Financial Information and Diverging Beliefs*

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Abstract

Standard Bayesians' beliefs converge when they receive the same piece of new information. However, when agents have uncertainty about the precision of a signal, their beliefs might instead diverge more despite receiving the same information. We demonstrate that this divergence leads to a unimodal effect of the absolute surprise in the signal on trading volume. We show that this prediction is consistent with the empirical evidence using trading volume around earnings announcements of US firms. We find evidence of elevated volume following moderate surprises and depressed volume following more extreme surprises, a pattern that is more pronounced when investors are more uncertain about earnings' precision. Because investors can disagree even further after receiving the same piece of news, the relationship between news and trading volume is not necessarily linear, suggesting that trading volume may not be an appropriate proxy for market liquidity.

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Disclosure Statement

We, the authors, Christopher S. Armstrong, Mirko S. Heinle, and Irina M. Luneva, have nothing to disclose. We have had no paid or unpaid positions as officer, director, or board member of relevant non-profit organizations or profit-making entities. No partner nor close relative has had a paid or unpaid position as officer, director, or board member of relevant non-profit organizations or profit-making entities. No third party had the right to review any part of this paper, prior to its circulation. This study does not require IRB approval.

1 Introduction

Information plays a key role in financial markets. Conventional wisdom in the finance and accounting literature suggests that when investors receive a common signal, their beliefs converge. For example, for a signal in between two investors' priors, an optimistic investor revises her belief downward while a pessimistic investor updates upward. Despite this perception, there is ample empirical and theoretical evidence (e.g., Gentzkow and Shapiro (2006), Filipowicz et al. (2018), Fryer et al. (2019), Kartik et al. (2021)) that individuals often reject information that does not conform to their priors, believing that the news is not informative. In this case, new information can actually cause their beliefs to diverge more.[1] We theoretically and empirically investigate the effect of information on the beliefs divergence. First we develop a theoretical model of investors in a firm and show that uncertainty over the precision of a signal implies that investors' beliefs can diverge more after observing a signal. Because beliefs cannot be observed in the data directly, we derive predictions about trading volume. We choose trading volume as our research setting because trading volume is "a product of the extent to which investors hold diverse opinions ... and the extent to which these opinions change on average" at the time when a new piece of information arrives (Verrecchia (2001)), in contrast to the price change, which is a function only of the average evolution of investors' beliefs. Finally, we test the predictions from our model around quarterly earnings disclosures of publicly traded U.S. firms.

In our model investors have different expectations about the firm's cash flow, are uncertain about the precision of a signal about the firm, and trade both before and after the signal is disclosed. Due to their uncertainty about the signal-precision, investors discount signal realizations that are far from their own prior beliefs. In addition, because investors have different priors, they also differ in the extent to which they believe signal realizations. Specifically, investors believe that a signal realization that deviates from their prior beliefs is imprecise and, therefore, do not update their beliefs as much as investors whose priors are closer to the realized signal. As a result, investors' (posterior) beliefs diverge more following some realizations of the signal. This feature makes the model realistic: in everyday life, we often observe how people, even if they are professionals, may disagree even more after they receive the same piece of information.[2] In financial markets, the

[1] See also Banerjee and Kremer (2010) and Banerjee et al. (2021) who argue that investors disagree about news.

[2] Examples include polarization of justices when new evidence is presented in court ("Supreme Court Justices Differ

divergence of beliefs results in trading volume that is an M-shaped function of the signal itself and unimodal in the absolute value of its information content (or "surprise"). In other words, trading volume is, at first, increasing in signal that produce a moderate surprise, but then decreases as the signal generates a more extreme surprise. And, this M-shape pattern is more pronounced when investors are more uncertain about the signal's precision.

We next assess the validity of our model by developing several empirical tests of its implications based on firms' quarterly earnings disclosures and systematic patterns in trading volume surrounding the disclosures. We formulate and test two predictions: (1) trading volume increases for intermediate levels of earnings surprise and dampens for extreme levels, and (2) the M-shape of trading volume is more pronounced under high signal-precision uncertainty. To test our first prediction, we analyze non-parametric and parametric shapes of trading volume. We show that for the sample of U.S. firms, trading volume's reaction is indeed weakened for extreme levels of an earnings surprise. If we impose a polynomial shape on trading volume, we find that trading volume as a function of the earnings surprise is most closely fitted by fourth-order polynomial regression. The estimated parameters exhibit a clear M-shape. We also estimate quantile regressions and show that trading volume increases for moderate absolute surprise levels, but does not increase for the upper 5% of surprises.

To test our second prediction, we develop a new measure of earnings-announcement-precision uncertainty that allows us to separate precision uncertainty from the overall market uncertainty: namely, the spread in analysts' earnings forecasts scaled by VIX. We verify our measure by showing that an S-shaped earnings response coefficient (ERC)[3] is more pronounced for observations with high earnings-precision uncertainty. Trading volume in subsamples with higher earnings-announcement-precision uncertainty demonstrates a clear M-shape, which gradually transforms into a traditional V-shape as we move to lower levels of earnings-announcement-precision uncertainty. This evidence is supported by both non-parametric and parametric tests.

Our study might lead researchers to reconsider their widespread usage of trading volume as a

on Boston Bomber's Death Sentence", Wall Street Journal), historians' different descriptions of a fact depending on historians' reference points ("Meghan at 40 examines two sides of a royal renegade on Channel 5", Financial Times), and government officials coming to contradicting conclusions after a meeting ("Analysts weigh in on Fed's contradictory inflation remarks", Financial Times).

[3] See, for example, Freeman and Tse (1992), Cheng et al. (1992), and Das and Lev (1994). Subramanyam (1996) shows in a theoretical model that because of signal variance uncertainty, ERC dampens for extreme levels of an earnings surprise.

proxy for market liquidity. Gabrielsen et al. (2012) describes a few weaknesses of trading volume as a measure of liquidity, including the difficulty of disentangling the liquidity component from the information asymmetry component. Datar (2000) demonstrates inconsistencies between two proxies for liquidity: the turnover ratio and the coefficient of elasticity of trading, measured as a percent change in trading volume over a percent change in firm price. Our study raises concerns about another disadvantage of trading volume: if investors are uncertain about the precision of financial information, trading volume is a non-monotonic function of signal surprise and can be low due to a signal being too far from the market's expectations rather than due to unavailability of liquid assets on the market.

We rely on and aim to extend multiple streams of literature. First, we contribute to the broad literature in Economics and Psychology that demonstrates the evidence and studies the implications of individuals putting more trust in information that confirms their prior beliefs (Gentzkow and Shapiro (2006), Filipowicz et al. (2018), Fryer et al. (2019), Kartik et al. (2021), Martel and Schneemeier (2021)). We demonstrate that such behavior is present in financial markets and leads to unusual patterns in trading volume, which are more pronounced as individuals become more uncertain about the precision of information they receive. We also propose a way to measure the degree of this signal-precision uncertainty.

Researchers have proposed several mechanisms to explain why investors' views diverge after they receive the same information. The two papers closest to our work are Banerjee and Kremer (2010) and Banerjee et al. (2021). Banerjee and Kremer (2010) develop a difference-of-beliefs model where investors exogenously disagree about their interpretations of a public signal. This gives rise to "belief-convergence" trade, when investors' beliefs converge after a prior disagreement and "idiosyncratic" trade, when investors disagree on the interpretation of new information. Banerjee et al. (2021) demonstrate how empirically descriptive deviations from the rational expectations framework arise endogenously when investors use "wishful thinking" – choose subjective beliefs to make themselves happier about the future. In Bordalo et al. (2021), investors overreact to positive news if they observed a stock growth in the past. Our paper differs from Bordalo et al. (2021) and Banerjee and Kremer (2010) in that in our model the differential reaction to signals arises endogenously, as opposed to being exogenously imposed. In our model, investors only underreact (namely when they observe very surprising signals) and never overreact. The greatest divergence of beliefs occurs when one type of investor underreacts and the other type reacts as if there were no signal-precision uncertainty. Another study that is closer to ours is Jia et al. (2017), which provides evidence that market segmentation may increase after an analyst recommendation because of the social connection between analysts and local investors: local investors react stronger to local analysts' recommendations than do foreign investors. If local investors' and local analysts' priors are closer or if local investors believe that local analysts are more precise than foreign analysts, the mechanism that we propose in our paper would also explain their evidence of increased market segmentation after analyst recommendations.

Our paper extends the theoretical literature on the trading volume effects of public signals (e.g., Karpoff (1986), Kim and Verrecchia (1991), Kondor (2012), Banerjee (2011)). Early work in this area suggests that trading volume is monotonically increasing in a signal's surprise, which is attributable to either differential information or information precision among traders before the signal's release (Kim and Verrecchia (1991)). Subsequent studies introduced differences-in-beliefs models and they typically provide inferences that are generally similar to those of their predecessors (Bamber et al. (2011)). To our knowledge, our study is the first to incorporate uncertainty about public signals' precision and show that it can result in a non-monotonic pattern between the magnitude of a signal's surprise and subsequent trade volume.

Finally, our study contributes to the parallel empirical literature that studies trading volume. Bamber (1987) finds that trading volume is increasing in the magnitude of an earnings surprise, and Choi (2019) shows that this relationship is amplified when markets are more volatile. We build on the latter, which studies uncertainty about the value of the signal (i.e., the first moment) by instead focusing on uncertainty about its precision (i.e., the second moment). Our paper is also a logical extension of antecedent work by Bamber et al. (1997), Irvine and Giannini (2012), Al-Nasseri and Menla Ali (2018), and Booker et al. (2018), who show that abnormal trading volume exhibits a positive relationship with changes in beliefs. Another related study by Giannini et al. (2019) suggests that both convergence and divergence of beliefs lead to increased trading volume around earnings announcements. Our paper differs from these by showing – both theoretically and empirically – a previously unknown effect of precision uncertainty: namely that trading volume exhibits different behavior across environments with low and high signal-precision uncertainty.

The rest of our paper is as follows. Section 2 develops a new model that incorporates both difference-of-beliefs and signal-precision-uncertainty and shows that the introduction of the latter can cause investors' beliefs to become more divergent after they receive the same signal. We then characterize the resulting equilibrium and derive empirical predictions for trading volume patterns. Section 3 describes our research design and develops multiple empirical tests of the two main predictions implied by our model. Section 4 summarizes our theoretical predictions, empirical evidence, and collective inferences from the synthesis of the two. We also discusses several potential drawbacks of our model and tests and suggest several promising avenues for future research.

