) 75% of the cumulative Consolidated Net Income of the Borrower and its Consolidated Subsidiaries during any period after the Closing Date (taken as one accounting period), calculated quarterly at the end of each Fiscal Quarter but excluding from such calculations of Consolidated Net Income for purposes of this clause (i), any Fiscal Quarter in which the Consolidated Net Income of the Borrower and its Consolidated Subsidiaries is negative; and (ii) 100% of the cumulative Net Proceeds of Capital Stock received during any period after the Closing Date, calculated quarterly at the end of each Fiscal Quarter.

Figure 20: An example of an amendment. An amendment to the Minimum Net Worth Covenant. The excerpt is taken from the Second Amendment, dated as of January 31, 2002, amending the Credit Agreement between Gerber Scientific, Inc. and Wachovia Bank, N.A. as Agent, dated as of March 14, 2001.

7.10 List of variables used in estimation

Variable	Notation	Definition
Probability of violating a covenant within the 1st year after origination	$pviol_0^1$	Covenant strictness, adapted from Demerjian and Owens (2016), measuring the probability of violating a covenant within 1 year after the loan origination date.
Probability of violating a covenant within the 2nd year after origination	$pviol_0^2$	Covenant strictness, adapted from Demerjian and Owens (2016), measuring the probability of violating a covenant in the period between the end of the 1st and the end of the 2nd years after the loan origination date.
Probability of violating a covenant within the 1st year after an amendment	$pviol_{am}^1$	Covenant strictness, adapted from Demerjian and Owens (2016), measuring the probability of violating a covenant within 1 year after the amendment effective date.
Covenant violation within the 1st year after origination	$viol_1$	An indicator variable equal to 1 if at least one of the company's financial ratios or characteristics for the 1st year violate a financial covenant effective for this year. Definitions of financial ratios and characteristics for financial covenants are from Demerjian and Owens (2016).
Covenant violation at any time	$viol_{ever}$	An indicator variable equal to 1 if at least one of the company's financial ratios or characteristics for a year violate a financial covenant effective for this year, for all years before the contract maturity date. Definitions of financial ratios and characteristics for financial covenants are from Demerjian and Owens (2016).
Covenant amendment within the 1st year after origination	am_1	An indicator variable equal to 1 if financial covenants are amended at least once within 1 year after the loan origination. Amendments are defined as described in section 2.1.
Covenant amendment at any time	am_{ever}	An indicator variable equal to 1 if financial covenants are amended at least once at any time before the contract maturity date. Amendments are defined as described in section 2.1.

7.11 Estimation procedure

7.11.1 Calculation of differences between empirical and theoretical moments

In this Appendix, I explain how the empirical moments used to fit the model are computed. The paper uses 6 moments, listed in Appendix 7.5: ex ante probabilities of violating a covenant (i.e., covenant strictness) within the first and the second year after the contract origination, unconditional probability of amendment and probability of amendment conditional on no covenant violation within the first year after loan origination, probabilities of violating a covenant (i.e., covenant strictness) within the first year after an amendment unconditional and conditional on no violation within the first year after loan origination.

The data series used in estimation are initial financial covenant levels from Dealscan database and hand-collected financial covenant schedules, hand-collected amended financial covenants, and firms' financial characteristics from the Compustat database. The variables are described in Appendix 7.10.

I treat my data as cross-sectional, i.e. every loan is an independent draw from a population of loans described by one distribution function. For each loan i, I have 5 columns:

1. Strictness of initial financial covenants for the first year after loan origination, $pviol^1_{0_i}$.
2. Strictness of initial financial covenants for the second year after loan origination, $pviol^2_{0_i}$.
3. Binary variable for whether a covenant was violated within the first year after loan origination, $viol_{1_i}$.
4. Binary variable for whether a covenant was amended within the first year after loan origination, am_{1_i}.
5. Strictness of amended financial covenants (if there was an amendment) for the first year after an amendment, $pviol^1_{am_i}$.

In the table 15 below, I provide formulas used to calculate differences between empirical and theoretical moments.

7.11.2 Parameter search algorithm

The objective of the GMM procedure is to minimize the distance between the theoretical moments, which are functions of the model parameters, and empirical moments, which are calculated from the data. In other words, the goal is to find a set of parameters $\hat{\theta}$ such that

$$\hat{\theta} = \text{argmin}_{\theta\in\mathbf{\Theta}} \quad (G(Y_i,\theta))^T \, \hat{W} \, (G(Y_i,\theta)) \,, \tag{23}$$

Table 15: Formulas to calculate differences between empirical and theoretical moments. N denotes the number of loans in the sample. Description of variables is provided in Appendix 7.10.