2 The Model

2.1 Model Setup

A continuum of investors competes for shares in two assets. The first asset is riskless, has infinite supply and a rate of return that is normalized to zero without loss of generality. The second asset — the shares of a firm — is risky and yields a random return of $\tilde{x}$. The entire firm is sold to new investors in the first trading period; supply of the risky asset at date $t = 1$ is 1. We assume that there are two types of investors and denote the type of investor by $i \in \{1, 2\}$. A fraction λ_1 of investors are of type 1 and a fraction $\lambda_2 = 1 - \lambda_1$ are of type 2. We assume that investors are risk-averse and have mean-variance utility over their terminal wealth:

$$U_i = E[W_i] - \frac{1}{2} r_i Var[W_i], \tag{2.1}$$

where W_i and r_i are investor i's terminal wealth and coefficient of risk aversion, respectively. Investors are initially endowed with wealth $W_{i0} = 0$.

There are three time periods:

$t = 1$. **Pre-announcement Period.** Investors trade in anticipation of the disclosure at $t = 2$.

$t = 2$. **Post-announcement Period.** The signal, $\tilde{y} = \tilde{x} + \tilde{u}$, is disclosed and investors trade for the second time.

$t = 3$. **Realization Period.** The risky asset's return, x, is realized and investors consume their terminal wealth, which is given by:

$$W_{i3} = d_{i2}x + q_{i2}, \tag{2.2}$$

where d_{i2} and q_{i2} are the amounts of risky and riskless assets held in $t = 2$, respectively.

We assume that investors have heterogeneous prior beliefs about the risky asset's return:

$$\tilde{x} \sim N\left(m_i, \frac{1}{\nu}\right),$$

where m_i is the expected return of the risky asset and $\frac{1}{\nu}$ is the variance of the return. Furthermore, we assume that $\tilde{u}$, the noise term of the signal $\tilde{y} = \tilde{x} + \tilde{u}$, is independent of $\tilde{x}$ and normally distributed with zero mean and unknown precision. This implies that investors have heterogeneous prior beliefs about the signal:

$$\tilde{y} \sim N\left(m_i, \frac{1}{\tilde{w}}\right),$$

where $\tilde{w}$ is a random variable. Following Subramanyam (1996), we assume that the true signal-precision, w, follows a truncated gamma distribution with support $[0, \nu]$ and parameters α and β[4].

2.2 Convergence and Divergence of Beliefs

At $t = 2$ all investors observe the realization of the signal, y. Investors update their beliefs about the mean and variance of the firm's cash flow based on the disclosed signal, $E[\tilde{x}|y]$ and $Var[\tilde{x}|y]$, which are given by:

$$E[\tilde{x}|y] = m_i + \hat{w}_i(y - m_i)\nu^{-1} \tag{2.3}$$

$$Var[\tilde{x}|y] = \nu^{-1}\left(1 - \hat{w}_i\nu^{-1}\right) \tag{2.4}$$

[4] The probability distribution of $\tilde{w}_i$ is given by: $f(\tilde{w}_i) = \frac{\beta^\alpha w_i^{\alpha-1} exp(-\beta w_i)}{\Gamma(\alpha)}$ where $\Gamma(\alpha) = \int_0^\nu \beta^\alpha w_i^{\alpha-1} exp(-\beta w_i)$.

where $\hat{w}_i = E[\tilde{w}_i|y]$ is investor i's estimated precision of the signal, conditional on observing y, given by (see Appendix A.1):

$$\hat{w}_i = \frac{\Gamma(\alpha + 1.5, [\frac{(y-m_i)^2}{2} + \beta]\nu)[\frac{(y-m_i)^2}{2} + \beta]^{-1}}{\Gamma(\alpha + 0.5, [\frac{(y-m_i)^2}{2} + \beta]\nu)} \tag{2.5}$$

Note that when investors have homogeneous prior beliefs about the firms cash flows, $m_i = m$, then all investors have the same posterior precision, $\hat{w}_i = \hat{w}$. Consequently, the combination of different prior beliefs and precision uncertainty is crucial for our results. Figure 1(a) plots investors' *ex ante* beliefs about the mean and variance of the firm's cash flows, $E_i[\tilde{x}]$ and $Var_i[\tilde{x}]$. Figure 1(b), plots the *ex-post* counterparts of these two moments, conditional on the realization of the signal y, $E_i[\tilde{x}|y]$ and $Var_i[\tilde{x}|y]$. Finally, Figure 1(c) plots the difference between investors' *ex-ante* and *ex-post* beliefs to illustrate the magnitude and direction in which the signal alters their beliefs. Before the signal is realized (Figure 1(a)), investors disagree: the first type of investor (call her investor H) expects firm cash flow to be 6, the second type of investor (call her investor L) expects it to be 0. In a setting where investors know the signal-precision (such that w is not random), investors' beliefs converge following the signal realization. That is, investors' *ex post* expectations differ by less than 6 and the solid line in Figure 1(c) is strictly below the dashed line. Given the same piece of information, Bayesians update their prior beliefs so that their posterior beliefs are more aligned than before the information was given.

This simple rule is no longer true when investors are uncertain about the precision of the signal. Even after observing the same piece of information — or public signal — investors' beliefs can become more divergent. The reason is that investors infer that the signal has a low precision when the signal does not correspond to their priors. Consequently, an optimistic investor (type H) will exhibit a stronger response to an optimistic signal than will a pessimistic investor (type L), who will exhibit a more muted response. Figure 1(b) demonstrates this phenomenon. In this case, for $y = 3$, investors agree more about the firm's expected cash flows after observing the signal. The solid line in Figure 1(c) is below the dashed line. Here, both investors come to the same estimate of the signal-precision and their beliefs converge towards 3. However, for $y = 10$, the signal increases investors' disagreement: H now expects the firm's cash flows to be around 9, whereas L expects cash flows of roughly 2. The solid line in Figure 1(c) is above the dashed line. In other words, while both investors update their beliefs towards the realized signal realization, the optimistic investor does so more strongly than does the pessimistic investor, which results in greater divergence.

Investors' beliefs about the variances also diverge for some values of y. Expected variances are equal before the signal (Figure 1(a)), but after the signal, they may be different (Figure 1(b)). After the signal is realized, investors revise their expectations in the same direction. However, an investor whose *ex-ante* expectation of cash flow is closer to the signal, moves her beliefs more than another investor. In our example, $y = 9$ is closer to investor H's prior expectation. As a result, the *ex-post* expectations are driven further away.

[Insert Figure 1 around here]

Similar situations occur in everyday life. When researchers get a result of a drug test, they may further debate the drug's effectiveness. Specifically, a researcher who does not expect that a drug should work and is uncertain about the conditions of the experiment may dismiss the results altogether. Similarly, political opinions in the news are often dismissed by people with opposing views. Finally, even more generic news in the media can easily be dismissed by appealing to the credibility of the source. In all these cases people disagree even more because they (1) disagree in the first place and (2) are not sure how precise the signal is (Jaynes (2005)). The same logic holds for investors: they have different prior beliefs and do not know exactly how precise the signal by a firm is.

2.3 Equilibrium Prices

In this section, we solve the model by backward induction. First, we derive investors' demands and the market-clearing price at $t = 2$. Next, anticipating their choices in $t = 2$, we solve for investors' demands and the price at $t = 1$. We measure trading volume as the absolute difference between demands at $t = 1$ and $t = 2$.

Let P_t denote the price of the risky asset at time t. The equilibrium is solved for in Appendix A.2. At $t = 2$ the signal y is disclosed and investors revise their expectations and the equilibrium price is:

$$P_2^* = [\psi_1(\hat{w}_1) + \psi_2(\hat{w}_2)]^{-1} [E_1[\tilde{x}|y]\psi_1(\hat{w}_1) + E_2[\tilde{x}|y]\psi_2(\hat{w}_2) - 1], \tag{2.6}$$

where $\psi_i(\hat{w}_i) = \frac{\lambda_i \nu}{r_i(1-\frac{\hat{w}_i}{\nu})}$, $E_i[\tilde{x}|y] = m_i + \hat{w}_i(y - m_i)\nu^{-1}$. Subscript i denotes investor i's expectation or variance, respectively. The function $\psi_i(\hat{w}_i)$ represents an investor i's confidence in the quality of the signal. It increases as an investor perceives the signal as more precise. Because investors are risk averse, a higher value of $\psi_i(\hat{w}_i)$ increases investor i's demand for risky asset.

At $t = 1$ investors anticipate that at $t = 2$ the signal will be realized and the price will be as in (1.6). Note that in $t = 1$ the signal, $\tilde{y}$, is not yet realized. We solve for the equilibrium in Appendix A.3. The equilibrium price is:

$$P_1^* = (\phi_1 + \phi_2)^{-1} \left(E_1[P_2^*]\phi_1 + E_2[P_2^*]\phi_2 - 1 \right), \tag{2.7}$$

where $\phi_i = \frac{\lambda_i}{r_i Var_i[P_2^*]}$, which is similar to $\psi_i(\hat{w}_i)$. Because the signal is not yet realized, investors cannot estimate its precision. Instead, they rely on the expected next period price variance, $Var_i[P_2^*]$.

Following Subramanyam (1996), we plot the return, $\frac{P_2^* - P_1^*}{P_1^*}$, as a function of the signal realization. Our model predicts an "S-shaped" form of returns. Figure 2 shows the return, defined as the difference between prices at times 2 and 1 scaled by price at time 1, as a function of the firm's signal, y, predicted by the model. The figure also shows that the S-shape of the return is less pronounced for higher values of the parameter β. That is, when the precision uncertainty decreases, the S shape in returns becomes less pronounced.

[Insert Figure 2 around here]

2.4 Trading Volume

Note that in the traditional framework with homogeneously informed investors (e.g., Kim and Verrecchia (1991)) there is no trading volume following the disclosure of a signal unless investors disagree about the signal-precision in the first place. In our model, this is not the case. While investors initially have homogeneous beliefs about the signal-precision, they have heterogeneous beliefs following the disclosure. This divergence of beliefs, in turn, generates trade following the disclosure.

Investor i's trading volume is the absolute difference between demands in pre-announcement and post-announcement periods:

$$V_i = |d_{i2}^* - d_{i1}^*| \tag{2.8}$$

In our model with two investor types $V_i = -V_j$ such that total trading volume is given by $2V_i$. Because of the updating over the uncertain precision, it is difficult to analyze V_i analytically. To develop some intuition, Figure 3(a) displays a graph of trading volume, V_i, as a function of the signal realization y. As the figure shows, trading volume is an M-shaped function of the signal. Specifically, trading volume first increases as the signal moves away from zero but as the signal gets extreme, trading volume starts to decrease.

To get intuition for the shape of the trading volume, we plot the difference in investors' beliefs before and after the signal realization in Figure 3(b) on the same scale. Post-announcement trading volume is the greatest when the divergence in investors' post-announcement beliefs increases. In contrast, trading volume is smallest when investors' beliefs converge after the disclosure. The forces that drive the M-shape in trading volume are the heterogeneous priors in combination with uncertainty about the signal-precision. As a result, even when investors receive the same public signal, disagreement about the firm's future cash flow may increase, which leads to more trade.

[Insert Figure 3 around here]

In what follows, we analyze how the parameters of the distribution of signal-precision, α and β, affect the non-linearity of trading volume (see Figure 4). As α increases (β decreases), the non-linearity of trading volume is more pronounced and for sufficiently low levels of α (high levels of β), the humps disappear altogether. To understand this finding, note that the expectation and variance of signal-precision are proportional to α and inversely proportional to β[5]. On the one hand, as α increases (β decreases), the expected precision increases. If the signal is perceived as more precise on average, the market reacts to the signal stronger: the increase in trading volume is more pronounced. On the other hand, as α increases (β decreases), the market is also more uncertain about the precision. Similar to Subramanyam (1996), a higher signal-precision uncertainty causes investors to use the signal more to update their beliefs about the signal-precision. This implies

[5]The mean and variance of $\tilde{w}$ are $E[w] = \frac{\Gamma(\alpha+1,\beta\nu)}{\beta\Gamma(\alpha,\beta\nu)}$ and $Var[w] = \frac{\Gamma(\alpha+2,\beta\nu)\Gamma(\alpha,\beta\nu)-\Gamma(\alpha+1,\beta\nu)^2}{\beta^2\Gamma(\alpha,\beta\nu)}$. As $\nu \to \infty$, $E[w] \to \frac{\alpha}{\beta}$ and $Var[w] \to \frac{\alpha}{\beta^2}$.

that, for example, a positive signal is more easily dismissed by a pessimistic investor and, therefore, the humps in trading volume are more pronounced.

[Insert Figure 4 around here]

3 Empirical Testing

3.1 Model Predictions

In this section, we empirically test the results of the model. While we cannot observe investors' beliefs, we can observe a model outcome: trading volume. We choose earnings announcements as the signal that investors receive and test two model predictions. The first prediction is about the general functional form of trading volume as a function of an earnings surprise:

Prediction 1. Trading volume is an M-shaped function of the earnings surprise.

The second prediction is linked to how trading volume's form changes with the parameters of the signal-precision uncertainty. As discussed in the previous section, when the variance of the signal precision increases, the M-shaped pattern for trading volume becomes more pronounced. Interpreting a high variance of the signal precision as a high uncertainty about this precision, we formulate the second empirical prediction as follows:

Prediction 2. The M-shape in trading volume is more pronounced when there is greater uncertainty about the precision of an earnings announcement.

3.2 Data and Measurement

To test our theoretical predictions, we gather data on quarterly earnings announcements of US firms from the first quarter of 1990 to the fourth quarter of 2019. We obtain data on released earnings, analyst EPS forecasts, and analyst price targets from the IBES database; prices and trading volume from the CRSP database; and firm characteristics from the Compustat database. Following Choi (2019), we delete firms whose actual EPS is less than zero. We keep only firms with prices at the end of a previous fiscal quarter greater than $5 to minimize the effect of market frictions (Ball et al. (1995)).

Following Landsman and Maydew (2002), Truong (2012), and Choi (2019), we measure abnormal trading volume in [0,1] days event window around the earnings announcement as:

$$AVOL_{i,q} = \sum_{t=0}^{t=1} \frac{VOL_{i,q,t} - mVOL_{i,q}}{\sigma(VOL)_{i,q}}, \tag{3.1}$$

where $VOL_{i,q,t}$ is the trading volume, $mVOL_{i,q}$ and $\sigma(VOL)_{i,q}$ are the mean and the standard deviation of the daily trading volume in $[-240, -5]$ days before the earnings announcement (Truong (2012), Choi (2019)); i, q and t denote the firm, the quarter and the day after the earnings announcement, respectively.

We measure the earnings surprise similar to Conrad et al. (2002) and Choi (2019) in the following way:

$$Surp_{i,q} = \frac{|\text{Actual EPS}_{i,q} - \text{Med. forecast}_{i,q}|}{PRC_{i,q-1}}, \tag{3.2}$$

where Actual $\text{EPS}_{i,q}$ is the actual value announced by the firm, Med. $\text{forecast}_{i,q}$ is the median analyst forecast of firm's EPS, $PRC_{i,q-1}$ is the firm's price at the end of a previous fiscal quarter. We use only the most recent forecasts by each analyst to calculate the median.

We include firm size and market-to-book ratio to control for differences in risk that are not already captured by the excess return (Fama and French (1992, 1993)). We measure firm size as the natural logarithm of the market value of equity and the market-to-book ratio as the market value of equity divided by the book value of equity. We also include analyst forecast dispersion before the announcement to control for firm-level disagreement ex ante (Choi (2019)). We control for market-wide liquidity levels by including Pástor and Stambaugh (2003) liquidity factor. To minimize the effect of outliers, we truncate the earnings surprise variable at the 5% level and all the other variables at the 1% level. As a result, we have a dataset of 87,944 firm-years from the first quarter of 1990 to the fourth quarter of 2019. The total number of firms in the sample is 4739. Table 1 describes how the sample size changed at each stage.

We present summary statistics in Table 2. Mean abnormal trading volume equals 0.75 with the standard deviation 1.15. Earnings surprises are 0.001 on average and vary from -0.01 to 0.01.

[Insert Tables 1 and 2 around here]

3.3 Functional Form of Trading Volume

The model predicts that trading volume is an M-shaped function of the earnings surprise. Trading volume increases for medium levels of surprises and decreases for extreme levels. We test this result in three ways. First, we look at scatterplots of trading volume as a function of earnings surprise with fitted non-parametric curves. Second, we estimate polynomial regressions of abnormal trading volume on earnings surprise and use an analysis-of-variance statistical test to choose the model that fits the data in the best possible way. Third, we estimate quantile regressions for different quantiles of an absolute earnings surprise.

3.3.1 Non-parametric Analysis

We begin our analysis of the functional form of trading volume by looking at scatterplots of trading volume as a function of earnings surprise with locally estimated scatterplot smoothing (LOESS) curves. LOESS is a local regression method that combines elements of simple linear least squares regression with elements of nonlinear regression. The method builds simple models for localized subsets of the data and then combines them to a function describing full data support. The advantage of this approach is that it does not require a researcher to pre-specify any functional form.

To control for other important factors that might affect trading volume, we first run a regression of abnormal trading volume on the set of control variables:

$$\begin{aligned} AVOL_{i,q} &= a_0 + a_1 \times Size_{i,q-1} + a_2 \times Market/Book_{i,q-1} \\ &+a_3 \times Dispersion_{i,m-1} + a_4 \times PSLiquidity_{i,q-1} + \epsilon_{i,q}, \end{aligned} \tag{3.3}$$

Next, we analyze scatterplots of the residuals of the regression above, $(AVOL_{i,q} - A\hat{V}OL_{i,q})$. This procedure allows us to concentrate on the part of trading volume that is orthogonal to other factors.

[Insert Figure 5 around here]

Figure 5 demonstrates the scatterplot of the residuals of abnormal trading volume as a function of an earnings surprise. While the classic V-shape is pronounced for intermediate levels of earnings surprises, trading volume does not increase but rather stays flat for more extreme surprises. The non-parametric analysis provides preliminary evidence that trading volume reactions are modestly increasing in earnings surprises.

3.3.2 Parametric Analysis: Polynomial Regression

For our second test, we estimate the following regression:

$$ln(AVOL_{i,q}) = a_0 + A_1' \times poly(Surp_{i,q}) + a_2 \times Size_{i,q-1} + a_3 \times Market/Book_{i,q-1}$$
$$+a_4 \times Dispersion_{i,m-1} + a_5 \times PSLiquidity_{i,q-1} + \epsilon_{i,q}, \qquad (3.4)$$

where $poly(Surp_{i,q})$ denotes the polynomials of $Surp_{i,q}$ from first to fifth order[6].

The results of the estimation are shown in Table 3. Polynomial terms of the earnings surprise are significant up to the fourth order. To choose the model that most closely describes the relation between trading volume and the earnings surprise, we conduct an analysis-of-variance test. The statistics are shown in Table 4. The analysis shows that polynomials of the earnings surprise up to fourth are incremental and the fourth-order specification fits the data the best possible way. We conclude that the relation between abnormal trading volume and the earnings surprise is non-linear, described by the fourth-order polynomial.

[Insert Tables 3 and 4 around here]

While the fourth-order polynomial with positive and negative coefficients suggests a non-linear effect of the earnings surprise on trading volume, the M-shape is not obvious. To illustrate the shape of the polynomial, Figure 6 shows the plot of the fourth-order polynomial using the estimated coefficients. The curve has two humps and a pronounced M-shape, similar to the model plots in Figure 3(a).

[Insert Figure 6 around here]

[6] Adding higher order polynomials does not significantly increase the predictive power of the model. The results are not reported in the paper to economize the space.

3.3.3 Parametric Analysis: Quantile Regression

As an alternative way to show the non-monotonicity in trading volume, we estimate quantile regressions. Specifically, we estimate the following regression:

$$ln(AVOL_{i,q}) = a_0 + a_1 \times |Surp_{i,q}| + a_2 \times Size_{i,q-1} + a_3 \times Market/Book_{i,q-1}$$
$$+a_4 \times Dispersion_{i,m-1} + a_5 \times PSLiquidity_{i,q-1} + \epsilon_{i,q}, \qquad (3.5)$$

separately for the upper 5% of *Surp* variable and the rest of the sample. Table 5 shows the results. Earnings surprise is positively associated with the trading volume for the lower 95% of the absolute earnings surprise. For the upper 5%, i.e. for the 2.5% of the lowest and 2.5% of the highest non-absolute earnings surprise, the association disappears. Extreme levels of earnings surprise do not invoke high reactions of trading volume, as predicted by the model.

[Insert Table 5 around here]

Our evidence from the three empirical tests speak in favor of the first model's prediction. When we do not impose any restrictions on the functional form, abnormal trading volume increases for small earnings surprises, but less so for larger surprises. Imposing a particular functional form further corroborates this finding: trading volume as a function of the earnings surprise is most closely approximated by the fourth-order polynomial, which is M-shaped. The estimated polynomial pattern is similar to its theoretical counterpart: trading volume is around zero for no earnings surprise, increases for modest surprises, and then decreases for extreme surprises.