Moment	**Formula for difference between empirical and theoretical moments**
Strictness of initial financial covenants for the first year after loan origination	$\frac{1}{N}\sum_{i=1}^{N} pviol^1_{0_i} - \left[\mu_0 \times \pi^{0*}_{m1} + (1-\mu_0) \times (1-\pi^{0*}_{b1})\right]$
Strictness of initial financial covenants for the second year after loan origination	$\frac{1}{N}\sum_{i=1}^{N} pviol^2_{0_i} - \left[\mu_0 \times \left(\mu \times \pi^{0*}_{m2} + (1-\mu) \times (1-\pi^{0*}_{b2})\right) + (1-\mu_0) \times \left(\beta \times (1-\pi^{0*}_{b2}) + (1-\beta) \times \pi^{0*}_{m2}\right)\right]$
Probability of amending a covenant within the first year after loan origination	$\frac{1}{N}\sum_{i=1}^{N} am_{1_i} - \left[\mu_0 \times (1-H_m) + (1-\mu_0) \times (1-H_b)\right]$
Probability of amending a covenant within the first year after loan origination, conditional on no violation within the first year after origination	$\frac{\sum_{i=1}^{N} am_{1_i}\left(1-viol_{1_i}\right)}{\sum_{i=1}^{N}\left(1-viol_{1_i}\right)} - \left[\frac{\mu_0 \times (1-\pi^{0*}_{m1}) \times (1-H_m) + (1-\mu_0) \times \pi^{0*}_{b1} \times (1-H_b)}{\mu_0 \times (1-\pi^{0*}_{m1}) + (1-\mu_0) \times \pi^{0*}_{b1}}\right]$
Strictness of amended financial covenants for the first year after an amendment	$\frac{\sum_{i=1}^{N} pviol^1_{am} am_{1_i}}{\sum_{i=1}^{N} am_{1_i}} - \Big[\frac{1}{\mu_0 \times (1-H_m) + (1-\mu_0) \times (1-H_b)} \times$ $\Big(\mu_0 \times (1-H_m) \times \left(\mu \times E\left[\pi^{1m*}_{m2} \mid f^m_m \notin \left(x^l_m, x^u_m\right)\right] + (1-\mu) \times \left(1 - E\left[\pi^{1m*}_{b2} \mid f^m_m \notin \left(x^l_m, x^u_m\right)\right]\right)\right) +$ $(1-\mu_0) \times (1-H_b) \times \left(\beta \times \left(1 - E\left[\pi^{1b*}_{b2} \mid f^b_b \notin \left(x^l_b, x^u_b\right)\right]\right) + (1-\beta) \times E\left[\pi^{1b*}_{m2} \mid f^b_b \notin \left(x^l_b, x^u_b\right)\right]\right)\Big)\Big]$
Strictness of amended financial covenants for the first year after an amendment, conditional on no violation within the first year after origination	$\frac{\sum_{i=1}^{N} pviol^1_{am} am_{1_i}\left(1-viol_{1_i}\right)}{\sum_{i=1}^{N} am_{1_i}\left(1-viol_{1_i}\right)} - \Big[\frac{1}{\mu_0 \times (1-\pi^{0*}_{m1}) \times (1-H_m) + (1-\mu_0) \times \pi^{0*}_{b1} \times (1-H_b)} \times$ $\Big(\mu_0 \times (1-H_m) \times (1-\pi^{0*}_{m1}) \times \left(\mu \times E\left[\pi^{1m*}_{m2} \mid f^m_m \notin \left(x^l_m, x^u_m\right)\right] + (1-\mu) \times \left(1 - E\left[\pi^{1m*}_{b2} \mid f^m_m \notin \left(x^l_m, x^u_m\right)\right]\right)\right) +$ $(1-\mu_0) \times (1-H_b) \times \pi^{0*}_{b1} \times \left(\beta \times \left(1 - E\left[\pi^{1b*}_{b2} \mid f^b_b \notin \left(x^l_b, x^u_b\right)\right]\right) + (1-\beta) \times E\left[\pi^{1b*}_{m2} \mid f^b_b \notin \left(x^l_b, x^u_b\right)\right]\right)\Big)\Big]$

where $G(Y_i, \theta) = m(d) - \hat{m}(\theta)$ is the vector of differences between moments computed from the data $m(d)$ – a function of data d – and their counterparts computed from the model $\hat{m}(\theta)$ the model – a function of the model's parameters θ. I show how each element of this vector is calculated in table 15 below. The matrix W is the weighting matrix.

The estimation is conducted in two steps. In the first step, the algorithm searches for $\hat{\theta}_1$ that minimizes 23 with an identity matrix as the weighting matrix $\hat{W}_1 = E$. Next, I take the obtained estimates $\hat{\theta}_1$, plug them into the vector $G(Y_i, \theta)$, and calculate the covariance matrix of this vector, $\hat{\Omega} \equiv G(Y_i, \theta)G(Y_i, \theta)'$, using bootstrap. I create 1000 randomly drawn subsamples of size 500 from my original dataset and calculate vectors of moment differences $G(Y_i^k, \hat{\theta}_1), k = 1, 2, ..., 1000$ for each of these subsamples. Next, I calculate the covariance matrix of moments based on these 1000 observations, $\hat{\Omega}$.