3.4 Cross-sectional Analysis

Our second prediction is the M-shaped pattern becomes more pronounced when uncertainty about the earnings precision is higher. To test this implication, we need to measure earnings-announcement-precision uncertainty in the sample. It is difficult to separate the earnings-announcement-precision uncertainty from the overall market uncertainty. We cannot use standard market uncertainty measures, such as VIX index, analyst forecast volatility, or market returns volatility because they may include both fundamental uncertainty and uncertainty about the earnings-announcement-precision.

3.4.1 Measure of Earnings-Precision Uncertainty

When analysts are not certain about whether the firm's earnings announcement is precise or not, the range for each analyst's forecasts is wider, and their resulting forecasts are more distant from each other. At the same time, the range of forecasts may be wide because of market-wide uncertainty, which is not specific to the firm. It is important to distinguish between these two different sources of uncertainty. To do so, we divide the measure that includes both types of uncertainty by the measure of the market-wide uncertainty. We measure the earnings-announcement-precision uncertainty in the following way:

$$PrecUnc_{i,q} = \frac{Analyst \quad forecast \quad spread_{i,q}}{VIX_{m-1}}, \tag{3.6}$$

where $Analyst \quad forecast \quad spread_{i,q}$ is the difference between the highest and the lowest analyst forecasts of the EPS of the firm i for the quarter q. VIX_{m-1} is the average monthly VIX from the daily data from the Chicago Board of Exchange website. The index is taken in the month before the disclosure month (Choi (2019)).

To validate our measure, we ask ourselves whether a common S-shaped ERC (e.g., Freeman and Tse (1992), Cheng et al. (1992), Das and Lev (1994)) is more pronounced for high levels of our measure, which would be consistent with the theory by Subramanyam (1996) (see Figure 2(b)). We calculate cumulative abnormal returns in the $[0, 1]$ event window in the following way:

$$CAR_{i,t} = \sum_{t=0}^{1} AR_{i,t}, \tag{3.7}$$

where $AR_{i,t}$ is the abnormal return calculated from one-factor model:

$$\hat{R}_{i,t} = \hat{\alpha} + \hat{\beta} R_{m,t}, \tag{3.8}$$

$$AR_{i,t} = R_{i,t} - \hat{R}_{i,t} \tag{3.9}$$

We partition our sample into four quartiles based on our measure and estimate quadratic polynomials of the cumulative abnormal returns as a function of the earnings surprise:

$$CAR_{i,t} = a_0 + a_1 \times Surp_{i,t} + a_2 \times Surp_{i,t}^2 + \epsilon_{i,t}, \tag{3.10}$$

for positive earnings surprises for each quartile of earnings-precision uncertainty. We do not estimate the regression for negative levels of earnings surprise because there are not enough observations with negative earnings surprise to yield sensible estimates of higher-order polynomial regression[7]. The results are presented in Table 6. The absolute value of the estimated coefficient before the quadratic term, or the degree to which the S-shape is more pronounced, is monotonically increasing from the first to the third quartile of the earnings-announcement-precision uncertainty measure. This implies that the S-shaped ERC is more pronounced for the higher levels of the measure. The evidence suggests that our measure, $PrecUnc$, almost correctly captures the theoretical construct we have in mind – the level of uncertainty about the precision of earnings announcements reported by firms.

[Insert Table 6 around here]

Having validated the measure, we proceed to estimate the functional form of trading volume for different levels of the earnings-announcement-precision uncertainty.

3.4.2 Non-parametric Cross-Sectional Analysis of Trading Volume

If the second model prediction is true and the measure of the earnings-announcement-precision uncertainty that we develop accurately captures the underlying theoretical construct, then trading volume's M-shape should be more pronounced for observations with high $PrecUnc$. We conduct both non-parametric and parametric tests of this prediction.

For our non-parametric analysis, we partition our sample into four groups based on the quartiles of our earnings-announcement-precision uncertainty measure. The scatterplots of the residuals of abnormal trading volume as a function of the earnings surprise for different quartiles of the earnings-announcement-precision uncertainty are presented in Figures 7-10. The pictures demonstrate a clear transition from the typical V-shape to the M-shape as we move from the first to the last

[7] Specifically, if we were to run regressions for quartiles of $PrecUnc$ for negative earnings surprises, we would have only about 1,000 observations for each regression. See Kleinbaum et al. (1978) for the reference.

quartile of the earnings-announcement-precision uncertainty. This evidence supports our theoretical prediction: namely the depressed trading volume at the extremes is more pronounced when there is greater market uncertainty about the signal-precision.

[Insert Figures 7-10 around here]

3.4.3 Parametric Cross-Sectional Analysis of Trading Volume

We begin our parametric analysis of our second theoretical prediction with a test of whether our data exhibits evidence of a structural shift using the procedure proposed by Aue et al. (2008). This procedure searches for a structural shift as follows. First, it partitions the data into two subsamples based on the magnitude of $PrecUnc$. Second, it estimates the coefficients in equation (2.4) for the first subsample. Third, it applies the estimated coefficients from the first subsample to fit the second subsample. Fourth, it compares the variance explained by the coefficient estimates between the two subsamples. Finally, it identifies the value of $PrecUnc$ that partitions the full sample so as to maximize the difference in explained variance between the two resulting subsamples. In other words, the algorithm searches for the value of $PrecUnc$ below which trading volume is well explained by one polynomnial function and above which the same function does not explain trading volume well. If the algorithm fails to identify a partition that results in a significant difference in the explained variance across the two subsamples, it concludes that there no evidence of a structural shift in the sample.

Applying this algorithm to our sample, we reject the null hypothesis of no structural shift. Instead, there is evidence of a structural shift starting from $PrecUnc = 2.05 \times 10^{-5}$, which is approximately the 11^{th} quantile of $PrecUnc$. The lower part, which we denote "low precision uncertainty," contains 11.3% of the full sample. The summary statistics for the low and high earnings-announcement-precision uncertainty subsamples are reported in Table 7 and show that the latter has both more trading volume and greater analyst forecast dispersion. And, although the average size of the firms in the two subsamples is similar, low precision uncertainty firms have a higher market-to-book ratio.

[Insert Table 7 around here]

To assess whether trading volume's shape changes significantly with the shift, we estimate regression (2.4) with fourth-order polynomial separately for the low precision uncertainty and the high precision uncertainty subsamples. The results are provided in Table 8. The estimated coefficients before $Surp^2$ and $Surp^4$ are greater in absolute values in a high precision uncertainty subsample. We plot the polynomials with the estimated coefficients in Figure 11. The M-shape of trading volume is pronounced for the high earnings announcement precision uncertainty subsample. The low precision uncertainty subsample does not have the left hump in trading volume, and the right hump is smaller than in the high precision uncertainty subsample. The data supports the prediction that the M-shape of trading volume is more pronounced in a high earnings-announcement-precision uncertainty environment.

[Insert Table 8 around here]

[Insert Figure 11 around here]

The empirical tests support the two model's predictions. First, trading volume is indeed an M-shaped function of the earnings surprise. Second, and more importantly, this M-shape occurs in an environment with high earnings-announcement-precision uncertainty. This suggests that investors trade without knowing exactly how precise earnings announcements are. As a result, trading volume is more complicated than a simple increasing function of the absolute earnings surprise. Specifically, the function is more M-shaped when investors' uncertainty about the earnings-announcement-precision is higher.

Conclusion

We provide initial evidence that investors' beliefs can further diverge even when they receive the same public signal. The source of this phenomenon lies in uncertainty about the precision of the financial information that investors receive, coupled with differential beliefs that they hold. We develop a model where investors with different beliefs about a firm's future cash flow trade in the firm's shares before and after the realization of a public signal. The novelty in the model is that investors are uncertain about the precision of this signal. Because of this uncertainty, investors' beliefs further diverge for some signal realizations. As a result of investors' posterior beliefs, trading volume is increasing for intermediate levels of signal surprise but is decreasing for extreme levels. The "M-shape" of trading volume is more pronounced when the uncertainty about the signal-precision is higher.

We test the predictions of our model using trading volume around quarterly earnings announcements of public U.S. firms. As a starting point in our empirical tests, we non-parametrically and parametrically show that total trading volume is described by a function that increases for the intermediate levels of the absolute earnings surprise and decreases (or stays flat) for the extreme levels.

We next develop a novel measure of the earnings-announcement-precision uncertainty and validate it by showing how the commonly-known S-shape of an earnings response coefficient changes with the different levels of our measure. As theory suggests, the S-shape is more pronounced for the high levels of the earnings-announcement-precision uncertainty.

As a final step, we non-parametrically and parametrically assess the functional form of trading volume for different levels of the earnings-announcement-precision uncertainty. The evidence further supports the model that we develop: as we move from low to high earnings-announcement-precision uncertainty, the M-shape, or declining trading volume for the extreme surprises, gets more pronounced.

Overall, we believe our paper is a small yet important piece of evidence for investors' beliefs divergence due to uncertainty about the quality of financial information.

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Appendix

A.1 Conditional Expectation of Signal Precision

This section is based on Subramanyam (1996). Conditional on precision w, $\tilde{y}$ is normally distributed, $\tilde{y} \sim N(m_i, w^{-1})$. Then,

$$h(y|w) = \sqrt{\frac{w}{2\pi}} exp\left[-\frac{w(y-m_i)^2}{2}\right] \tag{A11}$$

Compute the conditional expectation of $\tilde{w}$:

$$E[\tilde{w}|y] = \int wh(w|y)dw \tag{A12}$$

$$= \int \frac{1}{f(y)} wh_1(y|w)f(w)dw \tag{A13}$$

$$= \frac{\int wh_1(y|w)f(w)dw}{\int h_1(y|w)f(w)dw} \tag{A14}$$

$$= \frac{\int w^{1.5}(2\pi)^{-0.5} exp[-\frac{w(y-m_i)^2}{2}]f(w)dw}{\int w^{0.5}(2\pi)^{-0.5} exp[-\frac{w(y-m_i)^2}{2}]f(w)dw} \tag{A15}$$

Recall that $\tilde{w} \in [0, \nu]$ and substitute for $f(\tilde{w})$ to get:

$$E[\tilde{w}|y] = \frac{\Gamma(\alpha + 1.5, [\frac{(y-m_i)^2}{2} + \beta]\nu)[\frac{(y-m_i)^2}{2} + \beta]^{-1}}{\Gamma(\alpha + 0.5, [\frac{(y-m_i)^2}{2} + \beta]\nu)} \tag{A16}$$

A.2 Post-announcement Period Equilibrium

Investor i's budget constraint at time $t = 2$ is:

$$P_2 d_{i2} + q_{i2} = q_{i1}^* + P_2 d_{i1}^*, \tag{A17}$$

where q_{i1}^* and d_{i1}^* are the amounts of riskless and risky assets hold in equilibrium in $t = 1$, respectively. q_{i2} and d_{i2} are the amounts of riskless and risky assets hold in $t = 2$. Investor i solves:

$$max_{d_{i2},q_{i2}} \quad E_i[\tilde{x}d_{i2} + q_{i2}|y] - \frac{1}{2} r_i Var_i[\tilde{x}d_{i2} + q_{i2}|y] \tag{A18}$$

subject to (A9). The only random variable in the investor's utility is the return of the risky asset, $\tilde{x}$. Therefore, one can write $E_i[\tilde{x}d_{i2} + q_{i2}|y] = E_i[\tilde{x}|y]d_{i2} + q_{i2}$, $Var_i[\tilde{x}d_{i2} + q_{i2}|y] = Var_i[\tilde{x}|y]d_{i2}^2$. Rewrite the problem:

$$max_{d_{i2},q_{i2}} E_i[\tilde{x}|y]d_{i2} + q_{i2} - \frac{1}{2}r_i Var_i[\tilde{x}|y]d_{i2}^2 \tag{A19}$$

$$\text{s.t. } P_2 d_{i2} + q_{i2} = q_{i1}^* + P_2 d_{i1}^* \tag{A20}$$

Using the budget constraint (A9), express q_{i2}:

$$q_{i2} = q_{i1}^* + P_2 d_{i1}^* - P_2 d_{i2} \tag{A21}$$

Plug this expression into (A11):

$$max_{d_{i2}} E_i[\tilde{x}|y]d_{i2} + q_{i1}^* + P_2 d_{i1}^* - P_2 d_{i2} - \frac{1}{2}r_i Var_i[\tilde{x}]|y d_{i2}^2 \tag{A22}$$

q_{i1}^* and d_{i1}^* are chosen in $t = 1$ and are constant in our problem. Take the derivative of the (A14) with respect to d_{i2} and set it equal to zero:

$$E_i[\tilde{x}|y] - P_2 - r_i Var_i[\tilde{x}|y]d_{i2} = 0 \tag{A23}$$

Express d_{i2}:

$$d_{i2} = \frac{E_i[\tilde{x}|y] - P_2}{r_i Var_i[\tilde{x}|y]} \tag{A24}$$

or

$$d_{i2} = \frac{m_i + \hat{w}_i(y - m_i)\nu^{-1} - P_2}{r_i \frac{1}{\nu}(1 - \frac{\hat{w}_i}{\nu})} \tag{A25}$$

Use the market clearing condition to find an equilibrium price:

$$\lambda_1 d_{12} + \lambda_2 d_{22} = 1 \tag{A26}$$

$$\lambda_1 \frac{m_1 + \hat{w}_1(y - m_1)\nu^{-1} - P_2}{r_1 \frac{1}{\nu}(1 - \frac{\hat{w}_1}{\nu})} + \lambda_2 \frac{m_2 + \hat{w}_2(y - m_2)\nu^{-1} - P_2}{r_2 \frac{1}{\nu}(1 - \frac{\hat{w}_2}{\nu})} = 1 \tag{A27}$$

Solve for the price:

$$P_2^* = \left[\frac{\lambda_1 \nu}{r_1(1-\frac{\hat{w}_1}{\nu})} + \frac{\lambda_2 \nu}{r_2(1-\frac{\hat{w}_2}{\nu})}\right]^{-1}$$
$$\times \left[(m_1 + \hat{w}_1(y-m_1)\nu^{-1})\frac{\lambda_1 \nu}{r_1(1-\frac{\hat{w}_1}{\nu})} + (m_2 + \hat{w}_2(y-m_2)\nu^{-1})\frac{\lambda_2 \nu}{r_2(1-\frac{\hat{w}_2}{\nu})} - 1\right] \tag{A28}$$

The investor i's demand in equilibrium:

$$d_{i2}^* = \frac{\psi_i(\hat{w}_i)}{\lambda_i}\left[E_i[\tilde{x}|y] - c\left(E_i[\tilde{x}|y]\psi_i(\hat{w}_i) + E_j[\tilde{x}|y]\psi_j(\hat{w}_j) - (\psi_i(\hat{w}_i))^{-1}\right)\right], \tag{A29}$$

where $c = \frac{\psi_i(\hat{w}_i)}{\psi_i(\hat{w}_i)+\psi_j(\hat{w}_j)}$.

A.3 Pre-announcement Period Equilibrium

Investor i's problem:

$$max E_i[W_{i3}] - \frac{1}{2} r_i Var_i[W_{i3}] \tag{A30}$$
$$\text{s.t. } W_{i3} = x d_{i2}^* + q_{i2}^* \tag{A31}$$

From the budget constraint of the announcement period problem:

$$q_{i2}^* = q_{i1}^* + P_2^* d_{i1}^* - P_2^* d_{i2}^* \tag{A32}$$

The budget constraint in $t = 1$ is

$$P_1 d_{i1} + q_{i1} = W_{i0} = 0, \tag{A33}$$

where q_{i1} and d_{i1} are the amounts of riskless and risky assets hold in $t = 1$. Plug (A23) and (A24) into (A22), express the terminal wealth:

$$W_{i3} = (P_2^* - P_1)d_{i1} + (x - P_2^*)d_{i2}^* \tag{A34}$$

Rewrite the problem of investor i:

$$max_{d_{i1}} E_i[W_{i3}] - \frac{1}{2} r_i Var_i[W_{i3}] \tag{A35}$$

$$\text{s.t. } W_{i3} = (P_2^* - P_1)d_{i1} + (x - P_2^*)d_{i2}^* \tag{A36}$$

Plug (A27) into (A26):

$$max_{d_{i1}} E_i[(P_2^* - P_1)d_{i1} + (x - P_2^*)d_{i2}^*] - \frac{1}{2} r_i Var_i[(P_2^* - P_1)d_{i1} + (x - P_2^*)d_{i2}^*] \tag{A37}$$

Take the derivative of (A28) with respect to d_{i1} and set it equal to zero:

$$E_i[P_2^*] - P_1 - r_i Var_i[P_2^*]d_{i1} = 0 \tag{A38}$$

Express d_{i1}:

$$d_{i1} = \frac{E_i[P_2^*] - P_1}{r_i Var_i[P_2^*]} \tag{A39}$$

Use the market clearing condition to find an equilibrium price:

$$\lambda_1 d_{11} + \lambda_2 d_{21} = 1 \tag{A40}$$

$$\lambda_1 \frac{E_1[P_2^*] - P_1}{r_1 Var_1[P_2^*]} + \lambda_2 \frac{E_2[P_2^*] - P_1}{r_2 Var_2[P_2^*]} = 1 \tag{A41}$$

The equilibrium price is:

$$P_1^* = \frac{E_1[P_2^*]\frac{\lambda_1}{r_1 Var_1[P_2^*]}}{\frac{\lambda_1}{r_1 Var_1[P_2^*]} + \frac{\lambda_2}{r_2 Var_2[P_2^*]}} + \frac{E_2[P_2^*]\frac{\lambda_2}{r_2 Var_2[P_2^*]}}{\frac{\lambda_1}{r_1 Var_1[P_2^*]} + \frac{\lambda_2}{r_2 Var_2[P_2^*]}} - \frac{1}{\frac{\lambda_1}{r_1 Var_1[P_2^*]} + \frac{\lambda_2}{r_2 Var_2[P_2^*]}}, \tag{A42}$$

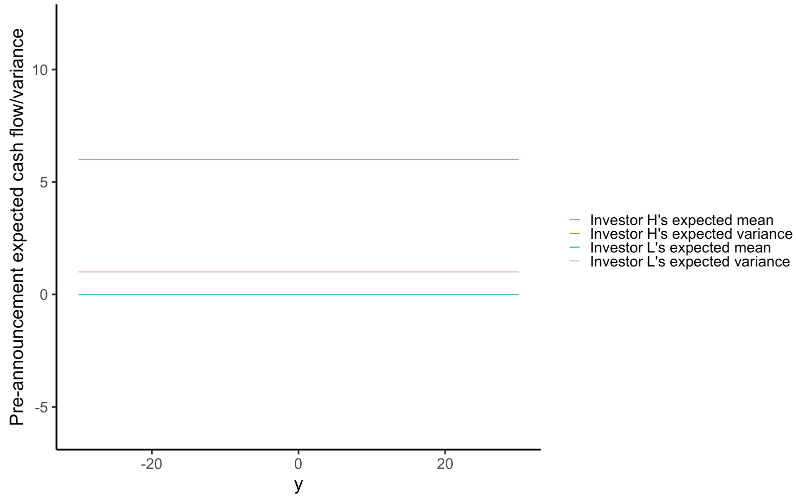
where investor i's expectation and variance of P_2^*, $E_i[P_2^*]$ and $Var_i[P_2^*]$, are of the following form:

$$E_i[P_2^*] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\frac{\nu+n}{\nu n}}} exp(-\frac{1}{2\frac{\nu+n}{\nu n}}(y - m_i)^2)P_2^* dy \tag{A43}$$

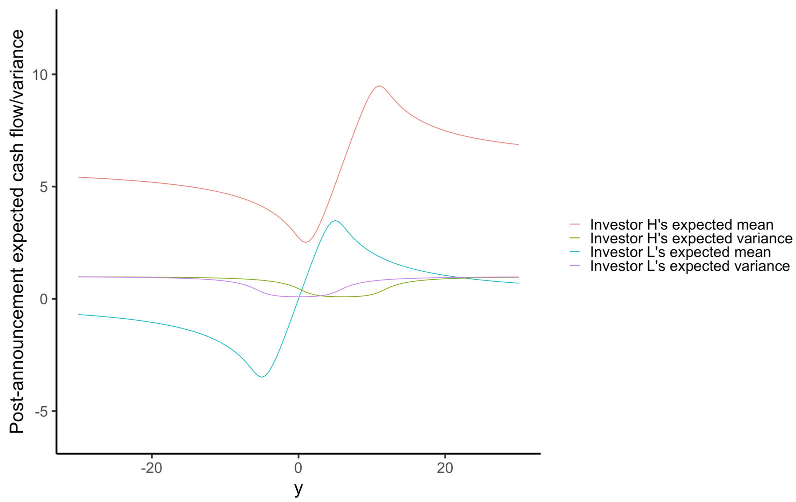
$$Var_i[P_2^*] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\frac{\nu+n}{\nu n}}} exp(-\frac{1}{2\frac{\nu+n}{\nu n}}(y - m_i)^2)[P_2^*]^2 dy - [E_i[P_2^*]]^2 \tag{A44}$$

The investor i's demand in equilibrium:

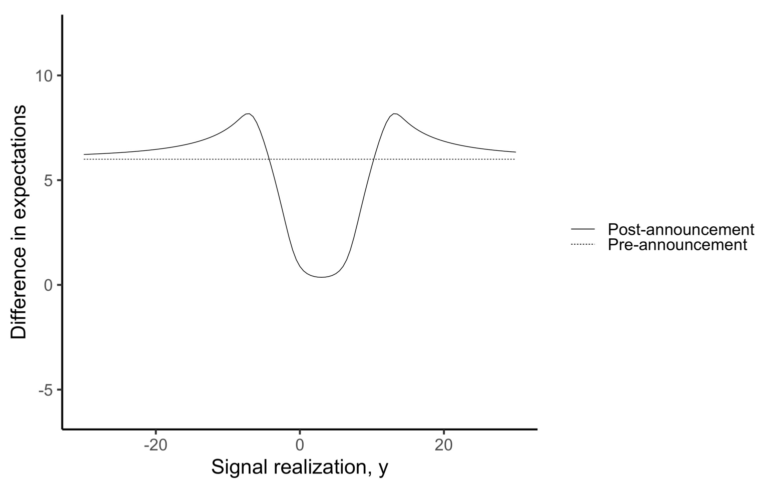
$$d_{i1}^{*} = \frac{E_i[P_2^{*}] - P_1^{*}}{r_i Var_i[P_2^{*}]} \tag{A45}$$



(a) Investors' expectations of firm cash flow and its variance before y is realized as a function of y.