In the second step, the algorithm searches for $\hat{\theta}_2$ that minimizes 23 where the weighting matrix is the inverse of the covariance matrix: $\hat{W}_2 = \hat{\Omega}^{-1}$. The parameter estimates obtained in the second step $\hat{\theta}_2$ are the ultimate estimates. I use the Nelder-Mead optimization method (Nelder and Mead (1965)) using R NLOPTR function (Johnson (2007)) to search for $\hat{\theta}$ in both steps.

I calculate standard errors of the estimates using the formula for the asymptotic covariance matrix of estimates:

$$\mathbf{V} \equiv \frac{1}{N}\left[\hat{G}\hat{\Omega}^{-1}\hat{G}^T\right]^{-1}, \tag{24}$$

where $\hat{G} \equiv \frac{\partial\left(\frac{1}{N}\sum_{i=1}^{N} g(Y_i,\theta)\right)}{\partial\theta}$ is the Jacobian matrix, evaluated at $\hat{\theta}_2$. The derivative of moment k with respect to parameter p, $\frac{\partial\left(\frac{1}{N}\sum_{i=1}^{N} g(Y_i,\theta)\right)_k}{\partial\theta_p}$, is calculated by varying the parameter $\hat{\theta}_p$ by 1% up and down (keeping other parameters constant) and dividing the difference between the new value of the moment at the 1% higher parameter and the new value of the moment at the 1% lower parameter $\hat{\theta}_p$, $\left(\frac{1}{N}\sum_{i=1}^{N} g(Y_i, \theta)\right)_k \left(1.01\hat{\theta}_p\right) - \left(\frac{1}{N}\sum_{i=1}^{N} g(Y_i, \theta)\right)_k \left(0.99\hat{\theta}_p\right)$ by 2% of $\hat{\theta}_p$.

Because the model is just-identified, I can not calculate the J-statistic.

7.12 Alternative specifications

7.12.1 Alternative cost persistence parameters

Because the persistence of the cost of leaving the "wrong" party in control (ρ) and the waiver friction (η_w) are difficult to separate, in the estimation, I assume $\rho = 0.9$. In this section, I investigate how robust the estimation results are to the choice of the persistence parameter. I estimate two alternative versions of the model: in one, ρ is set close to one, 0.999, and in the other, ρ is smaller than in the main specification, 0.8.

The estimates are shown in table 16. The conclusion based on estimated parameters does not change with different assumed cost persistence: the quality of contractible information is low, persistences of the non-contractible states are low, and the waiver friction considerably exceeds amendment cost. However, the waiver friction estimate goes up by about 40%.

7.12.2 Alternative data specification

The data allows flexibility in constructing financial covenant amendment variables. In particular, since the model is not about amendment frequencies but about a single decision to amend, I can look either at whether financial covenant schedules are amended within a year or whether they are ever amended in the entire life of a contract.

In the main body of the paper, I choose to only analyze amendment behavior within one year after contract origination. This section investigates how robust the parameter estimates are to specifications of amendment data. The results are presented in table 17.

Most of the parameters are close for the two data specifications. The most sensitive to the data is the estimated quality of contractible information, q. When I consider amendments at any time until a loan's maturity, the frequency of amendments in the data goes up, and the parameter search algorithm attributes it to a lower precision of contractible information.

Table 16: Parameter estimates for different cost persistence parameters

Parameter	Main specification, $\rho = 0.9$	$\rho = 0.999$	$\rho = 0.8$
Contractible information quality, q	0.687 (0.002)	0.699 (0.005)	0.615 (0.004)
Initial probability of the state m, μ_0	0.614 (0.000)	0.605 (0.000)	0.600 (0.000)
Persistence of the state m, μ	0.654 (0.000)	0.657 (0.004)	0.637 (0.001)
Persistence of the state b, β	0.501 (0.001)	0.521 (0.001)	0.500 (0.000)
Amendment cost, η_a	0.009 (0.000)	0.013 (0.001)	0.010 (0.000)
Waiver friction, η_w	0.700 (0.001)	0.997 (0.150)	0.998 (0.013)
Efficiency gain, %	2.323	1.094	2.707

Standard errors are in parentheses. In estimation, $\bar{c} = 1$.

Table 17: Parameter estimates for an alternative data specification

Parameter	Main specification, amendments within the 1st year	Amendments at all times
Contractible information quality, q	0.687 (0.002)	0.582 (0.004)
Initial probability of the state m, μ_0	0.614 (0.000)	0.607 (0.002)
Persistence of the state m, μ	0.654 (0.001)	0.618 (0.001)
Persistence of the state b, β	0.501 (0.001)	0.508 (0.002)
Amendment cost, η_a	0.009 (0.000)	0.007 (0.000)
Waiver friction, η_w	0.700 (0.001)	0.793 (0.034)
Contracting efficiency gain, %	2.323	4.039

Standard errors are in parentheses. In estimation, the parameters are set $\bar{c} = 1$, $\rho = 0.9$.