(b) Investors' expectations of firm cash flow and its variance after y is realized as a function of y.



(c) Differences in investors' expectations of firm cash flow and its variance before and after y is realized as a function of y.

Figure 1: $\lambda_H = \lambda_L = 0.5$, $r_H = r_L = 4$, $\alpha = 10$, $m_H = 6$, $m_L = 0$, $\nu = 1$, $n = 1$.

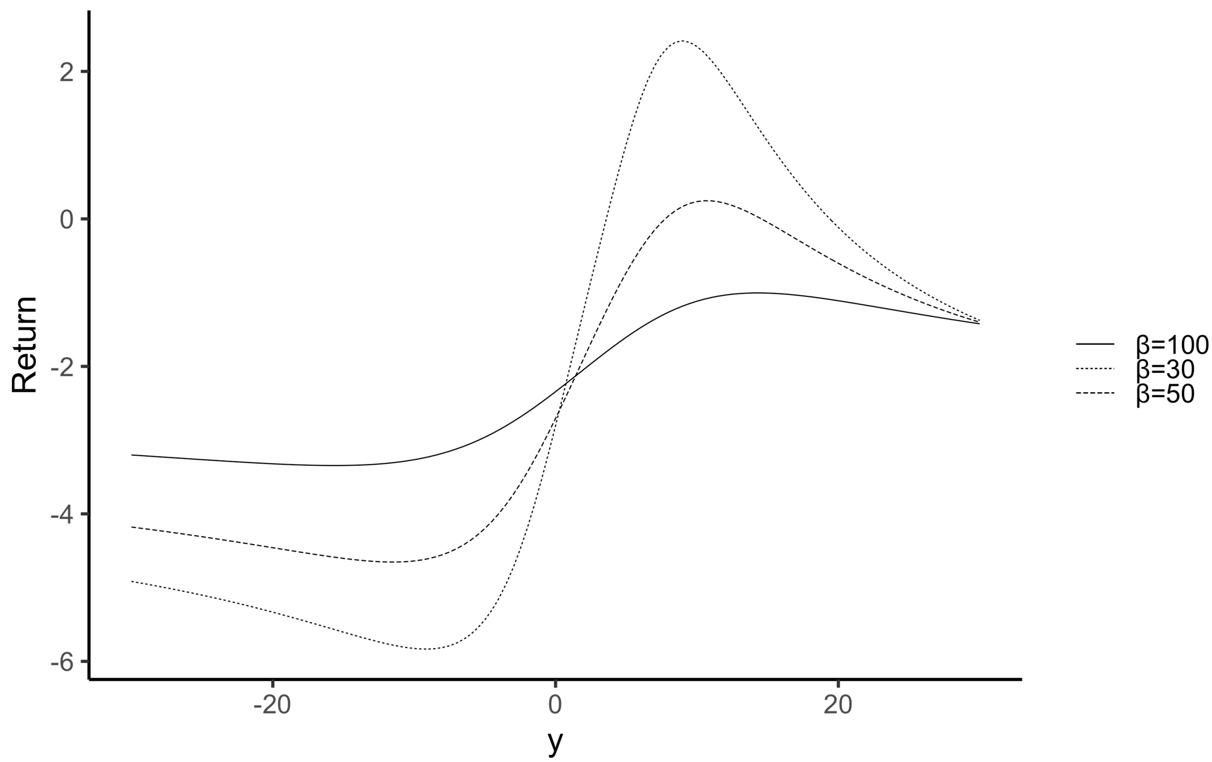
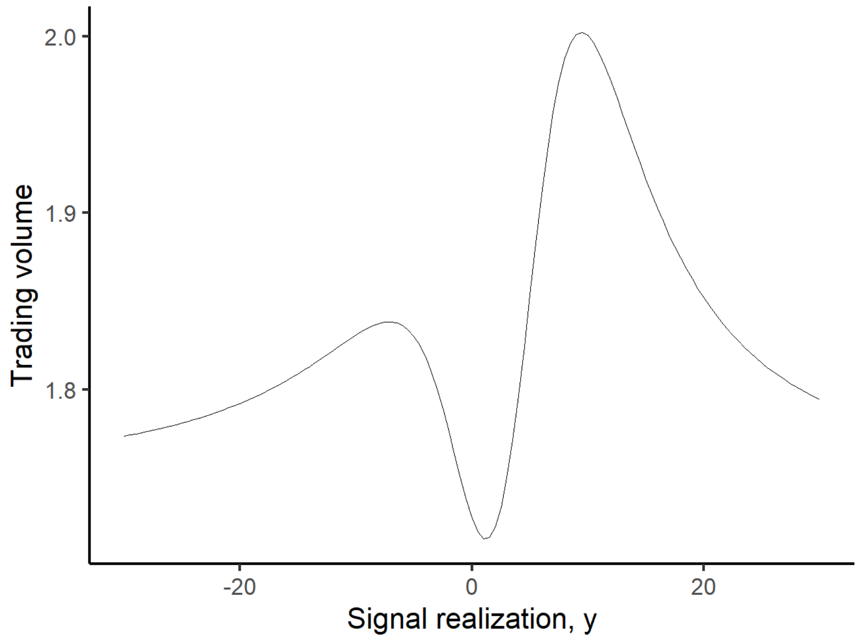
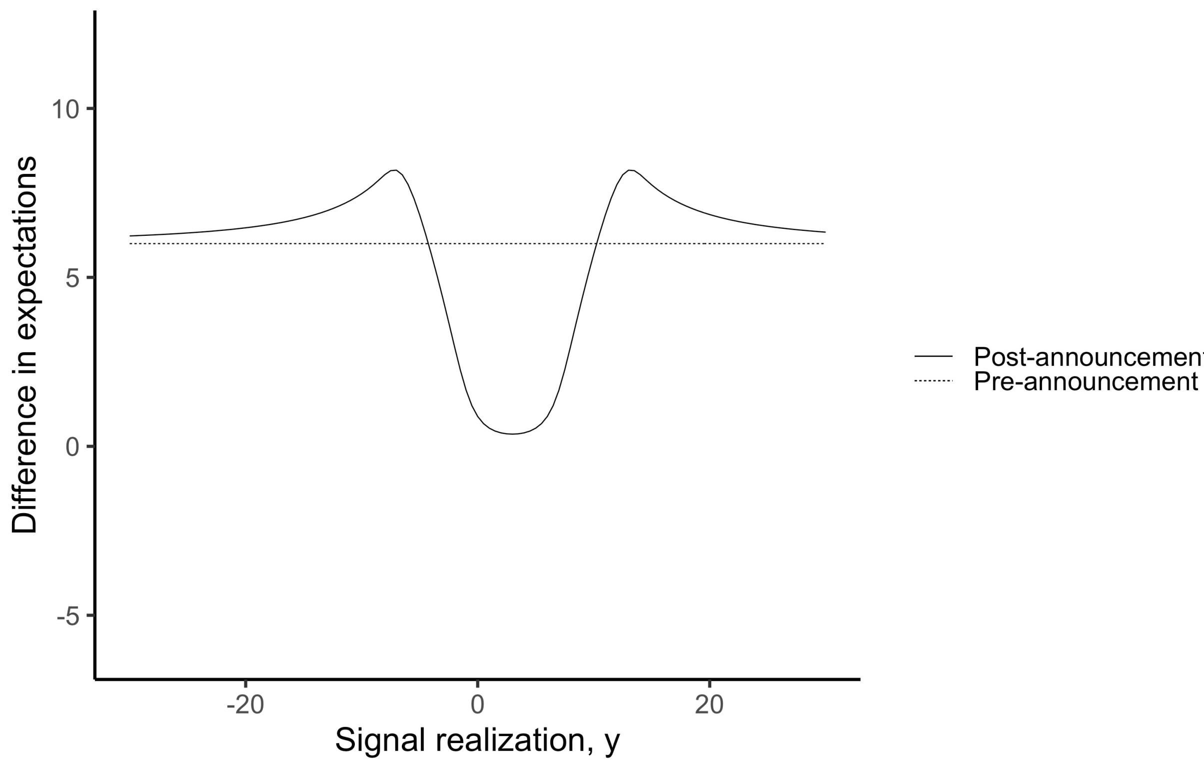


Figure 2: Return, $\frac{P_2^* - P_1^*}{P_1^*}$ as a function of y, for different levels of β.
$\lambda_1 = \lambda_2 = 0.5$, $r_1 = r_2 = 4$, $\alpha = 10$, $m_1 = 6$, $m_2 = 0$, $\nu = 1$, $n = 1$.

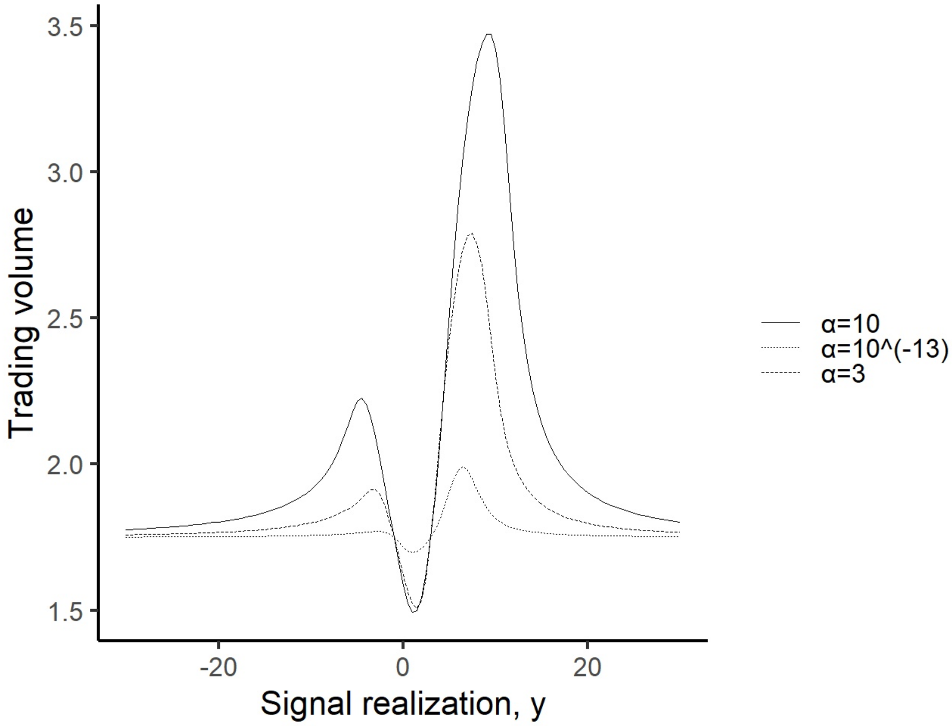


(a) A type-1 investor's trading volume as a function of the public signal, y.

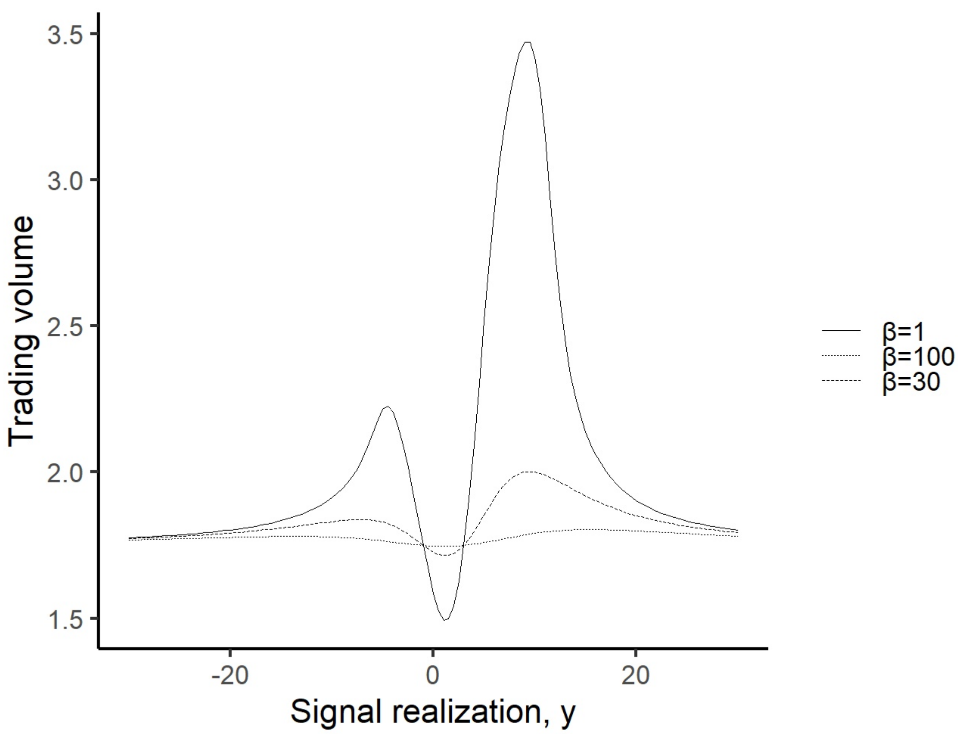


(b) Difference in investors' expectations of the firm's cash flow before and after disclosure as functions of y. $\lambda_1 = \lambda_2 = 0.5$, $r_1 = r_2 = 4$, $\alpha = 10$, $m_1 = 6$, $m_2 = 0$, $\nu = 1$, $n = 1$.

Figure 3



(a) $\beta = 1$.



(b) $\alpha = 10$.

Figure 4: A type-1 investor's trading volume as a function of the public signal, y. $\lambda_1 = \lambda_2 = 0.5$, $r_1 = r_2 = 4$, $m_1 = 6$, $m_2 = 0$, $\nu = 1$, $n = 1$.

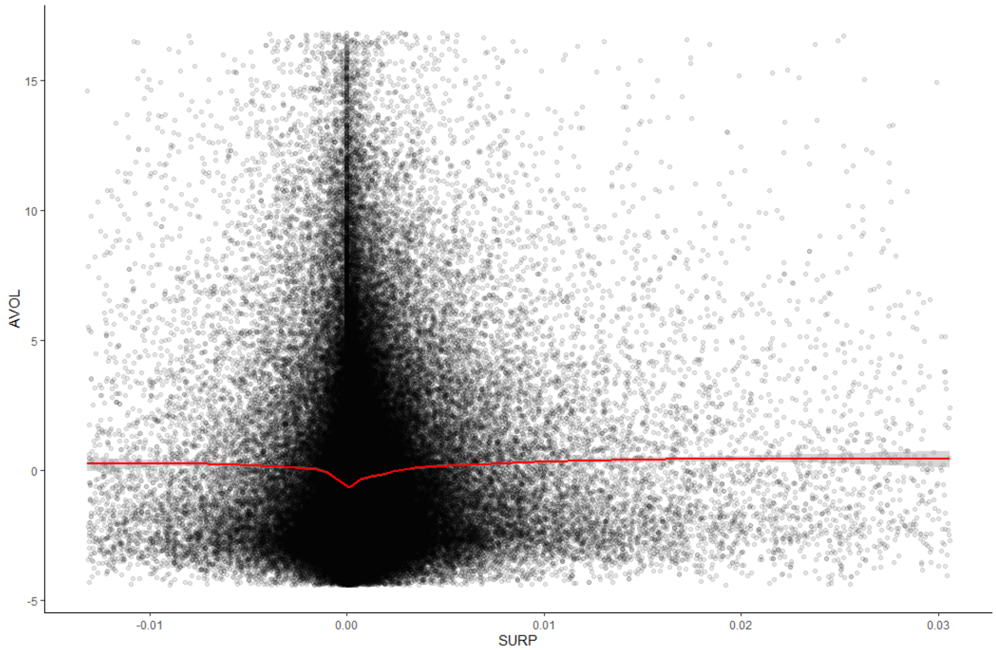


Figure 5: Scatterplot of residuals of abnormal trading volume with LOESS smoother

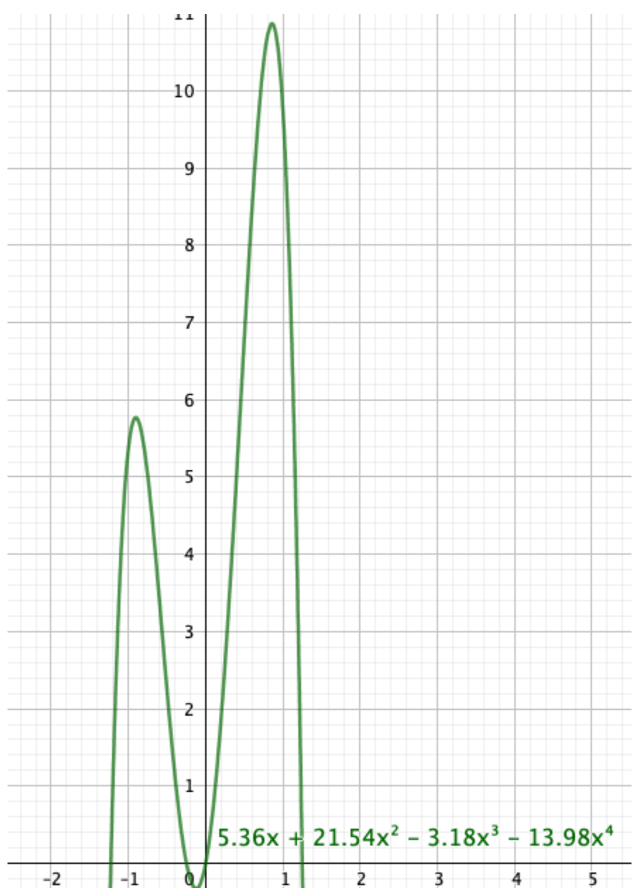


Figure 6: Functional form of trading volume with the estimated coefficients

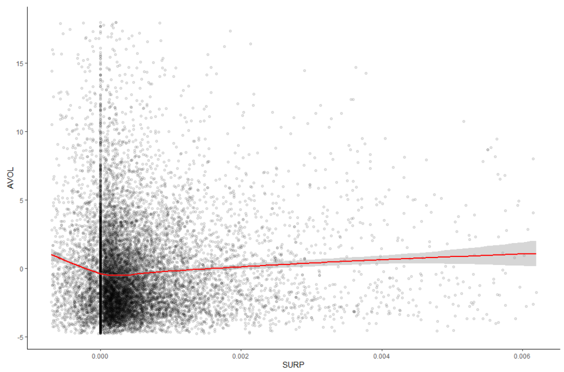


Figure 7: First quartile of $PrecUnc$

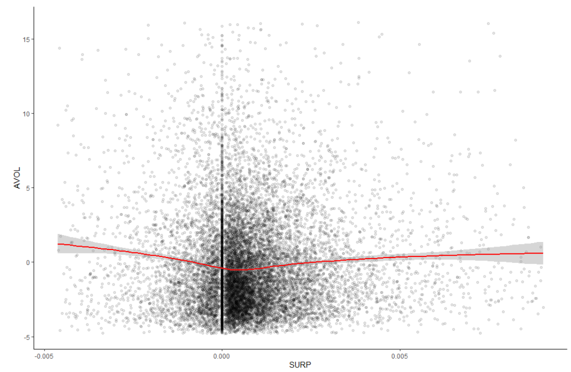


Figure 8: Second quartile of $PrecUnc$

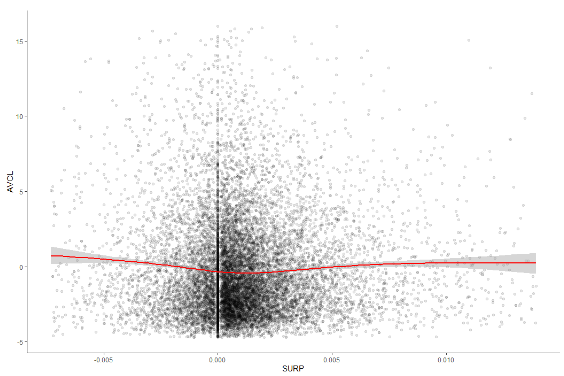


Figure 9: Third quartile of $PrecUnc$

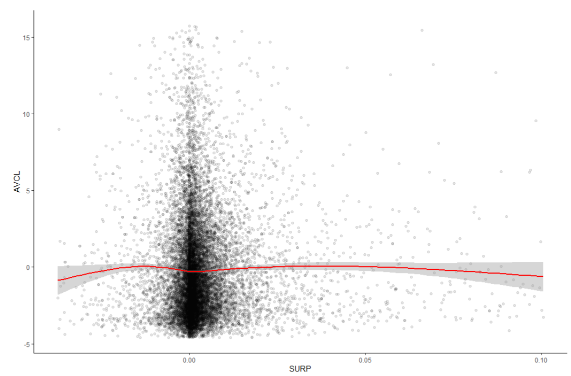


Figure 10: Fourth quartile of $PrecUnc$

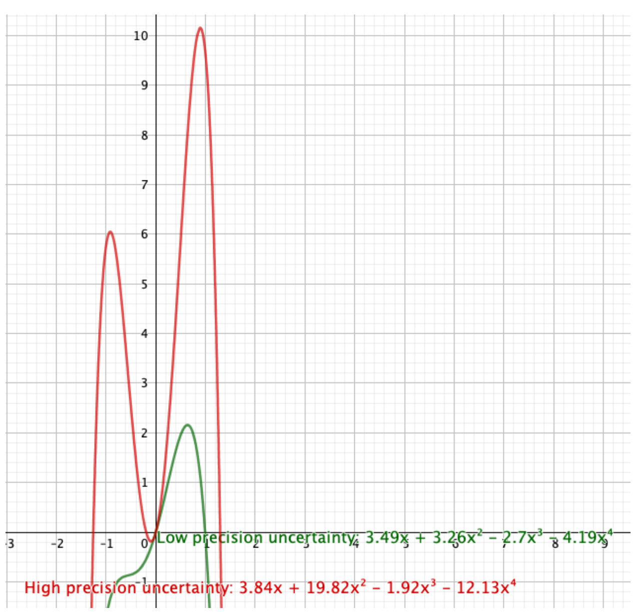


Figure 11: Functional form of trading volume for low and high precision uncertainty subsamples

Table 1: Sample selection procedure

Sample reduction reason	**Sample size**
Initial sample, price more than $5.00 and non-zero EPS	189,248
Firms that are in I/B/E/S, CRSP and Compustat	177,981
Firms with enough data to compute analyst dispersion	156,002
Firms with non-missing data on common/ordinary equity	153,017
Winsorize earnings surprise at 5% level and all the other variables at 1% level	87,944

Table 2: Summary statistics

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Abnormal Trading Volume (logs)	87,944	0.746	1.154	−3.274	0.082	1.571	3.072
Surp	87,944	0.001	0.003	−0.011	−0.0002	0.002	0.011
Control variables							
Size	87,944	7.537	1.455	4.230	6.476	8.488	11.564
Market-to-book ratio	87,944	3.162	2.627	0.645	1.592	3.690	21.285
Dispersion	87,944	0.038	0.059	0.0001	0.010	0.041	0.664
PS liquidity level	87,944	−0.059	0.106	−0.390	−0.112	0.018	0.103

Table 3: Polynomial regressions of abnormal trading volume on the earnings surprise

	Dependent variable:				
	AVOL				
	(1)	(2)	(3)	(4)	(5)
Surp	6.272***	5.238***	5.243***	5.361***	5.362***
	(1.460)	(1.146)	(1.145)	(1.145)	(1.145)
$Surp^2$		20.806***	20.832***	21.535***	21.538***
		(1.199)	(1.199)	(1.199)	(1.199)
$Surp^3$			−3.115***	−3.180***	−3.181***
			(1.144)	(1.143)	(1.144)
$Surp^4$				−13.977***	−13.979***
				(1.159)	(1.159)
$Surp^5$					0.418
					(1.143)
Size	0.057***	0.065***	0.065***	0.069***	0.069***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
Market-to-book	0.036***	0.039***	0.039***	0.040***	0.040***
	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)
Dispersion	0.015	−0.267***	−0.271***	−0.356***	−0.356***
	(0.067)	(0.069)	(0.069)	(0.069)	(0.069)
PS liquidity level	0.149***	0.158***	0.159***	0.161***	0.161***
	(0.036)	(0.036)	(0.036)	(0.036)	(0.036)
Constant	0.205***	0.151***	0.150***	0.123***	0.123***
	(0.021)	(0.021)	(0.021)	(0.021)	(0.021)
Observations	87,944	87,944	87,944	87,944	87,944
R^2	0.014	0.018	0.018	0.019	0.019
Adjusted R^2	0.014	0.018	0.018	0.019	0.019
Residual Std. Error	1.146 (df = 87938)	1.144 (df = 87937)	1.144 (df = 87936)	1.143 (df = 87935)	1.143 (df = 87934)
F Statistic	256.7*** (df = 5;87938)	264.8*** (df = 6;87937)	228.1*** (df = 7;87936)	218.1*** (df = 8;87935)	193.9*** (df = 9;87934)
Note:					*p<0.1; **p<0.05; ***p<0.01

Table 4: Analysis of variance: comparison of polynomial models

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1st order polynomial	87938	115465.35				
2nd order polynomial	87937	115071.29	1	394.06	301.65	0.0000***
3rd order polynomial	87936	115061.59	1	9.70	7.42	0.0064***
4th order polynomial	87935	114871.55	1	190.04	145.48	0.0000***
5th order polynomial	87934	114871.38	1	0.17	0.13	0.7149
Note:	*The analysis is performed by ANOVA statistical package*					

Table 5: Quantile regressions of abnormal trading volume for different levels of earnings surprise

	Dependent variable:	
	AVOL	
	Sample without upper 5%	Upper 5%
Surp (absolute)	65.176***	10.970
	(2.848)	(15.309)
Size	0.074***	0.045***
	(0.003)	(0.013)
Market-to-book	0.043***	0.027***
	(0.002)	(0.009)
Dispersion	−0.407***	−0.674***
	(0.075)	(0.194)
PS liquidity level	0.144***	0.428***
	(0.037)	(0.155)
Constant	−0.028	0.481***
	(0.023)	(0.159)
Observations	83,546	4,398
R^2	0.022	0.008
Adjusted R^2	0.022	0.007
Residual Std. Error	1.141 (df = 83540)	1.153 (df = 4392)
F Statistic	372.653*** (df = 5; 83540)	6.852*** (df = 5; 4392)
Note:		*p<0.1; **p<0.05; ***p<0.01

Table 6: Test of the functional form of cumulative abnormal returns for different quartiles of earnings-precision uncertainty: positive levels of earnings surprise

	Dependent variable:			
	CAR			
	1^{st} quartile of $PrecUnc$	2^{nd} quartile of $PrecUnc$	3^{rd} quartile of $PrecUnc$	4^{th} quartile of $PrecUnc$
$Surp$	$209,332^{***}$	$561,421^{***}$	$665,497^{***}$	$509,931^{***}$
	(76,018)	(76,042)	(75,274)	(74,957)
$Surp^2$	$-214,420^{***}$	$-288,399^{***}$	$-358,608^{***}$	$-194,470^{***}$
	(76,018)	(76,042)	(75,274)	(74,957)
Constant	$110,508^{***}$	$109,518^{***}$	$108,430^{***}$	$104,531^{***}$
	(897)	(893)	(890)	(890)
Observations	7,187	7,248	7,157	7,090
R^2	0.002	0.009	0.014	0.007
Adjusted R^2	0.002	0.009	0.014	0.007
Residual Std. Error	76,018 (df = 7184)	76,042 (df = 7245)	75,274 (df = 7154)	74,957 (df = 7087)
F Statistic	7.769^{***} (df = 2; 7184)	34.447^{***} (df = 2; 7245)	50.430^{***} (df = 2; 7154)	26.505^{***} (df = 2; 7087)
Note:				$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 7: Summary statistics for low and high precision uncertainty subsamples

Statistic	N		Mean		St. Dev.		Pctl(25)		Pctl(75)	
	LPU	HPU	LPU	HPU	LPU	HPU	LPU	HPU	LPU	HPU
ln(AVOL)	9,931	78,013	0.627	0.761	1.188	1.149	−0.073	0.102	1.467	1.583
Surp	9,931	78,013	0.0003	0.001	0.001	0.003	0.00004	−0.0003	0.0004	0.002
Size	9,931	78,013	7.501	7.541	1.417	1.459	6.491	6.472	8.421	8.497
Market-to-book ratio	9,931	78,013	4.481	2.995	3.301	2.478	2.264	1.542	5.529	3.482
Dispersion	9,931	78,013	0.005	0.042	0.004	0.061	0.003	0.012	0.006	0.046
PS liquidity level	9,931	78,013	−0.080	−0.056	0.116	0.105	−0.158	−0.098	0.010	0.018
Precision uncertainty	9,931	78,013	0.00001	0.0002	0.00001	0.0005	0.00001	0.0001	0.00002	0.0003

Note: LPU and HPU denote low and high precision uncertainty subsamples, respectively

Table 8: Polynomial regression for different signal-precision uncertainty subsamples

	Dependent variable:	
	AVOL	
	Low signal precision uncertainty	High signal precision uncertainty
Surp	3.486***	3.843***
	(1.182)	(1.138)
$Surp^2$	3.264***	19.822***
	(1.185)	(1.190)
$Surp^3$	−2.695**	−1.918*
	(1.180)	(1.138)
$Surp^4$	−4.191***	−12.125***
	(1.182)	(1.151)
Size	0.076***	0.067***
	(0.009)	(0.003)
Market-to-book	0.034***	0.046***
	(0.004)	(0.002)
Dispersion	−0.264	−0.496***
	(2.915)	(0.070)
PS liquidity level	−0.263***	0.196***
	(0.102)	(0.039)
Constant	−0.113*	0.152***
	(0.065)	(0.022)
Observations	9,931	78,013
R^2	0.023	0.020
Adjusted R^2	0.022	0.020
Residual Std. Error	1.175 (df = 9922)	1.137 (df = 78004)
F Statistic	29.523*** (df = 8; 9922)	199.781*** (df = 8; 78004)
Note:		*p<0.1; **p<0.05; ***p<0.01